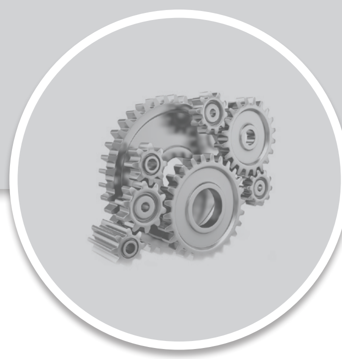
MECHANICAL ENGINEERING

Theory of Machines



Comprehensive Theory
with Solved Examples *and* Practice Questions

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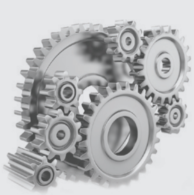

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EDITIONS

First Edition : 2015
Second Edition : 2016
Third Edition : 2017
Fourth Edition : 2018
Fifth Edition : 2019
Sixth Edition : 2020
Seventh Edition : 2021
Eighth Edition : 2022
Ninth Edition : 2023

Tenth Edition : 2024

CONTENTS

Theory of Machines

CHAPTER 5

CHAPTER 6

CHAPTER 7

CHAPTER 8

CHAPTER 9

Governors 303-345

CHAPTER 10

Mechanical Vibrations 346-422

CHAPTER 11

Gyroscope and Gyroscopic Effects 423-447

CHAPTER

2

Kinematic Analysis of Plane Mechanisms

2.1 INTRODUCTION

Analysis of mechanisms is the study of motions and forces concerning their different parts. The study of velocity analysis involves linear velocities of various points on different links of a mechanism as well as the angular velocities of the links. Velocity analysis involves determining “how fast” certain points on the links of a mechanism are travelling. Velocity is important because it associates the movement of a point on a mechanism with time. Often the timing in a mechanism analysis is critical.

Acceleration analysis involves determining the manner in which certain points on the links of a mechanism are either “speeding up” or “slowing down”. It is a critical property because of the inertial forces associated with it. An important part of mechanisms design is to ensure that the strength of the links and joints is sufficient to withstand the forces imposed on them. Acceleration analysis of a mechanism’s link must be performed to analyse the forces.

The velocity analysis is a perquisite for acceleration analysis which further leads to force analysis of various links of a mechanism.

2.2 DEFINITION OF VELOCITY AND ACCELERATION

2.2.1 Definition of Velocity

Velocity is defined as the rate of change of position with respect to time. Position (R) is a vector quantity and so is velocity. Velocity can be angular or linear. Angular velocity will be denoted as ω and linear velocity as V.

$$\omega = \frac{d\theta}{dt}\ ; \qquad V = \frac{dR}{dt} \quad ...(2.1)$$

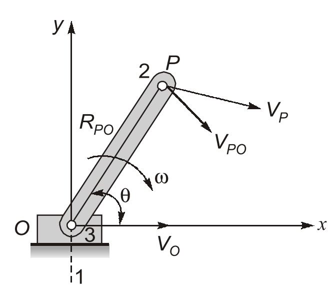


Fig. A link in pure rotation

Fig. shows a link OP in pure rotation, pivoted at point O in the $x\ y$ plane The position vector R_{PO} represented as a complex number in polar form,

$$R_{PO} = pe^{j\theta} \quad ...(2.2)$$

$$V_{PO} = \frac{dR_{PO}}{dt} = pj\,e^{j\theta}\frac{d\theta}{dt} = P\omega je^{j\theta} \quad ...(2.3)$$

We can see from Equations (2.2) and (2.3), that as a result of the differentiation, the velocity expression has been multiplied by the (constant) complex operator *j*. This causes a rotation of this velocity vector through 90° with respect to the original position vector R_{PO}. V_{PO} in given Fig. can be referred to as an absolute velocity since it is referenced to *O*, which is the origin of the global coordinate axes in that system. Fig. shows a different and slightly more complicated system in which *O* is no longer stationary.

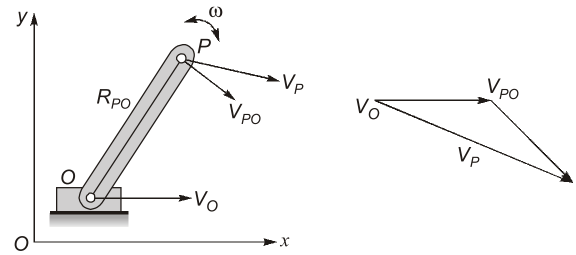


Figure: Velocity difference

It has a known linear velocity V_O as part of the translating carriage. In this case, V_{PO} can no longer be considered as an absolute velocity. It is now a velocity difference.

$$V_{PO} = V_P - V_O \quad ...(2.4)$$

or

$$V_P = V_O + V_{PO} \quad ...(2.5)$$

Equation (2.5) could also be written as :

Velocity : translation component + rotation component

2.2.2 Definition of Acceleration

Acceleration is defined as the rate of change of velocity with respect to time. Velocity (V, ω) is a vector quantity and so is the acceleration. Acceleration can be angular or linear. Angular acceleration will be denoted as α and linear acceleration as *A* or *a*.

$$\alpha = \frac{d\omega}{dt}; \qquad a \text{ or } A = \frac{dV}{dt} \quad ...(2.6)$$

$$R_{PO} = pe^{j\theta} \quad ...(2.7)$$

$$V_{PO} = \frac{dR_{PO}}{dt} = pje^{j\theta}\frac{d\theta}{dt} = p\omega je^{j\theta} \quad ...(2.8)$$

where *p* is scalar length of R_{PO}.

$$a_{PO} = \frac{dV_{PO}}{dt} = \frac{d}{dt}\left(p\omega je^{j\theta}\right) \quad ...(2.9)$$

$$a_{PO} = jp\left(e^{j\theta}\frac{d\omega}{dt} + \omega je^{j\theta}\frac{d\theta}{dt}\right)$$

$$a_{PO} = p\alpha je^{j\theta} - p\omega^2 e^{j\theta} \quad ...(2.10)$$

$$a_{PO} = a_{PO}^t + a_{PO}^n$$

Here a_{PO}^t is tangential component of acceleration

a_{PO}^n is normal or centripetal component of acceleration.

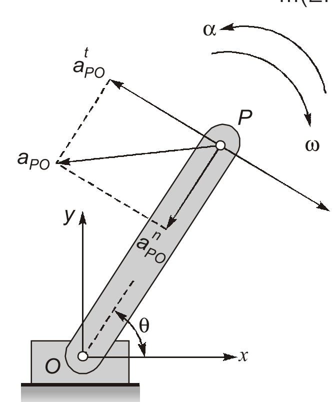


Fig. Acceleration of a link in pure rotation

From Equation (2.10), we can see that tangential acceleration is always in a direction perpendicular to the radius of rotation and is thus tangent to the path of motion as shown in given Fig. The normal or centripetal acceleration component is multiplied by j^2 or – 1. This directs the centripetal component at 180° to the angle θ of the original position vector i.e. towards the centre (Centripetal means toward the center). The acceleration a_{PO} in Fig. can be referred as an absolute acceleration since it is referred to *O*, which is the origin of the global co-ordinate system.

By equating, $100\,\omega_{BC} - 2400 = -500\,\omega_{DE}$

$$1200 + 800\,\omega_{BC} = -300\,\omega_{DE}$$

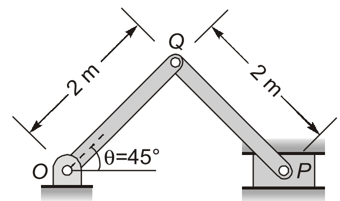
By solving above equation, we get $\omega_{DE} = 5.51$ rad/s, $\omega_{BC} = -3.57$ rad/s

$$v_C = -1330\,i - 941\,j \text{ (mm/s)}$$

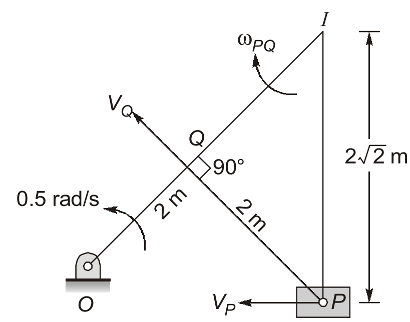
$$|v_C| = \sqrt{1330^2 + 941^2} = 1629.23 \text{ mm/s} = 1.63 \text{ m/s}$$

EXAMPLE : 2.13

The angle $\theta = 45°$, and the bar OQ as shown in given figure is rotating in the anti-clockwise direction at 0.5 rad/s. The velocity of the sleeve P is _____ m/sec. (Correct upto two decimal places)



Solution : 1.41 (1.20 to 1.80)



$$v_Q = \omega_{OQ} \times OQ$$
$$= 0.5 \times 2 = 1 \text{ m/s}$$

$$\omega_{PQ} = \frac{v_Q}{IQ}$$
$$= \frac{1}{2} = 0.5 \text{ rad/s} \quad \text{(CW)}$$

$$v_P = \omega_{PQ} \times IP = 2\sqrt{2} \times 0.5$$
$$= \sqrt{2} \text{ m/s} = 1.414 \text{ m/s}$$

■■■■

OBJECTIVE BRAIN TEASERS

1. Degree of freedom of a rigid body imply the
   (a) angles that it may turn through
   (b) total number of modes of displacement
   (c) constraints to its motion
   (d) angular motions the body can have

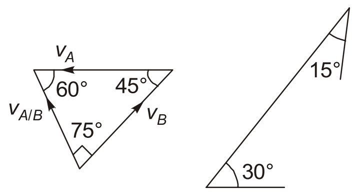
2. A rigid body in translation
   (a) must undergo plane motion only
   (b) cannot move on a circular path
   (c) may move along a straight or curved path
   (d) can only move in straight line

3. The instantaneous centre of rotation
   (a) must exists for any plane motion
   (b) can exist for any space motion
   (c) is a hypothetical concept
   (d) should also be the instantaneous centre of acceleration

4. For the motion of a rigid link in any mechanism,
   (a) the two ends of the link may have different component of velocity along the link
   (b) the acceleration of one end with respect to that of the other should be perpendicular to the link
   (c) the velocity of one with respect to that of the other should be perpendicular to the link
   (d) the velocity of one end should be zero

5. What is the sense of coriolis component as compared to the relative velocity vector?
   (a) 90° in the direction of rotation of the link containing the path

23. (d)



$$\frac{4}{\sin 75°} = \frac{V_{A/B}}{\sin 45°}$$

or $V_{A/B} = 2.93$ m/s

$V_{A/B} = 2.5\,\omega = 2.93$

$\Rightarrow$ $\omega = 1.17$ rad/s

24. (b)

I is the instantaneous centre

$$\frac{BI}{\sin 60°} = \frac{250}{\sin 75°}$$

$\Rightarrow$ $BI = 224.143$ mm

$IC = BI - BC = 224.143 - 150$

$= 74.143$ mm

$$\frac{AI}{\sin 45°} = \frac{250}{\sin 75°}$$

$\Rightarrow$ $AI = 183.012$ mm

$ID = 200 - 183 = 16.987$ mm

$$\frac{V_C}{IC} = \frac{V_D}{ID}$$

or $$V_D = V_C \frac{ID}{IC} = 1.5 \times \frac{16.987}{74.143}$$

$= 0.343$ m/s

25. (c)

$r_{A/O} = i$

$a_O = O$

$\omega = 20\,k$ rad/s

$\alpha = 6\,k$ rad/s^2

$a_A = a_0 + a \times r_{A/O} - \omega^2\, r_{A/O}$

$a_A = O + 6\,k \times i - (20)^2 \times i$

$a_A = -400\,i + 6\,j$ (m/s^2)

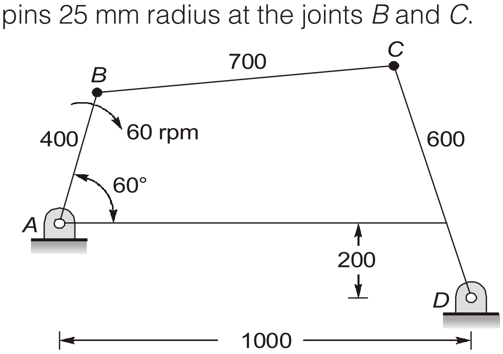
$|a_A| = \sqrt{400^2 + 6^2}$

$= 400.045$ m/s^2

■■■■

CONVENTIONAL BRAIN TEASERS

Q.1 The crank AB of a four-bar mechanism as shown in figure rotates at 60 rpm clockwise. Determine the relative angular velocities of the coupler to the crank and the lever to the coupler. Find also the rubbing velocities at the surface of pins 25 mm radius at the joints B and C.



(All dimensions in mm)

Solution :

$$\omega_2 = \frac{2\pi N}{60} = \frac{2\pi(60)}{60} = 6.28 \text{ rad/s}$$

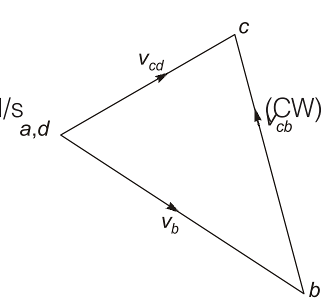
$v_b = \omega_2 \times AB = 6.28 \times 400$

$= 2513$ mm/s

Draw the velocity diagram as shown,

Here, $v_b = |ab| \perp AB$

$|bc| \perp BC$



Scale 10 mm = 500 mm/s
Velocity diagram

CHAPTER

11

Gyroscope and Gyroscopic Effects

11.1 INTRODUCTION

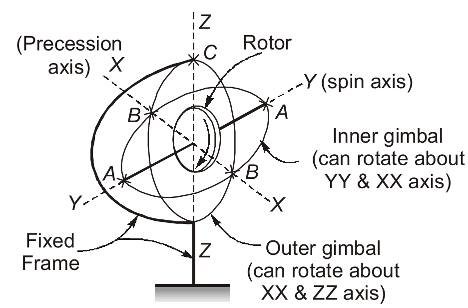


Fig. Gyroscope

A gyroscope is defined as a rigid body capable of three-dimensional rotation with high angular velocity about any axis that passes through a fixed point called the center, which may or may not be its centre of mass. A child's toy top fits this definition and is a form of gyroscope; its fixed point is the point of contact of the top with the floor or table on which it spins.

AA – Pin joint between rotor and inner gimbal rotor is rotating about *YY* axis

BB – Pin joint between inner gimbal and outer gimbal

CC – Pin joint between outer gimbal and fixed frame

The usual form of a gyroscope is a mechanical device in which the essential part is a rotor having a heavy rim spinning at a high speed about a fixed point on the rotor axis. The rotor is then mounted so as to turn freely about its centre of mass by means of double gimbals called a Cardan suspension.

The gyroscope has fascinated students of mechanics and applied mathematics for many years. Indeed, once the rotor is set spinning, a gyroscope seems to act like a device possessing intelligence. If we attempt to move some of its parts, it seems not only to resist this motion but even to evade it. It seemingly fails to conform to the laws of static equilibrium and of gravitation.

REMEMBER **Meaning of precession :** The motion of spinning body (as a spinning top) in which it wobbles so that the axis of rotation sweeps out cone.

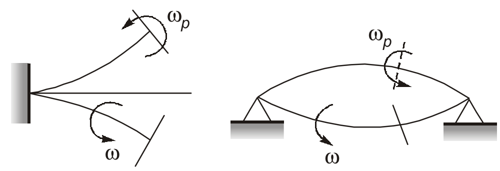
Application of Gyroscopes

(i) Used as turn and back indicators, artificial horizons and automatic pilots in aircraft and missiles.
(ii) Used in the gyro-compass
(iii) Serves as a stabilizer in naval ships and torpedoes
(iv) Gyroscopic effects are present when a motorcycle or bicycle is being rided.

(v) Used in inertial guidance control system for missiles and space travel.

(vi) Gyroscopic effects are encountered in the bearings of a jet engine shaft as the airplane changes direction.

(vii) For high speed rotors, precession (see figure below) becomes more and more predominant and should be accounted in the machine design calculations.



(a) Cantilever **(b) Simply supported**

Fig. Precessional motion of the rotor disk

Sometimes these effects are desirable, but more often they are undesirable and designers must account for them in their selection of bearings and rotating parts. It is certainly true that as machines speed increase to higher and higher values and as factors of safety decrease, we must stop neglecting gyroscopic forces in our machine design because their values will be more significant. The general equations for the motion of a gyroscope are, indeed, not simple. Fortunately, in designing machines, only a few simple and approximate solutions are necessary.

To provide a vehicle for the explanation of the simpler motions of a gyroscope is expedient to perform a series of experiments with the one of figure. In the following, we assume that the rotor is spinning and that the pivot friction can be neglected.

1. If the z-axis is kept in the vertical position, the gyroscope can be moved anywhere about a table or a room without altering the direction of the axis of spin. This is a consequence of the law of conservation of moment of momentum. If the axis of spin is to change its direction, the moment-of-momentum vector must also change its direction, but this requires an external torque, which in this experiment we have not supplied. While the rotor is still spinning, we might lift the inner gimbal out of its bearings and move it about. We then find that it can be translated anywhere but that we meet with definite resistance when we attempt to rotate the axis of spin.
2. With the inner gimbal back in the bearings, suppose that pressure is applied, say by a pencil, to the inner gimbal to make it turn about the x axis. Not only we meet with resistance to the pressure of the pencil but the outer gimbal begins to rotate slowly about the vertical z-axis and it continues this rotation until the pressure is released. The pressure of the pencil constitutes a torque on the inner gimbal with the parallel and opposite force of the couple coming from the pivots in the gimbal. In order to study these effects carefully we might cause the rotor to spin in the positive direction, i.e., with the angular-velocity vector pointing in the positive y direction. Then if we apply a positive torque to the inner gimbal (torque vector pointing in the positive x direction), the rotation of the outer gimbal is found to be in the negative z-direction. You should note that these effects occur in a right-hand coordinate system. Either a negative spin velocity or a negative torque will cause the gimbal to rotate in the positive z-direction for the set of axes shown. The rotation of the spin axis about an axis perpendicular to that of a torque applied to it is called precession, and so the application of a torque to the spinning rotor causes it to precess. In this example, the z-axis is called the axis of precession.
3. As a third experiment we might apply a torque to the outer gimbal in an attempt to cause it to rotate about the z-axis. Such an attempt meets with resistance and causes the inner gimbal with the spin axis to rotate. When the spin axis is in the vertical position, the gyroscope is in stable equilibrium though, and the outer gimbal can then be turned quite freely. Note in this as well as in the previous example that the moment-of-momentum vector is changing its direction because of an application of external torque.

OBJECTIVE BRAIN TEASERS

1. The magnitude of the gyroscopic couple applied to a disc of moment of inertia 5 kg. m^2, spinning with an angular velocity 120 rad/s and having an angular velocity of precession $\frac{1}{30}$rad/s is:
(a) 2 N.m (b) 200 N.m
(c) 0.2 N.m (d) 20 N.m

2. The axis of spin, the axis of precession and the axis of gyroscopic torque are in
(a) two parallel planes
(b) two perpendicular planes
(c) three perpendicular planes
(d) three parallel planes

3. What is meant by pitching of naval ship?
(a) up and down motion of bow and stern about transverse axis
(b) up and down motion of bow and stern about longitudinal axis
(c) up and down motion of port and starboard along longitudinal axis
(d) angular motion along the sea surface

4. What is the reactive gyroscopic couple acting on rotating disc which is inclined at an angle of 1.2? Mass moment of inertia about polar and diametral axis is 2.45 kg.m^2 and 0.5 kg. m^2 respectively. The disc rotates at an angular velocity of 130 rad/s.
(a) 776 N.m (b) 345 N.m
(c) 863 N.m (d) 690 N.m

5. Gyroscopic effect is NOT observed in which of the following actions performed by the ships?
(a) Rolling (b) Pitching
(c) Steering (d) All of the above

6. Degree of freedom for gyroscope rotor is
(a) 1 (b) 2
(c) 3 (d) 5

7. What is the effect of reactive gyroscopic couple when aeroplane takes right turn and propeller rotates in clockwise direction?
(a) The tail of the aeroplane is dipped and nose is raised.
(b) The tail of the aeroplane is raised and nose is dipped
(c) Reactive gyroscopic couple has no effect when propeller rotates in clockwise direction.
(d) Active gyroscopic couple is always correction couple to stabilise the rotor

8. The gyroscopic acceleration of a disc rotating at speed ω and uniform a acceleration is:
(a) $\frac{d\omega}{dt}$ (b) $w\frac{d\theta}{dt}$
(c) $\omega^2 r$ (d) $\theta\frac{d\omega}{dt}$

9. When a ship travels in sea, which of the following effects is more dangerous?
(a) steering (b) pitching
(c) rolling (d) all of the above

10. The total reaction of ground on wheels of a vehicle due to gyroscopic couple and centrifugal force while negotiating curve is:
(a) increased on inner wheels and decreased on outer wheels.
(b) decreased on inner wheels and increased on outer wheels.
(c) decreased on all the wheels
(d) increased on all the wheels.

■■■■

ANSWER KEY

1. (d)	2. (c)	3. (a)	4. (d)	5. (a)
6. (c)	7. (b)	8. (b)	9. (b)	10. (b)

HINTS & EXPLANATIONS

1. (d)

$$I = 5 \text{ kg.m}^2, \omega = 120 \text{ rad/s}$$

$$\omega_p = \frac{1}{30} \text{rad/s}$$

$$C = I\omega\omega_p = 5 \times 120 \times \frac{1}{30}$$

$$= 20 \text{ N.m}$$

4. (d)

Resultant gyroscopic couple on the disc

$$= \frac{1}{2}(I_p - I_d)\omega^2 \sin 2\theta$$

Polar MOI $I_P = 2.45 \text{ kg.m}^2$

Diametral MOI, $I_d = 0.5 \text{ kg.m}^2$

$$C = \frac{1}{2}(2.45 - 0.5) \times (130)^2 \times \sin 2.4° = 690 \text{ N.m}$$

■■■■

CONVENTIONAL BRAIN TEASERS

Q.1 The mass of a turbine rotor of a ship is 8000 kg and has a radius of gyration of 0.75 m. It rotates at 1800 rpm clockwise when viewed from the stern. Determine the gyroscopic effects in the following cases.

(a) If the ship travelling at 100 km/h steers to the left along a curve of 80 m radius.

(b) If the ship is pitching and the bow is descending with maximum velocity. The pitching is with simple harmonic motion with periodic time of 20 s and the total angular movement between extreme position is 10°.

(c) If the ship is rolling with an angular velocity of 0.03 rad/s clockwise when looking from stern. In each case, determine the direction in which the ship tends to move.

Solution :

(a) Given: $R = 80$ m, v = 100 km/h, $m = 8000$ kg, $K = 0.75$ m, $N = 1800$ rpm, $I = mK^2 = 8000 \times (0.75)^2 = 4500 \text{ kgm}^2$

$$\omega = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/s}$$

$$\omega_p = \frac{v}{r} = \frac{100}{3600 \times 60} = 0.347 \text{ rad/s}$$

$$C = I\omega\omega_p = 4500 \times 188.5 \times 0.347 = 294531 \text{ Nm}$$

The ship tends to move to the left.

(b)

$$t_p = 20\text{s}, A = 10° \text{ or } \frac{\pi}{18}\text{rad}$$

$$(\omega_p)_{max} = \frac{2\pi A}{t_p} = \frac{2\pi \times \pi}{18 \times 20} = 0.05483 \text{ rad/s}$$

$$C_{max} = I\omega(\omega_p)_{max} = 4500 \times 188.5 \times 0.05483 = 46510 \text{ N m}$$

The ship tends to move up.

(c) In case of rolling, $C = 0$ as $\omega_p = 0$.

Q.2 A four-wheeled trolley car has a total mass of 4000 kg. Each axle with its two wheels and gears has a total moment of inertia of 36 kg.m^2. Each wheel is of 500 mm radius. The centre distance between two wheels on an axle is 1.4 m. Each axle is driven by a motor with a speed ratio of 1 : 3. Each motor along with its gear has a moment of inertia of 18 kg.m^2 and rotates in the opposite direction to that of the axle. The centre of mass of the car is 1m above the rails. Calculate the limiting speed of the car when it has to travel around a curve of 250-m radius without the wheels leaving the rails.

Solution :

$$I_w = \frac{36}{2} = 18 \text{ kg.m}^2, \text{ m} = 4000 \text{ kg}$$

$$r = 0.5 \text{ m}, h = 1 \text{ m}, I = 18 \text{ kg m}^2$$

$$W = 1.4 \text{ m}, R = 250 \text{ m}$$

(i) Reaction due to weight, $R_w = \frac{W}{4} = \frac{mg}{4} = \frac{4000 \times 9.81}{4} = 9810 \text{ N}(\uparrow)$

(ii) Reaction due to gyroscopic couple, $C_w = 4\, I_w \frac{V^2}{rR} = \frac{4 \times 18 \times V^2}{0.5 \times 250} = 0.576\, v^2$

$$C_m = 2\, I \text{ m G}\omega_w \omega_p = 0.864 \text{ v}^2 \qquad \text{(As there are two motors)}$$

$$C_m = \frac{2 \times 18 \times 3 \times V^2}{0.5 \times 250} = 0.864\, V^2$$

$$C_G = C_w - C_m \qquad \text{...(as motors rotate in opposite direction)}$$

$$= 0.576\, V^2 - 0.864\, V^2 = -0.288\, V^2$$

Reaction on each outer wheel, $R_{GO} = \frac{C_G}{2W} = \frac{0.288\, V^2}{2 \times 1.4} = 0.1029\, V^2(\downarrow)$

Reaction on each inner wheel, $R_{Gi} = 0.1029\, V^2\, (\uparrow)$

(iii) Reaction due to centrifugal couple, $C_C = \frac{mv^2}{R} h = \frac{4000 \times v^2}{250} \times 1 = 16\, v^2$

Reaction due to centrifugal couple on outer wheel,

$$R_{CO} = \frac{C_C}{2W} = \frac{16 v^2}{2 \times 1.4} = 5.7143 v^2 (\uparrow)$$

Reaction due to centrifugal couple on inner wheel,

$$R_{Ci} = 5.7143 \text{ v}^2\, (\downarrow)$$

Total reaction m outer wheel $= 9810 - 0.1029 \text{ v}^2 + 5.7143 \text{ v}^2 = 9810 + 5.6114 \text{ v}^2$

Total reaction m inner wheel $= 9810 - 0.1029 \text{ v}^2 - 5.7143 \text{ v}^2 = 9810 - 5.8172 \text{ v}^2$

Thus, the reaction on the outer wheel is always positive (upwards). There are chances that the inner wheels leave the rails.

For maximum speed, $9810 - 5.8172 \text{ v}^2 = 0$

$$V = \sqrt{\frac{9810}{5.8172}} = 41.0655 \text{ m/s} = \frac{41.0655 \times 3600}{1000} = 147.836 \text{ km/h}$$

Q.3 A 2.2-tonne racing car has a wheel base of 2.4 m and a track of 1.4 m. The centre of mass of the car lies at 0.6 m above the ground and 1.4 m from the rear axle. The equivalent mass of engine parts is 140 kg with a radius of gyration of 150 mm. The back axle ratio is 5. The engine shaft and flywheel rotate clockwise when viewed from the front. Each wheel has a diameter of 0.8 m and a moment of inertia of 0.7 kg.m^2. Determine the load distribution on the wheels when the car is rounding a curve of 100 m radius at a speed of 72 km/h to the (i) left, and (ii) right.