MECHANICAL ENGINEERING

Strength of Materials

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Strength of Materials

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CHAPTER

1

Properties of Materials

1.1 INTRODUCTION

Strength of material is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading and internal forces developed due to these loading. A thorough understanding of mechanical behaviour is essential for the safe design of all structures, whether buildings, bridges, machines, motors, submarines or airplanes. Hence, strength of material is a basic subject in many engineering fields.

The objective of our analysis will be to determine the stresses, strains and deflections produced by the loads in different structures. Theoretical analysis and experimental results have equally important role in the study of strength of materials. So these quantities are found for all values of load upto the failure load, and then we will have a complete picture of the mechanical behaviour of the body.

The behaviour of a member subjected to forces depends not only on the fundamental law of Newtonian mechanics that govern the equilibrium of the forces but also on the mechanical characteristics of materials of which the member is fabricated. Sometimes, to predict the behaviour of material some necessary information regarding the characteristics of material comes from laboratory tests.

1.2 STRESS

The fundamental concept of stress can be understood by considering a prismatic bar that is loaded by axial force P at the ends as shown.

A prismatic bar is a straight structural member having constant cross-sectional area throughout its length. In the figure (a), axial force is acting away from the cross-section producing a uniform stretching of the bar, hence the bar is said to be in tension. Similarly in figure (c), axial force is acting towards the cross-section producing uniform compression of the bar, hence the bar is said to be in compression. To investigate the internal stresses produced in the bar by axial forces, we make an imaginary cut at section *mn* as shown in figure (b) and (d). This section is taken perpendicular to the longitudinal axis of bar. Hence it is known as cross-section.

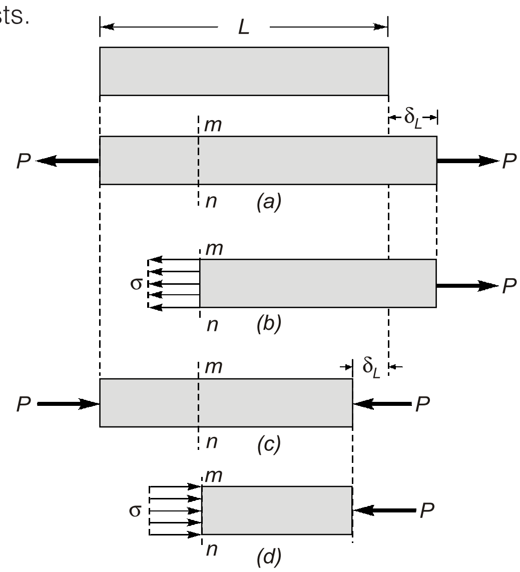


Fig. *Axial stress*

Now isolating the part of the bar to the right of the cut and considering the right of the cut as a free body. The force P has a tendency to move free body in the direction of load, so to restrict the motion of bar an internal force is induced which is uniformly distributed over cross-sectional area. The intensity of force developed, that is, internal force per unit area is called the **stress**.

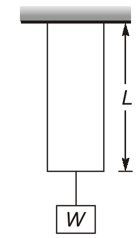
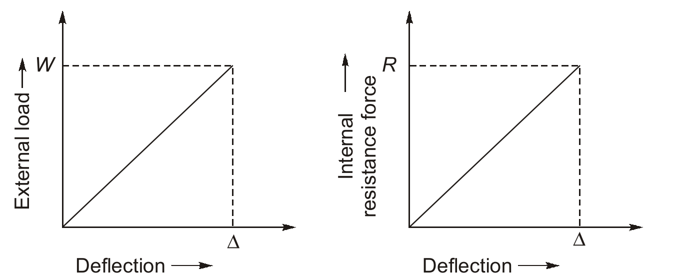


Fig. Axial load on bar

Stress differs from pressure because pressure is defined as the externally applied force on unit area while stress is internal resistive force on unit area. To have better understanding of difference between externally applied force and internal resistance. Consider a bar suspended from a fixed end and a weight *W* is gradually applied at its free end as shown in figure.

Case-I: Weight, W is applied gradually

Gradual loading means that value of load is zero at the starting time and gradually increases to value of *W*. Here, the bar gradually elongates with the increasing value of load. With increase in elongation, resistance forces say R will also increase gradually.

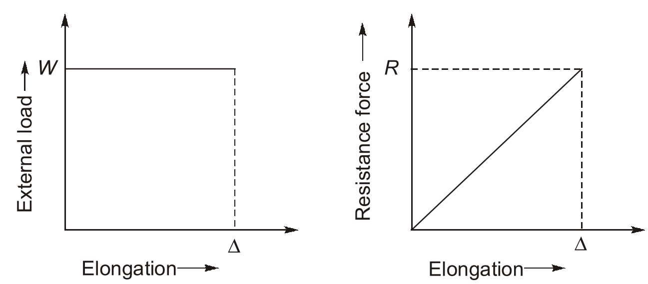


(a) Load deflection curve (b) Resistance deflection curve

Fig. External load is applied gradually

Case-II: Weight, W is applied suddenly

Here, external load variation with elongation of bar is such as that its value instantly increases to *W*. This sudden load will result into elongation of bar say Δ. When external load is applied suddenly, resistance force will be set up in bar, but unlike external load which is sudden, resistance force has always linear variation with elongation of bar.



(a) Load deflection curve (b) Resistance deflection curve

Fig. External load is applied suddenly

Now, as clear from figure (a) and (b), intensity of pressure is not equal to stress induced in bar.

Thus, stress can be defined as – **"Stress is the internal resistance of a material offered against deformation which is expressed in terms of force per unit area"**.

Stress induced in material depends upon the nature of force, point of application and cross-sectional area of material. Stress can be **tensile** or **compressive** in nature depending on the nature of load. Generally, stress is represented by the Greek letter σ. We can calculate stress mathematically as

$$\sigma = \frac{P}{A}$$

General Sign Convention:

Tensile stresses = +ve

Compressive stresses = –ve

Unit: (i) N/m^2 or Pa (SI unit)

(ii) N/mm^2 or MPa

NOTE

- Stresses are induced only when motion of bar is restricted either by some force or reaction induced. If body or bar is free to move or free expansion is allowed, then no stresses will be induced.
- Pressure has same unit but pressure is different physical quantity than stress. Pressure is external normal force distributed over surface.

CHAPTER

9 Pressure Vessels

9.1 THIN CYLINDRICAL SHELL

A cylindrical shell is said to be thin if the thickness of cylinder is less than $\frac{1}{10}$ to $\frac{1}{15}$ of the diameter of the cylinder.

9.1.1 Stresses in the Thin Cylindrical Shell

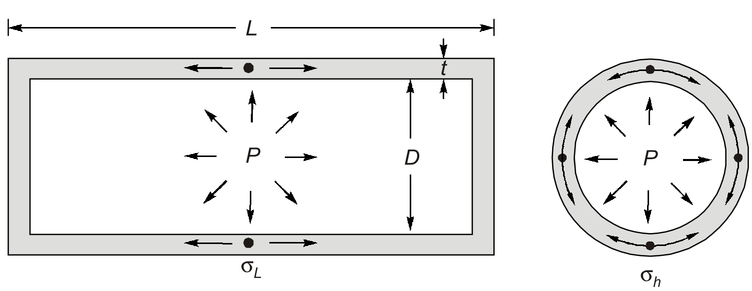
If the ends of cylindrical shell are closed and internal fluid pressure is applied then following three types of stresses may be developed.

(i) **Longitudinal stress (σ_L):** Due to internal fluid pressure, these are tensile but due to external fluid pressure these are compressive. In thin cylindrical shell, these are uniform across the thickness of shell.

(ii) **Hoop or circumferential stress (σ_h):** Due to the internal fluid pressure these are tensile. In thin cylindrical shell, these are assumed uniform.

(iii) **Radial stresses:** Magnitude is very small, hence neglected for all practical purpose.

9.2 ANALYSIS OF THIN CYLINDRICAL SHELL WITH CLOSED FLAT ENDS



(a) Longitudinal Stress (σ_L) (b) Hoop Stress (σ_h)

9.2.1 Hoop Stress or Circumferential Stress (σ_h)

Consider a thin cylindrical shell of diameter D, thickness t and length L subjected to an internal pressure P. Due to internal fluid pressure a bursting force is developed which tends to split the shell into two parts.

$\therefore$ Bursting force due to pressure, F_B = Pressure × projected area = $P \times (D \times L)$

Hoop stress induced in material resists the splitting of shell due to bursting force.

Resisting force, F_R = Hoop stress × Resisting area

$= \sigma_h \times (2 \times t \times L) = 2\,t\,L \times \sigma_h$

For equilibrium,

Resisting force, F_R = Bursting force, F_B

$\Rightarrow \quad 2\,t\,L \times \sigma_h = P \times (D \times L)$

$\Rightarrow \quad \boxed{\sigma_h = \frac{PD}{2t}}$ (Tensile)

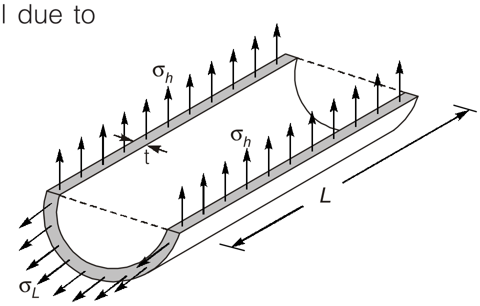


Fig. *Hoop stress on length of cylinder*

NOTE: Since, on the plane of σ_h there is no shear stress, hence σ_h is principal stress.

9.2.2 Longitudinal Stress (σ_L)

Consider a thin cylindrical shell subjected to internal pressure. Due to internal fluid pressure longitudinal stress will induce in the material of cylindrical shell. Since there is no bending therefore, longitudinal strain will be uniform in all longitudinal fibres. Hence longitudinal stress will constant across the thickness.

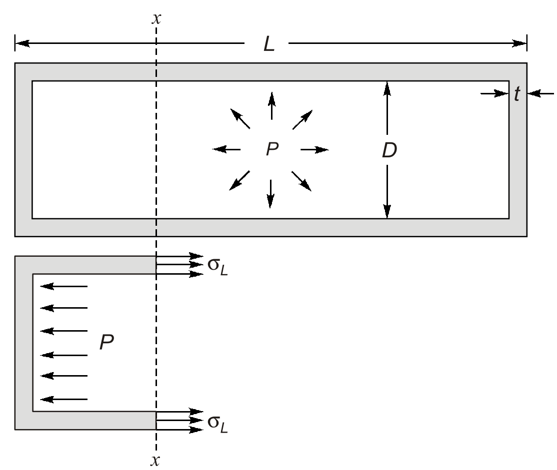


Fig. *Longitudinal stress*

Bursting force, $F_B = P \times \frac{\pi D^2}{4}$

and Resisting force, $F_R = \sigma_L \times (\pi D t)$

For equilibrium,

Resisting force, F_R = Bursting force, F_B

$\Rightarrow \quad \sigma_L \times (\pi D t) = P \times \frac{\pi D^2}{4}$

$\therefore \quad \boxed{\sigma_L = \frac{PD}{4t}}$ (Tensile)

NOTE

Since there is no shear stress on the plane of σ_L. Hence vertical plane is a principal plane and σ_L will be principal stress.

Since $\sigma_L < \sigma_h$, $\sigma_h = \frac{PD}{2t}$ will be major principal stress and $\sigma_L = \frac{PD}{4t}$ will be minor principal stress.

$\therefore \ \sigma_h = \frac{PD}{2t}$ (Major principal stress) and $\sigma_L = \frac{PD}{4t}$ (Minor principal stress)

Example 9.1 A cast iron pipe of 1 m diameter is required to withstand a 200 m head of water. If the limiting tensile stress of the pipe material is 20 MPa, then the minimum thickness of the pipe will be

(a) 25 mm (b) 50 mm

(c) 75 mm (d) 100 mm

OBJECTIVE BRAIN TEASERS

Q.1 A cylindrical shell of 500 mm diameter is required to withstand an internal fluid pressure of 5 N/mm^2. Find the minimum thickness of the cylindrical shell, if maximum tensile strength of the plate material is 400 N/mm^2 and the efficiency of the joint is 65%. (Assume factor of safety as 4)

(a) 20.1 mm (b) 19.23 mm
(c) 19 mm (d) 18.07 mm

Q.2 Circumferential stress and longitudinal stresses in a cylindrical steel boiler shell under internal pressure are 80 MPa and 40 MPa respectively. Young's modulus of elasticity and Poisson's ratio are respectively 2 × 10^5 MPa and 0.28. The magnitude of circumferential strain in the boiler shell will be

(a) 3.44×10^{-4} (b) 3.84×10^{-4}
(c) 4×10^{-4} (d) 4.56×10^{-4}

Q.3 A thick open ended cylinder is made of a material with permissible normal and shear stress 200 MPa and 100 MPa respectively. The ratio of permissible pressure based on the normal and shear stress is

(a) $\frac{9}{5}$ (b) $\frac{8}{5}$
(c) $\frac{7}{5}$ (d) $\frac{4}{5}$

Q.4 The purpose of compound cylinders is to

1. increase the pressure bearing capacity of single cylinder.
2. make the longitudinal stress distribution uniform.
3. increase the strength of cylinder along length.

Which of these statements is/are correct?

(a) only 1 (b) only 3
(c) both 1 and 2 (d) both 2 and 3

Q.5 Which of the following assumptions are made in Lame's theory of thick cylinders?

1. The material is stressed within the elastic limit
2. The material is homogeneous and isotropic
3. All the fibres of the material are free to expand or contract independently without being constrained by the adjacent fibres.

Select the correct answer using the codes given below:

(a) both 1 and 2 (b) both 1 and 3
(c) both 2 and 3 (d) 1, 2 and 3

Q.6 A thin walled long cylindrical tank of inner radius r is subjected to a axial compressive force F at its ends and the internal fluid pressure P simultaneously in order to produce state of pure shear in the wall of cylinder, the value of F should be

(a) πPr^2 (b) $2\pi Pr^2$
(c) $3\pi Pr^2$ (d) $4\pi Pr^2$

Q.7 A thin walled cylindrical pressure vessel having a radius of 0.5 m and wall thickness of 25 mm is subjected to an internal pressure of 700 kPa. The hoop stress developed is

(a) 14 MPa (b) 1.4 MPa
(c) 0.14 MPa (d) 0.014 MPa

Q.8 Two closed thin vessels, one cylindrical and the other spherical with equal internal diameter and wall thickness are subjected to equal internal fluid pressure. The ratio of hoop stresses in the cylindrical to that of spherical vessel is

(a) 4.0 (b) 2.0
(c) 1.0 (d) 0.5

Q.9 A thin cylindrical shell of internal diameter D and thickness 't' is subjected to internal pressure 'p'. The change in diameter is given by

(a) $\frac{pD^2}{4tE}(2-\mu)$ (b) $\frac{pD^2}{4tE}(1-2\mu)$
(c) $\frac{pD^2}{2tE}(1-2\mu)$ (d) $\frac{pD^2}{2tE}(2-\mu)$

$$= \frac{Pd}{4t} + \frac{(-F)}{\pi Dt}$$

$$= \frac{Pr}{2t} - \frac{F}{2\pi rt}$$

For the state of pure shear

$$\sigma_x + \sigma_y = \sigma_1 + \sigma_2 = o$$

$$\Rightarrow \quad \sigma_h + \sigma_L = o$$

$$\Rightarrow \left(\frac{Pr}{t}\right) + \left(\frac{Pr}{2t} - \frac{F}{2\pi rt}\right) = 0$$

$$\therefore \quad F = 3\pi pr^2$$

9. (a)

$$\frac{\text{Change in diameter}}{\text{Original diameter}} = \frac{\Delta D}{D} = \frac{1}{E}[\sigma_1 - \mu\sigma_2]$$

$$= \frac{1}{E}\left[\frac{pD}{2t} - \frac{\mu pD}{4t}\right] = \frac{pD}{4tE}(2-\mu)$$

$$\therefore \quad \Delta D = \frac{pD^2}{4tE}(2-\mu)$$

10. (b)

$p_1 = 80$ units, $p_2 = 90$ units, $S_{yt} = 240$ units

According to Maximum shear stress theory,

$$\sigma_1 \leq \frac{S_{yt}}{N}$$

$$p_0 \leq \frac{240}{N}$$

$$N = 3$$

12. (d)

$$\frac{pd}{2t} \leq 400 \text{ kg/cm}^2$$

$$\Rightarrow \quad \frac{1000 \times 200 \times 100 \times 160}{10^6 \times 2 \times t} \leq 400$$

$$\Rightarrow \quad t \geq 4 \text{ cm}$$

So, minimum value of $t = 4$ cm

14. (c)

$$\frac{pd}{2t} = \sigma_h$$

$$\Rightarrow \quad 100 = \frac{2 \times 80}{2 \times t}$$

$$\Rightarrow \quad t = 0.8 \text{ cm} = 8 \text{ mm}$$

CONVENTIONAL BRAIN TEASERS

Q.1 A compound cylinder, formed by shrinking one tube on to another, is subjected to an internal pressure of 50 MPa. Before the fluid is admitted, the internal and external diameters of the compound cylinder are 100 mm and 180 mm, and the diameter at the junction is 150 mm. If, after shrinking the radial pressure at the common surface is 8 MPa. Determine the total final stresses.

Solution:

Given: $(p_i)_f = 50$ MPa, $R = 90$ mm, $r = 50$ mm, $r_1 = 75$ mm, $(P_j)_s = 8$ MPa

For Inner tube, $$(\sigma_r)_x = a - \frac{b}{x^2}$$

$$(\sigma_h)_x = a + \frac{b}{x^2}$$

For outer tube, $$(\sigma_r)_x = A - \frac{B}{x^2}$$

$$(\sigma_h)_x = A + \frac{B}{x^2}$$

CHAPTER

10

Theory of Columns

10.1 COMPRESSION MEMBER

A compression member is a structural member which is straight and subjected to two equal and opposite compressive forces applied at its ends. It may be vertical (column) or inclined (strut). Column is a compression member subjected to predominantly axial loading. There are three mode of failure of columns:

1. **Crushing (yielding):** Generally short column fails in this mode.
2. **Buckling (elastic instability):** Generally long column fails in elastic bending i.e., buckling or elastic instability.
3. **Combined crushing and buckling:** This is the common case of intermediate column.

10.2 TYPES OF EQUILIBRIUM

(i) **Stable equilibrium:** A column is said to be in stable equilibrium when restoring moment is greater than overturning moment.

$$M_R > M_{OTM}$$

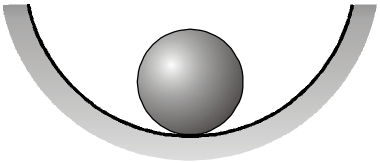


Fig. *Stable equilibrium*

(ii) **Neutral equilibrium:** A column is said to be in neutral equilibrium if the restoring moment is equal to overturning moment. This stage is called critical stage and corresponding load is called **critical load**.

$$M_R = M_{OTM}$$

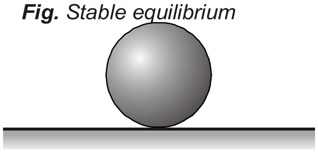


Fig. *Neutral equilibrium*

(iii) **Unstable equilibrium:** A column is said to be in unstable equilibrium if the restoring moment is less than overturning moment

$$M_R < M_{OTM}$$

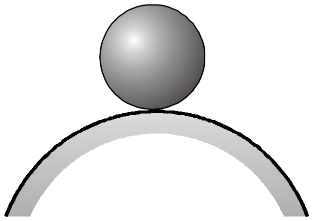


Fig. *Unstable equilibrium*

10.2.1 Elastic Instability and Critical Load

Restoring moment about A,

$$M_R = F_S \times L = (k\Delta) \times L \quad [\because F_S = k\Delta]$$

Overturning moment provided by P about A, $M_{OTR} = P\Delta$

When restoring moment is equal to overturning moment then that stage is called critical stage and corresponding load is called critical load. Hence, for critical load

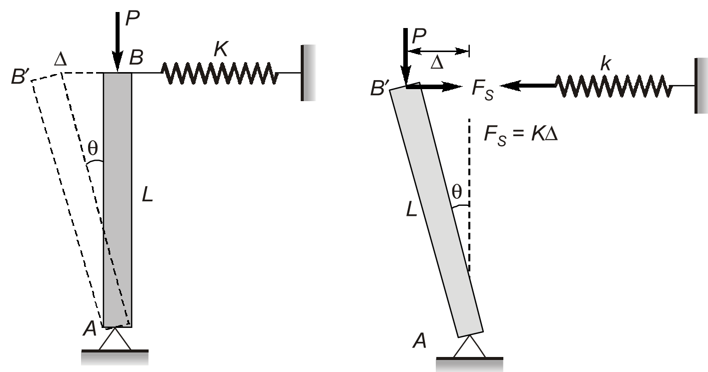


Fig. *Column spring system subjected to axial force*

$$M_{OTR} = M_R$$

$$\Rightarrow \quad P_{cr}\Delta = (k\Delta) \times L$$

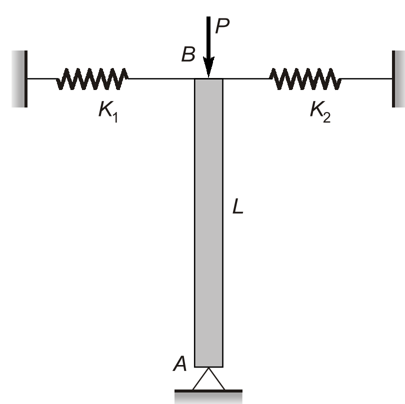
$$\Rightarrow \quad P_{cr} = kL$$

If, $P > P_{cr}$ = unstable equilibrium

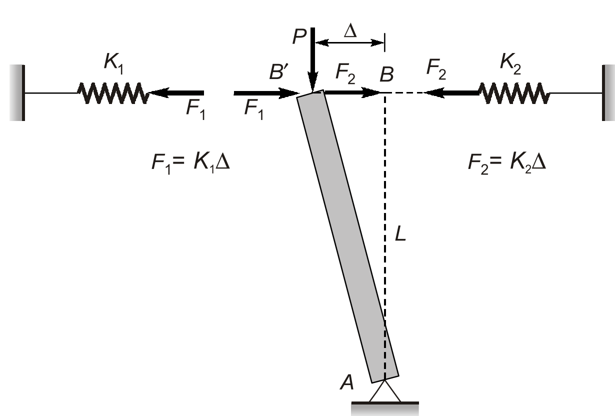
$P = P_{cr}$ = neutral equilibrium

$P < P_{cr}$ = stable equilibrium

Example 10.1 Find critical load for the loading shown in figure below:



Solution:



OBJECTIVE BRAIN TEASERS

Q.1 A steel rod 2 m long and 40 mm in diameter is used as a column with one end fixed and other end free. The crippling load by Euler's formula is? Take *EI* is 8×10^9 Nmm^2

(a) 4.93 kN (b) 5 kN
(c) 4.39 kN (d) 2.80 kN

Q.2 ISMB 200 rolled steel joist is used as column is 5 m long with one end fixed and other being hinged. The safe load on the column taking FOS of 3 is

Given: $\sigma_c = 320$ N/mm^2, $\alpha = \dfrac{1}{7500}$

Properties of section: $A = 4320$ mm^2, $I_{xx} = 4.125 \times 10^6$ mm^4, $I_{yy} = 3.021 \times 10^6$ mm^4

(a) 1271.30 kN (b) 1000 kN
(c) 136.16 kN (d) 1480 kN

Q.3 The length of a column which gives the same value of buckling load by Euler and Rankine Gordon formula, is equal to

(a) $\dfrac{\pi^2 EK}{\sigma_a - \pi^2 E_a}$ (b) $\sqrt{\dfrac{\pi^2 EK^2}{\sigma_a - \pi^2 E_a}}$

(c) $\sqrt{\dfrac{\pi^2 EK}{\pi^2 E_a - \sigma_a}}$ (d) None of these

Q.4 The greatest eccentricity which a load *W* can have without producing tension of the cross-section of a short column of external diameter *D* and internal diameter *d*, is

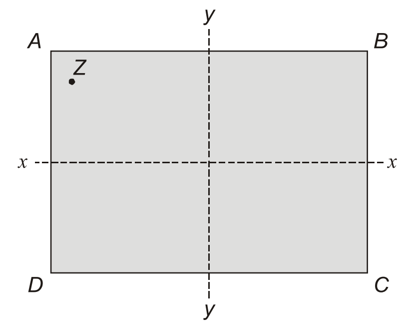
(a) $\dfrac{D+d}{8D}$ (b) $\pi\dfrac{(D^4 - d^4)}{32D^3}$

(c) $\dfrac{D^2+d^2}{8D}$ (d) $\dfrac{D^2-d^2}{8D}$

Q.5 A circular column of length 2 m has Euler's crippling load of 1.5 kN. If the diameter of column is reduced by 10%, the reduction in the crippling load will be

(a) 10% (b) 20%
(c) 30% (d) more than 30%

Q.6 A column *ABCD* of rectangular section is subjected to an eccentric load at z as shown in figure.



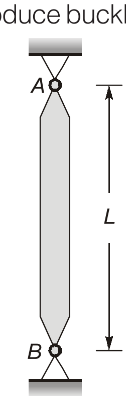
Under the compressive load, the direct stress is 15 t/m^2 and bending stresses are $\sigma_{xx} = 5$ t/m^2 and $\sigma_{yy} = 8$ t/m^2. The stress at the corner B will be

(a) 12 t/m^2 (compressive)
(b) 2 t/m^2 (tensile)
(c) 18 t/m^2 (tensile)
(d) 28 t/m^2 (compressive)

Q.7 For a circular column its both ends are hinged, the slenderness ratio is 180. The *l*/*d* ratio of column is

(a) 40 (b) 60
(c) 50 (d) 45

Q.8 A slender column AB with hinged at both ends is held between immovable supports as shown in figure below. Increase in temperature ΔT°C of the bar that will produce buckling at the Euler's load is



Given, $L = 3$ m
$\alpha = 12 \times 10^{-6}$/°C
Diameter, $d = 50$ mm

(a) 14.3°C (b) 12°C
(c) 15.3°C (d) 11.5°C

CONVENTIONAL BRAIN TEASERS

Q.1 A simply supported beam of length 4 metre is subjected to a uniformly distributed load of 30 kN/m over the whole span and deflects 15 mm at the centre. Determine the crippling load when this beam is used as a column with the following conditions:

(i) one end fixed and other end hinged

(ii) both the ends pin jointed

Solution :

Length, l = 4 m

UDL, w = 30 kN/m

Deflection at centre, δ = 15 mm = 0.015 m

For a simply supported beam carrying UDL over the whole span, the deflection at the centre is given by

$$\delta = \frac{5}{384}\frac{\omega l^4}{EI}$$

$$\therefore \quad EI = \frac{5}{384}\frac{w l^4}{\delta} = \frac{5}{384}\times\frac{30\times(4)^4}{0.015} = 6666.67 \text{ kN.mm}$$

(i) $$P_{cr} = \frac{2\pi^2 EI}{l^2} \quad \text{(for one end fixed and other hinged, } l_e = \frac{l}{\sqrt{2}}\text{)}$$

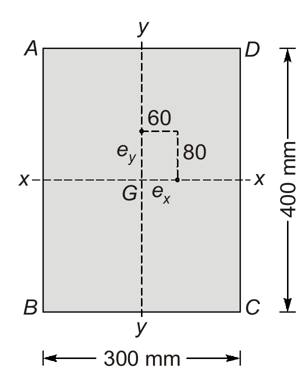
$$= \frac{2\times 3.14^2 \times 6666.67}{(4)^2} = 8216.33 \text{ kN}$$ Ans.

(ii) $$P_{cr} = \frac{\pi^2 EI}{l^2} \quad \text{(for both end hinged, } l_e = l\text{)}$$

$$= \frac{(3.14)^2\times 6666.67}{(4)^2} = 4108.167 \text{ kN}$$ Ans.

Q.2 Consider a column of rectangular section of dimension B = 300 mm, D = 400 mm, load W = 1200 kN at eccentricity e_x = 60 mm, e_y = 80 mm, x–x axis parallel to breadth. Find resultant stresses at four corners of the section of the column.

Solution :



W = 1200,000 N

Area = 300 × 400 = 12 × 10^4 mm^2

$$\sigma_d = \text{direct stress} = \frac{12\times10^5}{12\times10^4} = 10 \text{ MPa}$$

Moment, $M_x = We_y$ = 12 × 10^5 × 80 = 96 × 10^6 Nmm

$M_y = We_x$ = 12 × 10^5 × 60 = 72 × 10^6 Nmm

$$\sigma_{b1} = \pm\frac{6M_x}{BD^2} = \frac{6\times96\times10^6}{300\times400^2} = \pm 12 \text{ N/mm}^2$$

AD in compression, BC in tension

$$\sigma_{b2} = \pm\frac{6M_y}{BD^2} = \frac{6\times72\times10^6}{400\times300^2} = \pm 12 \text{ N/mm}^2$$