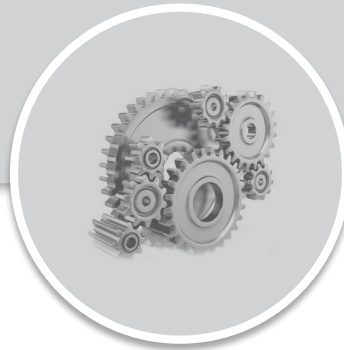


MECHANICAL ENGINEERING

Engineering Mechanics



Comprehensive Theory
with Solved Examples and Practice Questions





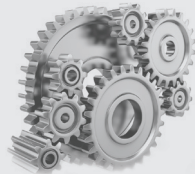
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Engineering Mechanics

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CONTENTS

Engineering Mechanics

CHAPTER 1

Basics of Vectors 1-9

1.1	Vectors and scalars.....	1
1.2	Equality of vectors.....	1
1.3	Addition of vectors.....	2
1.4	Multiplication of a vector by a number.....	3
1.5	Subtraction of vectors.....	3
1.6	Resolution of Vectors.....	3
1.7	Dot Product or Scalar Product of two vectors.....	4
1.8	Dot product of two vectors in terms of the components along the coordinate axes.....	4
1.9	Cross product or vector product of two vectors.....	5
	<i>Objective Brain Teasers</i>	8

CHAPTER 2

Motion in Space..... 10-20

2.1	Distance and displacement.....	10
2.2	Average speed and instantaneous speed.....	10
2.3	Average velocity and instantaneous velocity.....	11
2.4	Average Acceleration and instantaneous acceleration.....	11
2.5	Rectilinear motion.....	12
2.6	Curvilinear motion.....	15
	<i>Objective Brain Teasers</i>	17
	<i>Conventional Brain Teasers</i>	19

CHAPTER 3

Composition, Resolution and Equilibrium of Forces..... 21-35

3.1	Force.....	21
3.2	Force Systems.....	22
3.3	Resultant Force.....	23

3.4	Composition of Forces.....	23
3.5	Resolution of Forces.....	24
3.6	Equilibrium of Forces.....	24
3.7	Principle of Transmissibility.....	25
3.8	Free Body Diagram.....	26
3.9	Newton's Laws of Motion.....	30
	<i>Objective Brain Teasers</i>	31
	<i>Conventional Brain Teasers</i>	33

CHAPTER 4

Analysis of Simple Trusses..... 36-51

4.1	Plane Trusses.....	36
4.2	Perfect Truss.....	36
4.3	Type of Supports.....	36
4.4	Analysis of a Truss.....	37
	<i>Objective Brain Teasers</i>	47
	<i>Conventional Brain Teasers</i>	49

CHAPTER 5

Virtual Work..... 52-60

5.1	Concept of Virtual Work.....	52
5.2	Principle of Virtual Work.....	52
	<i>Objective Brain Teasers</i>	57
	<i>Conventional Brain Teasers</i>	58

CHAPTER 6

Friction 61-103

6.1	Introduction to Friction.....	61
6.2	Types of Friction.....	61
6.3	Laws of Friction.....	62

6.4	Coefficient of Friction.....	62
6.5	Angle of Friction (ϕ).....	63
6.6	Angle of Repose.....	63
6.7	Force Analysis of a Sliding Body.....	63
6.8	Wedge.....	66
6.9	Rolling Friction.....	69
6.10	Motion of Vehicles on rough Road.....	70
6.11	Belt, Rope and Pulley.....	76
6.12	Screw Jack.....	85
6.13	Two Mating Blocks.....	91
6.14	Friction in Wheels.....	92
	<i>Objective Brain Teasers</i>	98
	<i>Conventional Brain Teasers</i>	100

CHAPTER 7

Work, Power and Energy 104-117

7.1	Introduction.....	104
7.2	Rolling Motion under Gravity.....	107
7.3	Power.....	108
	<i>Objective Brain Teasers</i>	113
	<i>Conventional Brain Teasers</i>	114

CHAPTER 8

Center of Gravity and Moment of Inertia... 118-132

8.1	Center of Gravity.....	118
8.2	Centroid.....	118
8.3	Centroid of Given Lamina.....	118
8.4	Moment of Inertia.....	123
	<i>Objective Brain Teasers</i>	130
	<i>Conventional Brain Teasers</i>	131

CHAPTER 9

Impulse & Momentum 133-143

9.1	Momentum.....	133
9.2	Impulse.....	133
9.3	Impulse-Momentum Theorem.....	133
9.4	Law of Conservation of Linear Momentum.....	134
9.5	Elastic and Inelastic Impact.....	134
	<i>Objective Brain Teasers</i>	141
	<i>Conventional Brain Teasers</i>	142

CHAPTER 10

Rotational Motion 144-158

10.1	Introduction.....	144
10.2	Rotational kinematics.....	144
10.3	Relation between linear motion and rotational motion of a particle on a body.....	145
10.4	Torque of a Force.....	146
10.5	Body in Equilibrium.....	148
10.6	Angular momentum.....	148
10.7	Mass moment of inertia.....	149
10.8	Rolling Without sliding or pure rolling.....	151
	<i>Objective Brain Teasers</i>	153
	<i>Conventional Brain Teasers</i>	156

CHAPTER 11

Lagrangian Equation 159-167

11.1	Introduction.....	159
11.2	Generalized Coordinates.....	160
11.3	Degrees of Freedom.....	160
11.4	Constraints.....	160
11.5	Lagrangian Equations.....	161
11.6	Applications of Lagrange's Equation.....	164



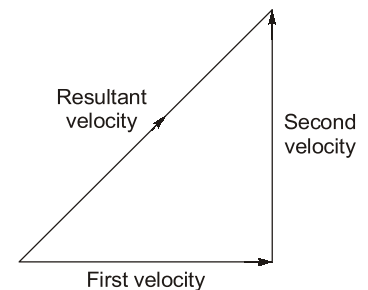
Basics of Vectors

1.1 VECTORS AND SCALARS

Many physical quantities are completely described by a numerical value alone and are added according to the ordinary rules of algebra. As an example the mass of a system is described by saying that it is 5 kg. If two bodies one having a mass of 5 kg and the other having a mass of 2 kg are added together to make a composite system, the total mass of the system becomes $5 \text{ kg} + 2 \text{ kg} = 7 \text{ kg}$. Such quantities are called scalars.

So, a scalar is any positive or negative physical quantity that can be completely specified by its magnitude. Other examples of scalar quantities are length, mass and time.

While the complete description of certain physical quantities requires a numerical value as well as a direction in space. Velocity of a particle is an example of this kind. The magnitude of velocity is represented by a number such as 5 m/s and tells us how fast a particle is moving. But the description of velocity becomes complete only when the direction of velocity is also specified. We can represent this velocity by drawing a line parallel to the velocity and putting an arrow showing the direction of velocity.



Further, if a particle is given two velocities simultaneously, its resultant velocity is different from the two velocities and is obtained by using a special rule known as triangle law.

The physical quantities which have magnitude and direction and which can be added according to the laws of vector addition are called vector quantities. Other examples of vector quantities are force, linear momentum, electric field, magnetic field etc.

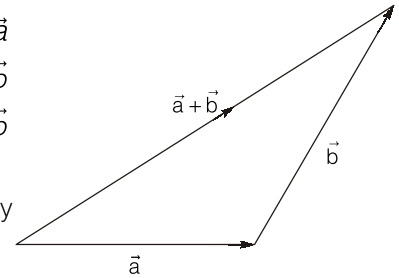
1.2 EQUALITY OF VECTORS

Two vectors (representing two values of the same physical quantity) are called equal if their magnitudes and directions are same. Thus, a parallel translation of a vector does not bring about any change in it.

1.3 ADDITION OF VECTORS

The triangle rule of vector addition is already described above. If \vec{a} and \vec{b} are two vectors to be added, a diagram is drawn in which the tail of \vec{b} coincides with the head of \vec{a} . The vector joining the tail of \vec{a} with the head of \vec{b} is the vector sum of \vec{a} and \vec{b} .

Figure shows the construction. The same rule may be stated in a slightly different way by parallelogram law.



In parallelogram law we draw vectors \vec{a} and \vec{b} with both the tails coinciding as shown in figure. Taking these two as the adjacent sides we complete the parallelogram. The diagonal through the common tails gives the sum of the two vectors.

Suppose the magnitude of $\vec{a} = a$ and that of $\vec{b} = b$. If the angle between \vec{a} and \vec{b} is θ , it is easy to see from figure that

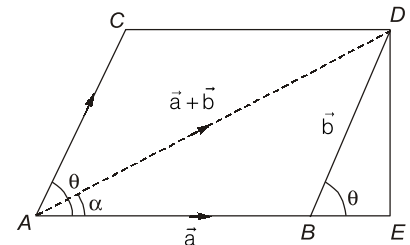
$$\begin{aligned} AD^2 &= (AB + BE)^2 + (DE)^2 \\ &= (a + b\cos\theta)^2 + (b\sin\theta)^2 \\ &= a^2 + 2ab\cos\theta + b^2 \end{aligned}$$

Thus, the magnitude of $\vec{a} + \vec{b}$ is

$$= \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

Its angle with \vec{a} is α where,

$$\tan\alpha = \frac{DE}{AE} = \frac{b\sin\theta}{a + b\cos\theta}$$

**Special cases:**

(a) When two vectors are acting in same direction, then,

$$\theta = 0^\circ$$

$$\therefore |\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab} = a + b$$

and

$$\tan\alpha = \frac{b \times \sin 0^\circ}{a + b \cos 0^\circ} = 0$$

\Rightarrow

$$\alpha = 0^\circ$$

Thus, the magnitude of sum of vectors \vec{a} and \vec{b} is equal to the sum of magnitudes of two vectors acting in same direction and their resultant acts in direction of \vec{a} and \vec{b} .

(b) When two vectors acts in opposite directions:

Then,

$$\theta = 180^\circ$$

$$\therefore |\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 - 2ab} = a - b$$

and

$$\tan\alpha = \frac{b \times \sin(180^\circ)}{a + b \times \cos(180^\circ)} = 0$$

\Rightarrow

$$\alpha = 0^\circ \text{ or } 180^\circ$$

Thus, the magnitude of sum of the vectors \vec{a} and $(-\vec{b})$ is equal to the difference of magnitudes of two vectors and their resultant acts in direction of bigger vector.

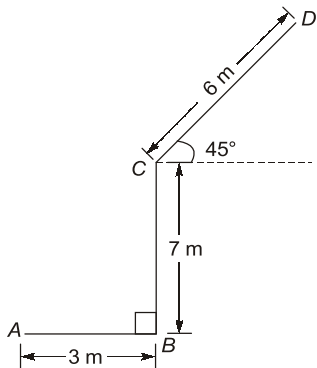


OBJECTIVE BRAIN TEASERS

Q.1 A particle whose speed is 25 m/sec moves along the line from $A(2, 1)$ to $B(9, 25)$. The velocity vector of the particle in the form $a\hat{i} + b\hat{j}$ is

- (a) $(7\hat{i} + 7\hat{j})$ m/s (b) $(7\hat{i} + 24\hat{j})$ m/s
(c) $(24\hat{i} + 12\hat{j})$ m/s (d) $(24\hat{i} + 7\hat{j})$ m/s

Q.2 A particle moves along a path ABCD as shown in the figure. The magnitude of net displacement of the particle from A to D is



- (a) 9.26 m (b) 13.37 m
(c) 10.42 m (d) 8.38 m

Q.3 If $(\vec{a} + \vec{b})$ is perpendicular to \vec{b} and $\vec{a} + 2\vec{b}$ is perpendicular to \vec{a} . If $|\vec{a}| = a$ and $|\vec{b}| = b$, then

- (a) $a = b$ (b) $a = 2b$
(c) $b = 2a$ (d) $a = b\sqrt{2}$

Q.4 Two forces are acting on a body, $\vec{F}_1 = 2\hat{i} + 3\hat{j}$ and it does 8J of work, $\vec{F}_2 = 3\hat{i} + 5\hat{j}$ and it does -4J of work on body. The magnitude of displacement traversed by the body is

- (a) 57.27 m (b) 30.53 m
(c) 54.40 m (d) 61.06 m

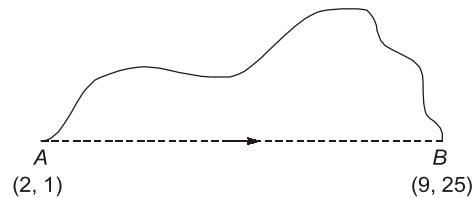
ANSWER KEY

1. (b) 2. (b) 3. (d) 4. (d)

HINTS & EXPLANATIONS

1. (b)

Velocity vector is given by the product of magnitude of velocity (speed) multiplied by the unit vector along velocity.



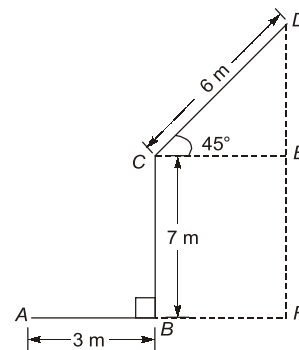
Unit vector along velocity,

$$\widehat{AB} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{(9-2)\hat{i} + (25-1)\hat{j}}{\sqrt{7^2 + 24^2}}$$

$$\begin{aligned}\vec{V} &= |\vec{V}| \widehat{AB} = 25 \left(\frac{7\hat{i} + 24\hat{j}}{25} \right) \\ &= (7\hat{i} + 24\hat{j}) \text{ m/s}\end{aligned}$$

2. (b)

The displacement of the particle from A to D is given by AD



$$\begin{aligned}AD &= \sqrt{AF^2 + DF^2} = \sqrt{(AB + BF)^2 + (DE + EF)^2} \\ &= \sqrt{(3 + 6\cos 45^\circ)^2 + (7 + 6\sin 45^\circ)^2} \\ &= \sqrt{(7.24)^2 + (11.24)^2} \\ &= \sqrt{52.41 + 126.33} = \sqrt{178.74} \\ &= 13.37 \text{ m}\end{aligned}$$

3. (d)

From the given conditions, $(\vec{a} + \vec{b})$ is perpendicular to \vec{b} , therefore their dot product is zero.

$$(\vec{a} + \vec{b}) \cdot \vec{b} = 0$$

$$\vec{a} \cdot \vec{b} + |\vec{b}|^2 = 0$$

$$\vec{a} \cdot \vec{b} + b^2 = 0 \quad \dots (i)$$

Similarly,

$$(\vec{a} + 2\vec{b}) \cdot \vec{a} = 0$$

$$|\vec{a}|^2 + 2\vec{a} \cdot \vec{b} = 0$$

$$a^2 + 2\vec{a} \cdot \vec{b} = 0 \quad \dots (ii)$$

From equations (i) and (ii), we get

$$-\frac{a^2}{2} = -b^2$$

$$\therefore a = \pm\sqrt{2}b$$

Neglecting negative sign as magnitude cannot be negative, therefore $a = \sqrt{2}b$.

4. (d)

Let the displacement of the body is given by

$$\vec{r} = x\hat{i} + y\hat{j}$$

work done by first force,

$$W_1 = \vec{F}_1 \cdot \vec{r} = 8$$

$$\therefore (2\hat{i} + 3\hat{j}) \cdot (x\hat{i} + y\hat{j}) = 8$$

$$2x + 3y = 8 \quad \dots (i)$$

Work done by second force,

$$W_2 = \vec{F}_2 \cdot \vec{r} = -4$$

$$(3\hat{i} + 5\hat{j}) \cdot (x\hat{i} + y\hat{j}) = -4$$

$$3x + 5y = -4 \quad \dots (ii)$$

Solving (i) and (ii) equation, we get

$$x = 52, y = -32$$

$$\therefore \vec{r} = 52\hat{i} - 32\hat{j}$$

$$|\vec{r}| = \sqrt{(52)^2 + (-32)^2} = \sqrt{3728} \\ = 61.06 \text{ m}$$



Rotational Motion

10.1 INTRODUCTION

Consider a pulley fixed at a well on which a rope is wound with one end attached to the bucket. When the bucket is released, pulley starts rotating. If we take pulley as a system. The centre of mass of pulley is at its centre which remains at rest but the other particles move and are accelerated. So, the pulley is said to be in rotational motion.

10.2 ROTATIONAL KINEMATICS

Consider a rigid body rotating about a given fixed axis. Suppose the angular position of the body at any time t is θ . During a time Δt , it further rotates $\Delta\theta$, so its angular position becomes $\theta + \Delta\theta$.

So, the average velocity during time interval Δt is given by

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$$

and the instantaneous angular velocity at any time t is given by

$$\omega = \frac{d\theta}{dt}$$

If the body rotates through equal angles in equal time intervals, then we can say that it rotates with uniform angular velocity. In this case,

$$\omega = \frac{d\theta}{dt} = \text{Constant}$$

So, $\theta = \omega t$

If it is not the case, then the body is said to be rotationally accelerated and its angular acceleration will be

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt} \left(\frac{d\theta}{dt} \right) = \frac{d^2\theta}{dt^2}$$

But if angular acceleration is constant, then, $\omega = \omega_0 + \alpha t$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

Where ω_0 is angular velocity at time $t = 0$

EXAMPLE : 10.1

A wheel rotates with a constant acceleration of 4 rad/s^2 . If the wheel starts from rest, the revolutions made by it in the first 20 seconds are:

- (a) 176 (b) 156
(c) 128 (d) 64

Solution: (c)

The angular displacement in the first 20 seconds is given by

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 0 + \frac{1}{2} \times 4 \times 20^2 = 800 \text{ rad}$$

As the wheel turns by 2π radians in each revolution, the number of revolutions in 20 seconds is

$$n = \frac{800}{2\pi} = 127.32 \approx 128 \text{ revolutions}$$

EXAMPLE : 10.2

The wheel of a motor, accelerated uniformly from rest, rotates through 5 radians during the first second. Find the angle rotated during the next second.

- (a) 5 rad (b) 15 rad
(c) 20 rad (d) 10 rad

Solution: (b)

As the angular acceleration is constant, we have,

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = \frac{1}{2} \alpha t^2 \quad (\omega_0 = 0)$$

Thus,

$$5 \text{ rad} = \frac{1}{2} \times \alpha \times (1)^2$$

$$\alpha = 10 \text{ rad/s}^2$$

The angle rotated during the first two seconds is

$$= \frac{1}{2} \times \alpha \times t^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ rad}$$

Thus, the angle rotated during the 2nd second is $= 20 - 5 = 15$ radians

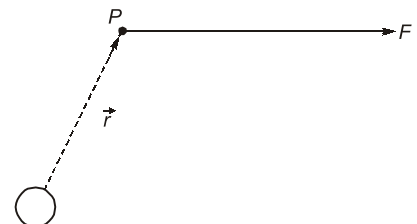
10.3 RELATION BETWEEN LINEAR MOTION AND ROTATIONAL MOTION OF A PARTICLE ON A BODY

Let \vec{r} is the position vector of a particle P experiencing the force, then torque by force \vec{F} about O

$$\vec{\tau} = \vec{r} \times \vec{F}$$

But if there are more than one forces acting on a body, then total torque on body about a given axis is

$$\vec{\tau} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \dots$$



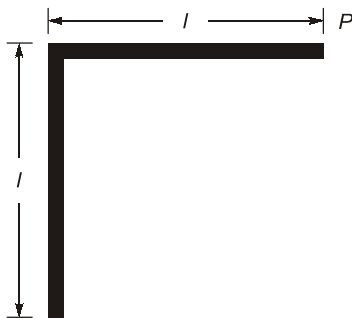


**OBJECTIVE
BRAIN TEASERS**

- Q.1** The motor of an engine is rotating about its axis with an angular velocity of 100 rev/minutes. It comes to rest in 15 seconds, after being switched off. Assuming constant angular deceleration, the number of revolutions made by it before coming to rest is
(a) 12.5 (b) 25
(c) 6.25 (d) 18.75

- Q.2** Four particles each of mass m are kept at the four corners of a square of edge a . What is the moment of inertia of the system about a line perpendicular to the plane of the square and passing through the centre of the square?
(a) $2ma^2$ (b) $2\sqrt{2}ma^2$
(c) $4ma^2$ (d) $4\sqrt{2}ma^2$

- Q.3** Two uniform rods having mass m each and length l are joined at a right angle as shown in figure. What is the moment of inertia of this system about an axis passing through point P and perpendicular to the plane of the system?



- (a) $\frac{8}{3}ml^2$ (b) $3ml^2$
(c) $\frac{5}{3}ml^2$ (d) $2ml^2$

- Q.4** A uniform rod of mass m and length L can rotate freely on a smooth horizontal plane about a vertical axis hinged at point H . A point mass having same mass m coming with an initial speed u perpendicular to the rod, strikes the rod inelastically at its free end. The angular velocity of the rod just after the collision is:



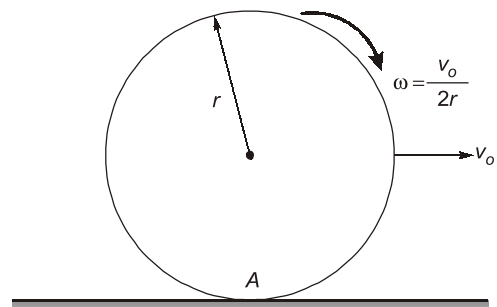
- (a) $\frac{u}{3L}$ (b) $\frac{u}{2L}$
(c) $\frac{2u}{3L}$ (d) $\frac{3u}{4L}$

- Q.5** A uniform sphere of mass 200 gms rolls without slipping on a plane surface so that its centre moves at a speed of 2 cm/sec. What is the kinetic energy of the sphere?
(a) 1.2×10^{-5} J (b) 3.6×10^{-5} J
(c) 2.8×10^{-5} J (d) 5.6×10^{-5} J

- Q.6** A cylinder is released from rest from the top of an incline of inclination θ and length l . If the cylinder rolls without slipping, what will be its speed when it reaches the bottom?

- (a) $\sqrt{\frac{4gl \sin \theta}{3}}$ (b) $\sqrt{\frac{2gl \sin \theta}{3}}$
(c) $\sqrt{\frac{2gl \sin \theta}{5}}$ (d) $\sqrt{\frac{gl \sin \theta}{3}}$

- Q.7** A cylinder of M and radius r shown in figure slips on a rough horizontal plane. At some instant, it has translational velocity v_0 and rotational velocity about the centre $\frac{v_0}{2r}$. What will be the translational velocity of the cylinder after it starts pure rolling?



- (a) $\frac{2}{5}v_0$ (b) $\frac{5}{6}v_0$

$$\omega(t) = \omega_o + at = \frac{v_o}{2r} + \frac{2ft}{Mr}$$

Pure rolling will start then

$$v(t) = r\omega(t)$$

i.e.
$$v(t) = r\left(\frac{v_o}{2r} + \frac{2ft}{Mr}\right)$$

$$v(t) = \frac{v_o}{2} + \frac{2ft}{M} \quad \dots (ii)$$

Finding the value of t from equation (i) and (ii),

$$v_o - \frac{f}{M}t = \frac{v_o}{2} + \frac{2f}{M}t$$

$$\therefore \frac{v_o}{2} = \frac{3f}{M}t$$

$$\therefore t = \frac{v_o M}{6f} \quad \dots (iii)$$

Putting this value of time (t) in equation (ii), we get,

$$\begin{aligned} v(t) &= \frac{v_o}{2} + \frac{2f}{M} \times \frac{v_o M}{6f} \\ &= \frac{v_o}{2} + \frac{v_o}{3} = \frac{5v_o}{6} \end{aligned}$$

8. (b)

$$T = mg \times \frac{L}{2}$$

$$I_o = \frac{mL^2}{3}$$

$$\alpha = \frac{T}{I_o} = \frac{mgL}{2} \times \frac{3}{mL^2} = \frac{1.5g}{L}$$

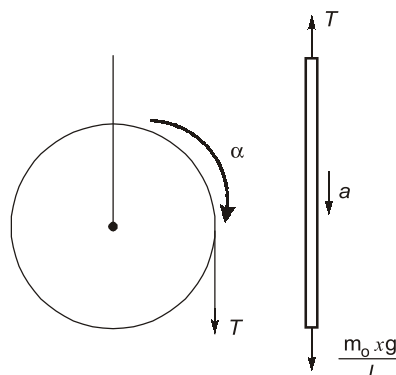
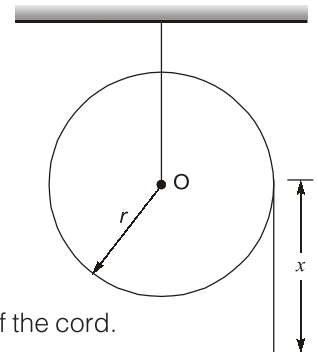


CONVENTIONAL BRAIN TEASERS

Q.1 A uniform cylinder of radius ' r ' and mass ' m ' can rotate freely about a fixed horizontal axis. A thin cord of length ' l ' and mass ' m_o ' is wound on the cylinder in a single layer. Find the angular acceleration of the cylinder as a function of the length x of the hanging part of the cord. The wound part of the cord is supposed to have its centre of gravity on the cylinder axis as shown in figure.

Solution:

Figure below shown the free body diagram of the cylinder and hanging part of the cord.



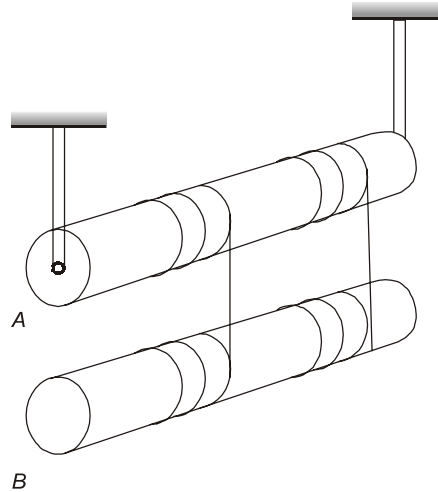
If α is the angular acceleration of the cylinder, then from $\tau = I\alpha$, we have,

$$Tr = \left[\frac{mr^2}{2} + \frac{m_o}{l}(l-x)r^2 \right] \alpha$$

\therefore Net moment of inertia,

$$I = I_1 - I_2 = \frac{2MR^2}{3} - \frac{MR^2}{8} = \frac{16MR^2 - 3MR^2}{24} = \frac{13MR^2}{24}$$

- Q.3** The arrangement shown in figure consists of two identical, uniform, solid cylinder each of mass m on which two light threads are wound symmetrically. Find the tensions on each thread in the process of motion. The friction in the axle of the upper cylinder is assumed to be absent.



Solution:

Let the lower cylinder go down by x when the upper cylinder A rotates through θ and the lower one rotates through θ' . Since both cylinders unwind themselves, therefore,

$$x = R(\theta' + \theta)$$

Difference this equation twice, we get

$$a = R(\alpha' + \alpha)$$

Where, a = Downward acceleration of B

α = Angular acceleration of A

α' = Angular acceleration of B

Considering rotational motion of A,

$$\tau = 2TR = \frac{1}{2}mR^2\alpha \Rightarrow \alpha = \frac{4T}{mR}$$

Considering rotational motion of B,

$$\tau = 2TR = \frac{1}{2}mR^2\alpha' \Rightarrow \alpha' = \frac{4T}{mR}$$

Considering downward motion of B,

$$mg - 2T = ma$$

$$mg - 2T = mR(\alpha + \alpha')$$

$$mg - 2T = mR\left(\frac{4T}{mR} + \frac{4T}{mR}\right) = 8T$$

$$10T = mg$$

$$T = \frac{mg}{10}$$

