

# POSTAL BOOK PACKAGE 2024

## MECHANICAL ENGINEERING

CONVENTIONAL Practice Sets

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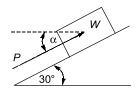
### **ENGINEERING MECHANICS**

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## **Equilibrium of Forces and Moment**

#### **Practice Questions**

Determine the magnitude and direction of the smallest force P, which will maintain the body of weight W = 300 N on an inclined smooth plane as shown in figure is in equilibrium.



#### **Solution:**

The body is acted upon by three forces, namely the action of gravity force W, the applied force P and the reaction R. Since these three forces are in equilibrium, the vectors representing them must build a closed triangle, we begin with the known vector  $\overline{bc}$  representing to a certain scale, the weight of the body, and then draw the line as parallel to the R.

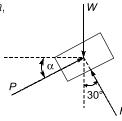
The side  $\overline{cd}$  will be minimum if it is perpendicular to line aa,

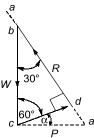
that is P will be minimum, if it is perpendicular to aa.

From the triangle bcd, 
$$\angle c = 90^{\circ} - 30^{\circ} = 60^{\circ}$$

$$\alpha = 90^{\circ} - 60^{\circ} = 30^{\circ}$$

and using the triangle bcd, we obtain,





$$P = W \sin 30^{\circ} = \frac{W}{2} = 150 \text{ N}$$

Alternate solution: After drawing the free-body diagram of the body of above, then applying the Lami's theorem to the free-body diagram of the body as shown in figure we get

$$\frac{W}{\sin(90^{\circ} - \alpha + 30^{\circ})} = \frac{P}{\sin(\pi - 30^{\circ})} = \frac{R}{\sin(90^{\circ} + \alpha)}$$

Using the first two of the equation we obtain

$$\frac{W}{\cos(30^{\circ} - \alpha)} = \frac{P}{\sin 30^{\circ}}$$

$$P = \frac{W \sin 30^{\circ}}{\cos(30^{\circ} - \alpha)}$$

From equation, P will be minimum, if the denominator is maximum, i.e.

$$\cos(30^{\circ} - \alpha) = 1$$

$$\Rightarrow 30^{\circ} - \alpha = 0$$

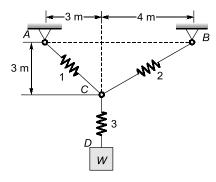
$$\Rightarrow \alpha = 30^{\circ}$$

and substituting this value into equation, we get the value of

$$P = W \sin 30^{\circ} = 150 \text{ N}$$
, as before



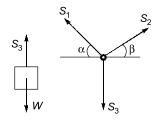
Q2 Determine the stretch in each spring for equilibrium of the weight W = 40 N block as shown in figure. The springs are in equilibrium position. The stiffness of each spring is given as:  $k_1 = 40 \text{ N/m}$ ,  $k_2 = 50 \text{ N/m}$ , and  $k_3 = 60 \text{ N/m}$ 



#### **Solution:**

Draw the free-body diagram of the body as shown in figure.

Only two forces are acting on the body, gravity force W and the reactive force caused by the spring  $S_3$ . Since the body is in equilibrium, from the law of equilibrium of two forces,



$$S_3 = W$$

Now, draw the free-body diagram of the point C. At the joint, C three forces are acting all are reactive forces caused by the springs. The angles that springs  $S_1$  and  $S_2$  make with the horizontal are calculated as below:

$$\tan \alpha = \frac{3}{3} = 1 \Rightarrow \alpha = 45^{\circ}$$

$$\tan \beta = \frac{3}{4} \Rightarrow \beta = 36.87^{\circ}$$

Since the joint C is in equilibrium, applying Lami's theorem, we obtain

$$\frac{S_1}{\sin\left(\frac{\pi}{2} + \beta\right)} = \frac{S_2}{\sin\left(\frac{\pi}{2} + \alpha\right)} = \frac{S_3}{\sin(\pi - \alpha - \beta)}$$

From equation we get

$$S_1 = \frac{S_3 \cos \beta}{\sin(\alpha + \beta)} = \frac{W \cos \beta}{\sin(\alpha + \beta)}$$

$$S_2 = \frac{S_3 \cos \alpha}{\sin(\alpha + \beta)} = \frac{W \cos \alpha}{\sin(\alpha + \beta)}$$

$$EF = EC + CF = r_1 + r_2 = 100 + 50 = 150 \text{ mm}$$
and
$$EH = OI - OG - BI$$

$$OI = a = 200 \text{ mm}$$

$$OG = r_2 = 50 \text{ mm}$$

$$BI = EI \sin \frac{\alpha}{2} \left[ \because EI = \frac{BE}{\cos \frac{\alpha}{2}} = \frac{r_1}{\cos 30^\circ} = \frac{100}{\cos 30^\circ} = 115.47 \text{ mm} \right]$$

$$\therefore BI = 115.47 \sin 30^\circ = 57.74 \text{ mm} \text{ and}$$

$$\therefore EH = 200 - 50 - 57.74 = 92.26 \text{ mm}$$

$$EH = 200 - 50 - 57.74 = 92.26 \text{ mm}$$

$$EH = 220 - 50 - 57.74 = 92.26 \text{ mm}$$

$$\cos \beta = \frac{EH}{EF} = \frac{92.26}{150} = 0.615$$



$$\beta = 52.05^{\circ}$$

$$R_c \cos \beta = R_d$$

$$R_c \sin \beta = Q$$

Substituting the values for  $\beta$  and Q in the above equations and solving for  $R_c$  and  $R_{d'}$  we obtain

$$R_c = \frac{Q}{\sin\beta} = \frac{800}{\sin 52.05} = 1014.52 \,\mathrm{N}$$

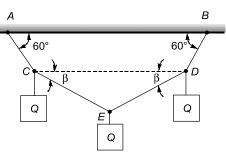
$$R_d = R_c \cos \beta = 1014.52 \times \cos 52.05^\circ = 623.9 \text{ N}$$

$$R_a = R_c \frac{\cos \beta}{\sin \alpha} = 1014.52 \times \frac{\cos 52.05}{\sin 60} = 720.42 \text{ N}$$

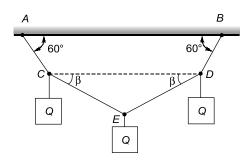
$$R_b = R_c \sin \beta + P - R_a \cos \alpha$$

= 
$$1014.52 \times \sin 52.05^{\circ} + 2000 - 720.42 \cos 60^{\circ} = 2439.79 \text{ N}$$

Q3 On the string ACEDB are hung three equal weights Q symmetrically placed with respect to the vertical line through the mid-point E. Determine the value of the angles b if the other angles are as shown in the figure.

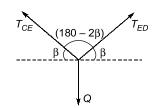


#### **Solution:**



At point *E*,

By symmetry,

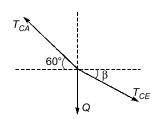


...(ii)

$$T_{CE} = T_{ED}$$
Lami's theorem
$$\frac{T_{CE}}{Sin(90 + \beta)} = \frac{T_{ED}}{\sin(90 + \beta)} = \frac{Q}{\sin(180 - 2\beta)}$$

$$T_{CE} = \frac{Q \cos \beta}{\sin 2\beta} = \frac{Q}{2 \sin \beta} \qquad ...(i)$$

At point C:



$$\frac{T_{CA}}{\sin(90-\beta)} = \frac{T_{CE}}{\sin 150} = \frac{Q}{\sin(120+\beta)}$$

$$T_{CE} = \frac{Q \times \sin 150}{\sin(120 + \beta)}$$



By equation (i) and (ii) 
$$\frac{Q}{2\sin\beta} = \frac{Q\times 1/2}{\sin(120+\beta)}$$
 
$$\sin(120+\beta) = \sin\beta$$
 
$$\sin[90+(30+\beta)] = \sin\beta$$
 
$$\cos\beta \cdot \cos 30 - \sin\beta \cdot \sin 30 = \sin\beta$$
 
$$\cos\beta \times \frac{\sqrt{3}}{2} = \sin\beta + \frac{1}{2}(\sin\beta)$$
 
$$\frac{\cos\beta}{\sin\beta} = \frac{\frac{3}{2}}{\left(\frac{\sqrt{3}}{2}\right)}$$
 
$$\left(\frac{\cos\beta}{\sin\beta}\right) = \sqrt{3}$$
 
$$\tan\beta = \frac{1}{\sqrt{3}}$$
 
$$\beta = 30^{\circ}$$

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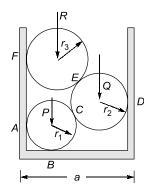
#### Alternate:

$$\sin [180 - (120 + \beta)] = \sin \beta$$

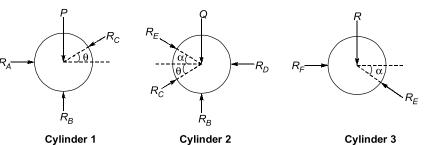
comparing on both sides

$$180 - 120 - \beta = \beta$$
$$60 = 2 \beta$$
$$\beta = 30^{\circ}$$

Q.4 The smooth cylinders rest in a horizontal channel having vertical walls, the distance between which is a. Find the pressures exerted on the walls and floor at the points of contact A, B, D and F. the following numerical data are given: P = 200 N, Q = 400 N, R = 300 N,  $r_1 = 120 \text{ mm}$ ,  $r_2 = 180$  mm,  $r_3 = 150$  mm and a = 540 mm.



#### **Solution:**



For cylinder 2:

$$\cos \alpha = \frac{540 - 180 - 150}{180 + 150}$$

$$\alpha = 50.47^{\circ}$$

$$\frac{R}{\sin(180 - 50.47)^{\circ}} = \frac{R_E}{\sin 90^{\circ}} = \frac{R_F}{\sin(90 + 50.47)^{\circ}}$$

...(i)

...(ii)

$$R_E = \frac{R}{\sin(129.53^\circ)} = 388.96 \, \mathrm{N}$$

$$R_F = \frac{R \times \sin(140.47^\circ)}{\sin(129.53^\circ)} = 247.565 \, \mathrm{N}$$
For cylinder 1: 
$$\cos \theta = \frac{540 - 120 - 180}{120 + 180}$$

$$\theta = 36.87^\circ$$
Now 
$$R_C \cos \theta = R_A$$

$$P + R_C \sin \theta = R_B$$
Now, 
$$R_C = 1.25 \, R_A$$

$$P + 0.6 \, R_C = R_B \Rightarrow P + 0.75 \, R_A = R_B$$
For cylinder 2: 
$$Q + R_E \sin \alpha = R_C \sin \theta$$
Now, 
$$R_C = \frac{400 + 388.96 \times \sin50.47^\circ}{\sin36.87^\circ}$$

$$R_C = \frac{400 + 388.96 \times \sin50.47^\circ}{\sin36.87^\circ}$$

$$R_C = 1166.67 \, \mathrm{N}$$

$$R_D = R_E \cos \alpha + R_C \cos \theta$$

$$R_D = 388.96 \cos 50.47^\circ + 1166.67 \cos 36.87^\circ$$

$$R_D = 1180.9 \, \mathrm{N}$$
For cylinder 1: 
$$R_A = \frac{R_C}{1.25} = \frac{1166.67}{1.25} = 933.336 \, \mathrm{N}$$

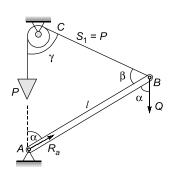
 $R_B = P + 0.75 R_A$ 

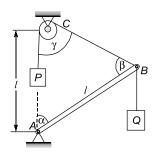
 $R_{\rm R} = 900 \, \rm N$ 

 $= 200 + 0.75 \times 933.336$ 

Q5 A prismatic bar AB of negligible weight and length I is hinged at A and supported at B by a string that passes over a pulley C and carries a load P at its free end. Assuming that the distance h between the hinge A and the pulley C is equal to the length I of the bar, find the angle α at which the system will be in equilibrium.

#### **Solution:**





The triangle ABC is isosceles

$$\beta = \gamma = \frac{\pi - \alpha}{2} = 90^{\circ} - \left(\frac{\alpha}{2}\right)$$

Taking point A as the moment center (thus eliminating consideration of the unknown reaction at A), we obtain



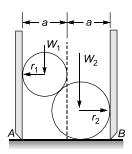
$$(l)P\sin\left(90^{\circ} - \frac{\alpha}{2}\right) - (l)(Q\sin\alpha) = 0$$

$$P\cos\frac{\alpha}{2} - (Q\sin\alpha) = 0$$

$$P\cos\frac{\alpha}{2} - \left(Q \times 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right) = 0$$

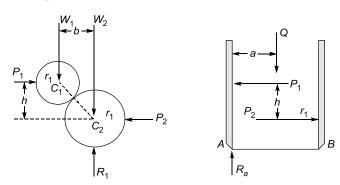
$$\Rightarrow \sin\frac{\alpha}{2} = \frac{P}{2Q}$$

A hollow right circular cylinder of radius a is open at both ends and rests on a smooth horizontal plane as shown in figure. Inside the cylinder there are two spheres having weights  $W_1$  and  $W_2$  and radii  $r_1$  and  $r_2$  respectively. The lower sphere also rests on the horizontal plane. Neglecting friction, find the minimum weight Q of the cylinder in order that it will not tip over.



#### **Solution:**

Let us consider first a free-body for the two sphere, assuming that they are joined together at their point of contact as shown in figure. By virtue of the assumptions of smooth surfaces, we conclude that the reactive forces  $P_1$  and  $P_2$  exerted on the spheres by the walls of the cylinder are horizontal forces as shown and likewise that the reaction  $P_2$  on the bottom of the lower sphere is a vertical force.



Thus the two spheres are in equilibrium under the action of the five coplanar forces shown in figure. Using the first and last of equation with  $C_2$  as a moment center, we obtain

$$P_1 - P_2 = 0,$$
  
 $W_1 b - P_1 h = 0$ 

from which

$$P_1 = P_2 = \frac{W_1 b}{h}$$

Taking moment about point A,

$$Qa = W_1b$$