MADE EASY PUBLICATIONS

POSTAL BOOK PACKAGE 2024

MECHANICAL ENGINEERING

. . . .

CONVENTIONAL Practice Sets

CONTENTS

HEAT TRANSFER

1 Introduction and Basic Concepts

CHAPTER

Practice Questions : Level-I

Q.1 The ratio of radius of the earth's orbit to that of sun is 216 : 1. The solar insolation on the earth is 1.4 kW/m^2.

Find the surface temperature of the sun if it assumed to be an ideal radiator (black body).

Solution:

Given data: $\dfrac{R}{r}$ = 216; where r is radius of the sun and T is the surface temperature of the sun. Therefore,

Total radiation from the sun, $\qquad Q_r = 1.4 \times 4\pi R^2$; $\qquad$ where R is the radius of the earth's orbit.

$\quad$ Total radiation emitted by the sun, $Q_r = \sigma 4\pi r^2 \, T^4$

Therefore, $\qquad\qquad\qquad \sigma 4\pi r^2 \, T^4 = 1.4 \times 4\pi R^2$

$\therefore \qquad\qquad\qquad\qquad T^4 = \dfrac{1.4 \times 10^3}{5.67 \times 10^{-8}} \times (216)^2 = 0.1152 \times 10^{16} \text{ K}^4$

$\qquad\qquad\qquad\qquad\quad T = 5826 \text{ K}$

Q.2 A pipe 2 cm in diameter at 40°C is placed in (i) an air flow at 50°C, with h = 20 W/m^{2}K and in (ii) water at 30°C with h = 70 W/m^{2}K. Find the heat transfer rate per unit length of the pipe.

Solution:

Given data: $\quad D = 2$ cm, $\quad T_w = 40°C$

The definition of the mean heat transfer coefficient gives

$\qquad\qquad\qquad Q = hA(T_w - T_\infty)$

Here, $\qquad\qquad\qquad T_w = 40°C$, and since 1 m length of pipe ie being considered

$\qquad\qquad\qquad A = \pi DL = \pi \times 0.02 \text{ m}^2$

$\therefore \qquad\qquad\qquad Q = h\pi \times 0.02 \times (40 - T_\infty)$

For case (i),

$\qquad\qquad\qquad h = 20 \text{ W/m}^2\text{K}, \ T_\infty = 50°C$

$\qquad\qquad\qquad Q = 20 \times \pi \times 0.02 \times (40 - 50) = -12.57 \text{ W}$

The negative sign indicates that the heat transfer is from the air to the cylinder.

For case (ii),

$\qquad\qquad\qquad h = 70 \text{ W/m}^2\text{K}, \ T_\infty = 30°C$

$\qquad\qquad\qquad Q = 70 \times \pi \times 0.02 \times (40 - 30) = 43.98 \text{ W}$

This result is positive which indicates the heat transfer to be occurring from the cylinder to the water.

Q.3 The outer surface temperature of a refrigerator is 16°C where h = 10 W/m²K and the room temperature is 20°C. The sides are 3 cm thick and k = 0.1 W/mK. Find the net heat flow and inside temperature of the refrigerator.

Solution:

Given data: $T_{s,0} = 16°$; $T_\infty = 20°C$, L = 0.03 cm, h = 10 W/m²K, k = 0.1 W/mK

Convective heat flux to the surface

$$\boxed{q = \frac{Q}{A} = h(T_{s,0} - T_\infty)} = 10(16 - 20) = -40 \text{ W/m}^2$$

Since this must be equal to the heat conducted through the sides,

$$q = -k\frac{dT}{dx} = -k\frac{T_{s,0} - T_{s,i}}{L}$$

∴ $$T_{s,i} = -\frac{qL}{k} + T_{s,0} = -\frac{40 \times 0.03}{0.1} + 16 = 4°C$$

Q.4 A steam pipe (O.D. = 10 cm, T_s = 500 K, ε = 0.8) passing through a large root at 300 K. The pipe loss heat by natural convection (h = 15 W/m²K) and radiation.
Find: (i) the surface emissive power of the pipe, (ii) the total radiation falling upon the pipe, and (iii) the total rate of heat loss from the pipe.

Solution:

Given data: D_0 = 10 cm; T_S = 500 K, T_R = 300 K, ε = 0.8

(i) Surface emissive power of the pipe,
$$E = \varepsilon\sigma T_S^4 = 0.8 \times 5.67 \times 10^{-8} \times (500)^4 = 2835 \text{ W/m}^2$$

(ii) Total radiation falling upon the pipe surface = Total radiation leaving the surface,
$$G = \sigma T_R^4 = 5.67 \times 10^{-8} \times (300)^4 = 459.27 \text{ W/m}^2$$

(iii) Heat loss from pipe by radiation,
$$Q_r = \varepsilon A\sigma(T_S^4 - T_R^4) = 0.8 \times \pi \times 0.1 \times 5.67 \times 10^{-8} \times (500^4 - 300^4) = 775.21 \text{W/m}^2$$

Heat loss by natural convection,
$$Q_c = h_c A(\Delta T) = 15 \times \pi \times 0.1 \times (500 - 300) = 942.5 \text{ W/m}$$

Thus, total rate of heat loss,
$$Q = Q_c + Q_r = 942.5 + 775.21 = 1717.71 \text{ W/m}$$

Q.5 A 0.2 m thick infinite black plate having a thermal conductivity of 3.96 W/m-K is exposed to two infinite black surfaces at 300 K and 400 K as shown in the figure. At steady state the surface temperature of the plate facing the cold side is 350 K. The value of Stefan-Boltzmann constant σ is
$$5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$
Assuming 1-D heat conduction, find the magnitude of heat flux through the plate (in W/m²).
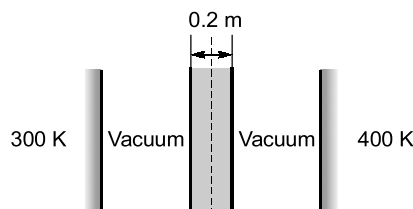
Solution:

Given data: $T_c = 300$ K; $T_h = 400$ K; $T_s = 350$ K

Under steady state condition, all rate of heat transfer i.e. from surface at 400 K to black plate (via radiation), inside black plate (via conduction) and from black plate to surface at 300 K (via radiation) are equal.

So, heat flux through wall = Radiation flux from wall to surface at 300 K

$$= \frac{\sigma(T_s^4 - T_c^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} \qquad \therefore \varepsilon_1 = \varepsilon_2 = 1$$

$$= \frac{5.67 \times 10^{-8}(350^4 - 300^4)}{\dfrac{1}{1} + \dfrac{1}{1} - 1} = 391.584 \text{ W/m}^2$$

Q.6 A coolant fluid at 30°C flows over a heated flat plate maintained at a constant temperature of 100°C. The boundary layer temperature distribution at a given location on the plate may be approximated as $T = 30 + 70 \exp(-y)$ where y (in m) is the distance normal to the plate and T is in °C. If thermal conductivity of the fluid is 1.0 W/mK, the local convective heat transfer coefficient (in W/m²K) at that location will be

Solution:

At $y = 0$; $q_{cond} = q_{conv}$

$$-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0} = h\Delta T = h(T_s - T_\infty)$$

$$-(1)\frac{\partial}{\partial y}\left[30 + 70\, e^{-y}\right]\Big|_{y=0} = h(100 - 30)$$

$$-\left[0 + 70(-1)\, e^{-y}\right]\Big|_{y=0} = h(70)$$

$$70\, e^{-0} = h(70)$$

or $h = 1$ W/m²K

Alternative :

Given data:

$T_\infty = 30$°C, $T_s = 100$°C, $k_f = 1$ W/mK,

$$T = 30 + 70 \exp(-y)$$

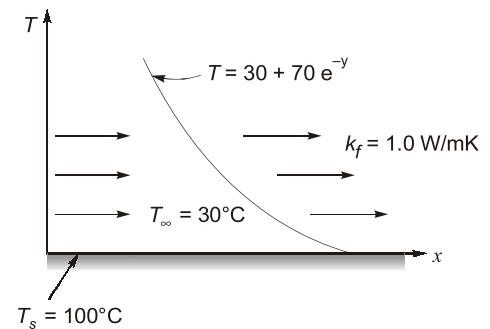Differentiating w.r.t y, we get

$$\frac{dT}{dy} = -70\, e^{-y}$$

At $y = 0$

$$\left(\frac{dT}{dy}\right)_{y=0} = -70$$

We know that local convective heat transfer coefficient:

$$\boxed{h_x = \frac{-k_f \left(\dfrac{dT}{dy}\right)_{y=0}}{T_s - T_\infty}} = \frac{-1 \times (-70)}{100 - 30} = 1 \text{ W/m}^2\text{K}$$

Q.7 A solid sphere of diameter 10 cm is heated to 1000°C and suspended in a room whose walls are at 30°C. Compute the following:

(i) Rate of heat transfer due to radiation only neglecting other losses.

(ii) Time taken by the sphere to cool to 500°C assuming emissivity for the sphere = 0.1 and density 8.68 gm/cc. Specific heat 0.098 J/kgK.

Solution:

Given data: Emissivity, ϵ = 0.1;   Density, ρ = 0.868 gm/cc = 868 kg/m³

               Specific heat, c_p = 0.098 J/kgK;   Diameter = 10 cm, T = 1000 + 273 = 1273 K

               Room temperature, T_0 = 30 + 273 = 30°C

Radiation heat transfer,

(i) $Q = \epsilon\sigma A(T^4 - T_0^4) = 0.1 \times 5.67 \times 10^{-8} \times 4\pi \times (0.05)^2 \times (1273^4 - 303^4) = 466.2$ W

(ii) Performing energy balance,

    Rate of change of Internal energy = Heat loss rate due to radiation

$$-mc\frac{dT}{dt} = \sigma\epsilon A(T^4 - 303^4)$$

$$-\frac{mc}{\sigma\epsilon A}\int_{1273}^{773}\frac{dT}{(T^4 - 303^4)} = \int_0^t dt$$

$$\Rightarrow -\frac{868 \times 4/3\pi(0.05)^3 \times 0.098 \times 10^3}{5.67 \times 10^{-8} \times 0.1 \times 4\pi \times (0.05)^2} \times \int_{1273}^{773}\frac{dT}{(T^4 - 303^4)} = t$$

$$-2.5 \times 10^{12} \times (-5.67 \times 10^{-10}) = t$$

$$\Rightarrow \qquad\qquad\qquad\qquad t = 141.84 \text{ seconds}$$

Practice Questions : Level-II

Q.8 The sun may be regarded as a black body with a surface temperature of 5600 K at a mean distance of 15×10^{10} m from the earth. The diameter of the sun is 1.4×10^9 m and that of the earth is 12.8×10^6 m. Make calculations for
(a) the total energy radiated by the sun,
(b) the energy received per m² just outside the earth's atmosphere,
(c) the total energy the earth would receive if no energy were blocked by the earth's atmosphere,
(d) the energy received by a 1.25 × 1.25 m solar collector whose perpendicular is inclined at 35° to the sun. The energy loss through the atmosphere is 35% and the diffuse radiation is 15% of direct radiation.

Solution:

Given data: T = 5600 K

(a) For the sun: ϵ = 1 (Black body)   and   Surface area = $4\pi r^2 = 4\pi(0.7 \times 10^9)^2$

    $\therefore$ Energy radiated by the sun,

$$\boxed{Q = \epsilon\sigma_b AT^4} = 1 \times (5.67 \times 10^{-8}) \times 4\pi (0.7 \times 10^9) \times (5600)^4 = 3.43 \times 10^{26} \text{ W}$$

(b) The sun may be regarded as a point source at a distance of 15×10^{10} from the earth. The mean area over which the radiation is distributed becomes $4\pi(15 \times 10^{10})^2$

    $\therefore$ Radiation received at this distance

$$= \frac{3.43 \times 10^{26}}{4\pi(15 \times 10^{10})^2} = 1.213 \times 10^3 \text{ W/m}^2$$

(c) The earth is nearly spherical and as such the energy received by it will be proportional to the perpendicular projected area, i.e., that of a circle.

$\therefore$ Energy received by the earth = $1.213 \times 10^3 \times \pi (6.4 \times 10^6)^2 = 1.56 \times 10^{17}$ W

(d) Direct energy reaching the earth,

$$= \left(1 - \frac{35}{100}\right) \times 1.213 \times 10^3 = 788.45 \times 10^3 \text{ W/m}^2$$

Diffused radiation, $= \dfrac{15}{100} \times 0.788 \times 10^3 = 118.2675 \text{ W/m}^3$

Total radiation reaching the plate,

$$788.45 + 118.2675 = 906.7175$$

Since the plate surface is not oriented perpendicular to the incoming radiations, the relevant area is equivalent to the projected perpendicular surface area.

Projected plate area = $A \cos\theta = 1.25 \times 1.25 \times \cos 35 = 1.28 \text{ m}^2$

$\therefore$ Energy received by the plate = $906.7174 \times 1.28 = 1160.6$ W

A reduction in energy received due to inclination explains the variation in solar intensity with season and much reduced solar intensity at the poles of earth.

Q.9 Assuming the sun as a black body, it emits maximum radiation at 0.5 µm wavelength. Calculate
(i) the surface temperature of the sun,
(ii) its emissive power,
(iii) the energy received by the surface of the earth and
(iv) the energy received by a 2 m × 2 m solar collector whose normal is inclined at 60° to the sum. Take the diameter of the sun as 1.4×10^9 m, diameter of the earth as 13×10^6 m and the distance of the earth from the sun as 15×10^{10} m.

Solution:

Given data: $\lambda_{max} = 0.5 \text{ µm};\quad \theta = 60°;\qquad A = 2 \times 2 = 4 \text{ m}^2$

$$\lambda_{max} = 0.5 \text{ µm}$$

From Wien's displacement law,

$$\lambda_{max} T = 2898 \times 10^{-6} \text{ mK}$$

$\therefore$ Surface temperature of the sun,

$$T = \frac{2898 \times 10^6}{0.5 \times 10^{-6}} = 5796 \text{ K} \qquad \dots \text{(i)}$$

Emissive power of the sun, a black body, is obtained from Stefan - Boltzmann law:

$$E_b = \sigma T^4 = 5.67 \times 10^{-8} (5796)^4 = 63987.7 \text{ kW/m}^2 \qquad \dots \text{(ii)}$$

Radiation reaching the earth would be = Emissive power of the sun $\times \left(\dfrac{\text{Radius of the sun}}{\text{Distance from the earth}}\right)^2$

$$= 63987.7 \times \left(\frac{0.7 \times 10^9}{15 \times 10^{10}}\right)^2 = 1.39 \text{ kW/m}^2 \qquad \dots \text{(iii)}$$

Surface area fo the solar collector in the direction normal to solar radiation

$$= A\cos\theta = 4\cos 60° = 2 \text{ m}^2$$

$\therefore$ Energy received by the solar collector = **1.39 × 2 = 2.78 kW**

■■■■