SSC-JE 2025

Staff Selection Commission

Junior Engineer Examination

Mechanical Engineering

Engineering Mechanics

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

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Engineering Mechanics

Contents

Chapter 01

Composition, Resolution and Equilibrium of Forces

1.1 Force

Force is the action of one body on another. It may be defined as an action which changes or tends to change the state of rest or of uniform motion of body. For representing the force acting on the body, the magnitude of the force, its point of action and direction of its action should be known. There are different types of forces such as gravitational, frictional, magnetic, inertia or those caused by mass and acceleration.

According to Newton's second law of motion, we can write force as

$$F = ma = \text{mass} \times \frac{\text{length}}{\text{time}^2}$$

One Newton force is defined as that which gives an acceleration of 1 m/s^2 to a body of mass of 1 kg in the direction of force.

Thus, $1 \text{ N} = 1 \text{ kg} \times 1 \text{ m/s}^2 = 1 \text{ kg-m/s}^2$

The action of one body and another, which changes or tends to change the state of rest or of uniform motion of body is called as force.

The three requisites for representing the force acting on the body are:

- Magnitude of force
- Its point of action, and
- Direction of its action

1.2 Effects of a Force

A force may produce the following effects in a body, on which it acts:

1. It may change the motion of a body i.e. if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate or retard it.
2. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.
3. It may give rise to the internal stresses in the body, on which it acts.

1.3 Characteristics of a Force

To know the effect of force on a body, the following elements of force should be known.

1. Magnitude (i.e. 2 N, 5 kN, 10 kN etc.)
2. Direction or line of action.
3. Sense or nature (push or pull).
4. Point of application.

1.4 Force Systems

A force system is collection of forces acting on a body in one or more planes. According to the relative position of the lines of action of the forces, the forces may be classified as follows:

1. **Collinear:** The forces whose lines of action lie on the same line are known as collinear forces.
2. **Concurrent:** The forces, which meet at one point, are known as concurrent forces. Concurrent forces may or may not be collinear.
3. **Coplanar:** The forces whose line of action lie on the same plane are known as coplanar forces.
4. **Coplanar concurrent:** The forces, which meet at one point and their line of action lie on the same plane, are known as coplanar concurrent forces.
5. **Non-coplanar concurrent:** The forces, which meet at one point but their lines of action do not lie on the same plane, are known as coplanar non-concurrent forces.
6. **Coplanar non-concurrent:** The forces, which do not meet at one point but their line of action lie on the same plane, are known as coplanar non-concurrent forces.
7. **Non-coplanar non-concurrent:** The forces, which do not meet at one point and their line of action do not lie on the same plane, are known as non-coplanar non-concurrent forces.

1.5 Resultant Force

A single force which produces same effect on the body as the system of forces is called as resultant force.

1.6 Parallelogram Law of Forces

This law is used for finding the resultant of two forces acting at a point.

If two forces F_1 and F_2 are acting at a point and are represented in magnitude and direction by two sides of a parallelogram, then their resultant is represented by the diagonal of the parallelogram both in magnitude and direction.

Consider a parallelogram *OACB* as shown in figure 1.1 where sides *OA* and *OB* represent the forces F_1 F_2 acting at a point *O*. According to the parallelogram law of forces, the resultant *R* is represented by a diagonal *OC*.

Let θ be the angle between the forces F_1 and F_2 and α be the angle made by *R* with force F_1.

From the figure 1.1 we can write

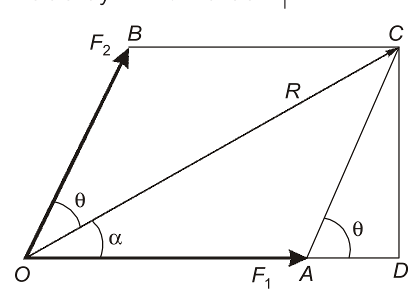


Figure 1.1

$$BC = OA = F_1$$
$$AC = OB = F_2$$
$$\angle BOA = \theta = \angle CAD$$

and ΔODC and ΔADC are right angle triangles.

From triangle *ADC*, we can write

$$AD = AC\cos\theta = F_2\cos\theta$$
$$CD = AC\sin\theta = F_2\sin\theta$$

From triangle *ODC*, we can write

$$OC^2 = OD^2 + CD^2 = (OA + AD)^2 + CD^2$$
$$R^2 = (F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2$$
$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2\cos^2\theta + F_2^2\sin^2\theta$$
$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2(\cos^2\theta + \sin^2\theta)$$
$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2$$
$$R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2} \quad \text{... (i)}$$

From triangle *ODC*,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta} \quad \text{... (ii)}$$

Thus

$$R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2}$$

and

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

1.7 Triangle Law of Forces

This law states that:

If two forces acting simultaneously on a body are represented in magnitude and direction by two sides of a triangle taken in order then their third side will represent the resultant of two forces in the direction and magnitude taken in opposite order.

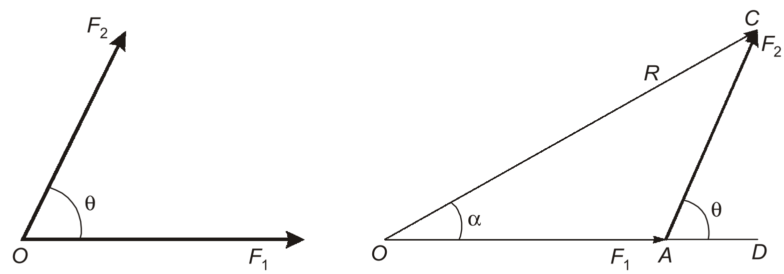


Figure. 1.2

If three forces are acting on a body and they are represented by three sides of the triangle in magnitude and direction, then the body will be in equilibrium condition.

1.8 Polygon Law of Forces

When two more forces are acting on the body, the triangle law can be extended to polygon law.

If a number coplanar concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, then their resultant can be represented by closing side of the polygon in magnitude and direction in the opposite order.

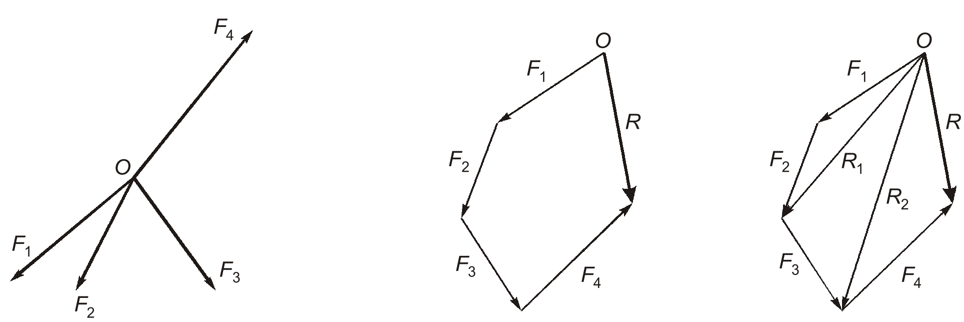


Figure. 1.3

Consider the forces F_1, F_2 and F_3 acting at a point O as shown in figure 1.3. As per the polygon law of forces the resultant force R is as shown in figure 1.3. According to parallelogram law, then the resultant of F_1 and F_2 is represented by R_1 and resultant of R_1 and F_3 is represented by R_2. The resultant R is the resultant of F_4 and R_2. This procedure can be extended to any number of forces acting at a point in a plane.

1.9 Composition of Forces

Conversion of system of forces into an equivalent single force system is known as the composition of forces. The effect of single equivalent force will be same as the effect produced by number of forces action on a body.

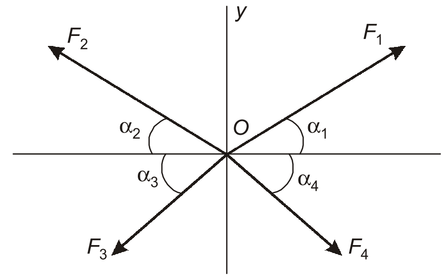


Figure 1.4

Let the forces F_1, F_2, F_3, F_4 are acting on a body in a plane making angle α_1, α_2, α_3 and α_4 with x-axis as shown in figure 1.4. Let R be the resultant force of all the forces acting at the point making an angle θ with horizontal as shown in figure. Resolving the forces along x-axis and y-axis, we get

$$\Sigma F_x = F_1 \cos \alpha_1 - F_2 \cos \alpha_2 - F_3 \cos \alpha_3 + F_4 \cos \alpha_4$$
$$\Sigma F_y = F_1 \sin \alpha_1 + F_2 \sin \alpha_2 - F_3 \sin \alpha_3 - F_4 \sin \alpha_4$$

Component of R along x-axis = $R\cos\theta$

Component of R along y-axis = $R\sin\theta$

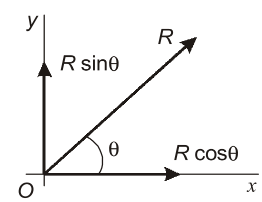


Figure 1.5

$$R\cos\theta = \Sigma F_x$$

and $$R\sin\theta = \Sigma F_y$$

$$R^2(\sin^2\theta + \cos^2\theta) = (\Sigma F_x)^2 + (\Sigma F_y)^2$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and $$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

A body which is under co-planar system of concurrent forces is in equilibrium if $R = 0$ or

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

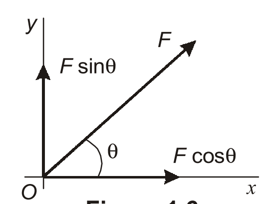


Figure 1.6

1.10 Resolution of Forces

Replacing force F by two forces along x and y axis acting on the same body is called resolution of forces. Resolution is the reverse process of composition.

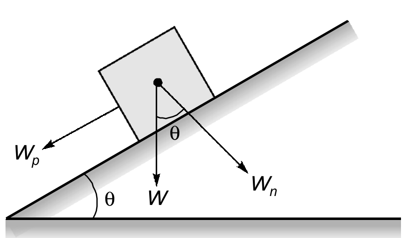


Figure 1.7

Case I: A force F acting at a point 'O' making angle θ with horizontal as shown in figure 1.6. Then its components along x and y axis are given by

$F_x = F\cos\theta$ and $F_y = F\sin\theta$

Case II: The resolution of force W when the body is on an inclined plane. The components of the body force W are given by

$W_n = W\cos\theta$ and $W_p = W\sin\theta$

where W_n is normal component to inclined plane and W_p is parallel component to inclined plane.

1.11 Equilibrium of Forces

If a body is moving at a constant velocity or the body is at rest then the body is said to be in equilibrium in a state. If a number of forces are acting on the body and its resultant comes out to be zero, then the body is said to be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces.

1.12 Principles of Equilibrium

Three important principles of equilibrium are:

1. **Two force principle.** If a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.
2. **Three force principal.** If body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force or in other words forces must be coplanar and concurrent.
3. **Four force principle.** If a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

1.13 Lami's Theorem

If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two.

Mathematically, $$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

where, P, Q and R are three forces and α, β and γ are the angles as shown in figure 1.8.

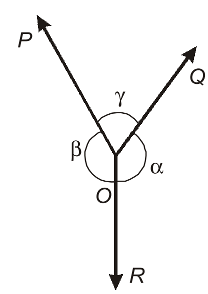


Figure. 1.8

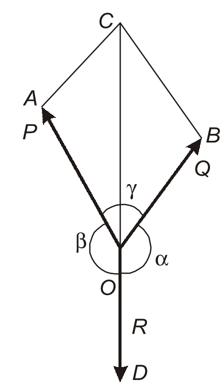


Figure. 1.9

Proof of Lami's Theorem

Consider three coplanar forces P, Q and R acting at a point O as shown in figure 1.8. Now complete the parallelogram $OACB$ with OA and OB as adjacent sides as shown in the figure 1.9. The resultant of two forces P and Q is diagonal OC both in magnitude and direction of the parallelogram $OACB$.

Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

From the geometry of the figure,

$$BC = P \text{ and } AC = Q$$

$$\angle AOC = (180° - \beta)$$

and $$\angle ACO = \angle BOC = (180° - \alpha)$$

$$\angle CAO = 180° - (\angle AOC + \angle ACO) = 180° - [(180° - \beta) + (180° - \alpha)]$$

$$= 180° - 180° + \beta - 180° + \alpha$$

$$\angle CAO = \alpha + \beta - 180° \quad \text{... (i)}$$

But $$\alpha + \beta + \gamma = 360°$$

or $$\alpha + \beta + \gamma - 180° = 360° - 180° = 180°$$

$$(\alpha + \beta - 180°) + \gamma = 180° \quad \text{... (ii)}$$

From equation (i) and (ii) we get,

$$\angle CAO = 180° - \gamma$$

We know that in triangle AOC

$$\frac{OA}{\sin\angle ACO} = \frac{AC}{\sin\angle AOC} = \frac{OC}{\sin\angle CAO}$$

$$\frac{OA}{\sin(180° - \alpha)} = \frac{AC}{\sin(180° - \beta)} = \frac{OC}{\sin(180° - \gamma)}$$

or $$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$ Hence Proved

1.14 Free Body Diagram

A body may consist of more than one element and supports. Each element or support can be isolated from the rest of the system by incorporating the net effect of the remaining system through a set of forces. This diagram of the isolated element of a portions of the body along with the net effects of the system on it is called free body diagram.

The diagram shows all forces applied to the system by mechanical contact with other bodies, which are imagined to be removed. If appreciable body force are present, such as gravitational or magnetic attraction, then these force must also be shown on the free-body diagram of the isolated system.

The free-body diagram is the most important single step in the solution of problems in mechanics.

Example 1.1 Consider the following statements:

For a particle in plane in equilibrium

1. sum of the forces along X-direction is zero.
2. sum of the force along Y-direction is zero.
3. sum of the moments of all forces about any point is zero.

Of these statements

(a) 1 and 3 are correct (b) 2 and 3 are correct

(c) 1 and 2 are correct (d) 1, 2 and 3 are correct

Solution: **(d)**

Example 1.2 Consider the following statements:

A particle starting from rest is accelerating along a straight line with an acceleration *kt* where *k* is a constant and *t* is the time elapsed after time '*t*'.

1. its velocity is given by kt^2.
2. its velocity is given by 1/2 kt^2.
3. the distance covered is given by 1/2 kt^3.
4. the distance covered is given by 1/6 kt^3.

Of these statements

(a) 1 and 3 are correct (b) 2 and 4 are correct

(c) 1 and 4 are correct (d) 2 and 3 are correct

Solution: **(b)**

$$a = kt$$

$$\frac{d^2x}{dt^2} = kt\,; \qquad \frac{dx}{dt} = \frac{kt^2}{2} + C_1$$

At time $t = 0$, particle starts from rest, so $\dfrac{dx}{dt} = 0$, so $C_1 = 0$

$$\frac{dx}{dt} = \frac{kt^2}{2}$$

$$x = \frac{kt^3}{6} + C_2$$

At $t = 0$, $x \to 0$, $C_2 = 0$, $\qquad x = \dfrac{kt^3}{6}$

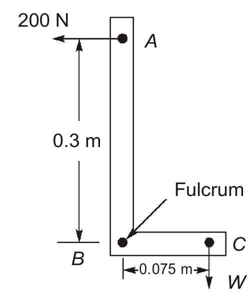
Example 1.3 If two forces P and Q at angle θ, the resultant of these two forces would make an angle α with P such that

(a) $\tan\alpha = \dfrac{Q\sin\theta}{P - Q\sin\theta}$ (b) $\tan\alpha = \dfrac{P\sin\theta}{P + Q\sin\theta}$

(c) $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$ (d) $\tan\alpha = \dfrac{P\sin\theta}{Q - P\cos\theta}$

Solution: **(c)**

Example 1.4 A horizontal force of 200 N is applied at 'A' to lift the weight 'W' at C as shown in the given figure. The value of weight 'W' will be



(a) 200 N (b) 400 N

(c) 600 N (d) 800 N

Solution: **(d)**

Taking moment about fulcrum:

$$200 \times 0.3 = W \times 0.075$$
$$W = 800 \text{ N}$$

Example 1.5 If the maximum and minimum resultant forces of the two forces acting on a particle are 40 kN and 10 kN respectively, then the two forces in question would be

(a) 25 kN and 15 kN (b) 20 kN and 20 kN

(c) 20 kN and 10 kN (d) 20 kN and 5 kN

Solution: **(a)**

Let P and Q be the two forces inclined at angle θ.

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

Maximum value of $\cos\theta = 1$; Minimum value of $\cos\theta = -1$

$$R_{max}^2 = P^2 + Q^2 + 2PQ \Rightarrow R_{max} = P + Q$$
$$R_{min}^2 = P^2 + Q^2 - 2PQ \Rightarrow R_{min} = P - Q$$
$$40 = P + Q \quad \text{...(i)}$$
$$10 = P - Q \quad \text{...(ii)}$$

Solving equation (i) and (ii), we get, $P = 25$ kN, $Q = 15$ kN

Example 1.6 Consider the following statements:

1. Two couples in the same plane can be added algebraically
2. Coplanar and concurrent forces are the ones which do neither lie in one plane nor meet at a point
3. Non-concurrent forces are the ones which do not meet at a point
4. A single force may be replaced by a force and couple

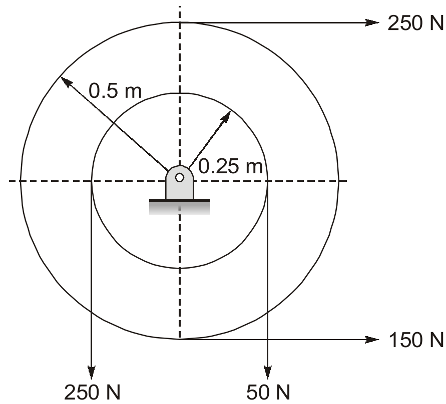
Which of these statements are correct?

(a) 1, 2 and 4 (b) 2, 3 and 4

(c) 1, 2 and 3 (d) 1, 3 and 4

Solution: **(d)**

Example 1.7 A differential pulley is subjected to belt tensions as shown in the diagram. The resulting force and moment when transferred to the centre of the pulley are, respectively



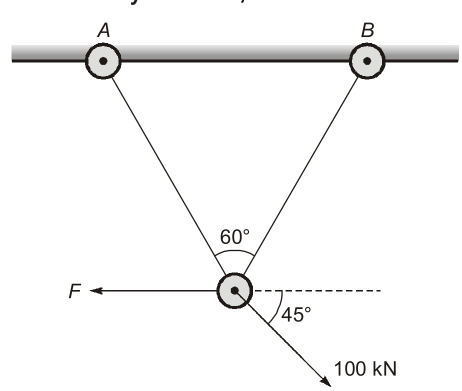
(a) 400 N and 0 Nm (b) 400 N and 100 Nm
(c) 500 N and 0 Nm (d) 500 N and 100 Nm

Solution: **(c)**

$$\Sigma F_x = 250 + 150 = 400 \text{ N}$$
$$\Sigma F_y = 250 + 50 = 300 \text{ N}$$

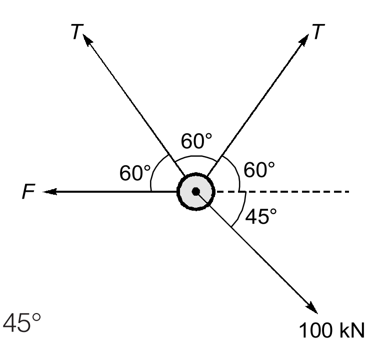
Resultant force,
$$R = \left(F_x^2 + F_y^2\right)^{1/2} = (400^2 + 300^2)^{1/2} = 500 \text{ N}$$
$$\Sigma M_o = -250 \times 0.5 - 50 \times 0.25 + 150 \times 0.5 + 250 \times 0.25$$
$$= -125 - 12.5 + 75 + 62.5 = 0 \text{ Nm}$$

Example 1.8 The force F such that both the bars AC and BC (AC and BC are equal in length) as shown in the figure are identically loaded, is



(a) 70.7 N (b) 100 N
(c) 141.4 N (d) 168 N

Solution: **(a)**



$$\Sigma F_x = 0$$
$$F + T\cos 60° = T\cos 60° + 100 \times \cos 45°$$
$$F = 100\cos 45° = 70.7 \text{ kN}$$