SSC-JE 2025

Staff Selection Commission

Junior Engineer Examination

Mechanical Engineering

Theory of Machines

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY PUBLICATIONS

Theory of Machines

Contents

CHAPTER 01

Mechanism and Machines

1.1 Introduction

If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motions to the others it is known as a **mechanism**. A **machine** is a mechanism or a combination of mechanism which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work. It is neither a source of energy nor a producer of work but helps in proper utilization of the same.

Example : A slider crank chain is a mechanism that converts reciprocating motion into rotary motion of the crank or vice-versa. But when it is used in an automobile by adding valve mechanism, CAM mechanism etc., it becomes a machine.

1.2 Types of Constrained Motion

(i) **Completely Constrained Motion :** When motion between two elements of a pair is in a definite (***single***) direction irrespective of the direction of force applied. It is known as completely constrained motion.
e.g., sliding pair.

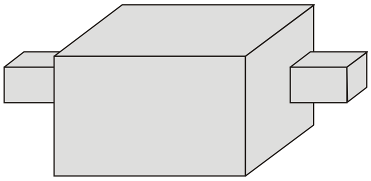


Figure 1.1

(ii) **Successfully Constrained Motion :** When motion between two element of a pair is ***possible*** in ***more than one*** direction but is made to have motion ***only in one*** direction by using some ***external*** means, it is called successfully constrained motion.

e.g., a piston in a cylinder of an internal combustion engine is made to have only reciprocating motion due to constrain of the piston pin (external), cam and follower, shaft in foot step bearing.

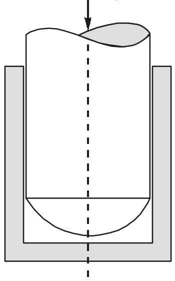


Figure 1.2

(iii) **Incompletely Constrained Motion :** When the motion between the elements of a pair is possible in ***more than one direction*** and depends upon the ***direction of force applied***, it is known as incompletely constrained motion e.g. cylindrical shaft in round bearing.

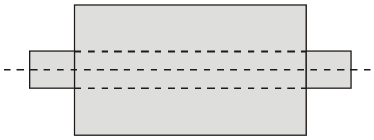


Figure 1.3

Rigid Body, Resistant Body

- Rigid body: It does not suffer any distortion, under the action of force.
- Resistant body: Those body which are rigid for the purpose they have to serve for e.g. belt drive, where belt is rigid when subjected to tensile forces.

1.3 Link

A link is defined as a member of mechanism, connecting other member and having motion relative to them.

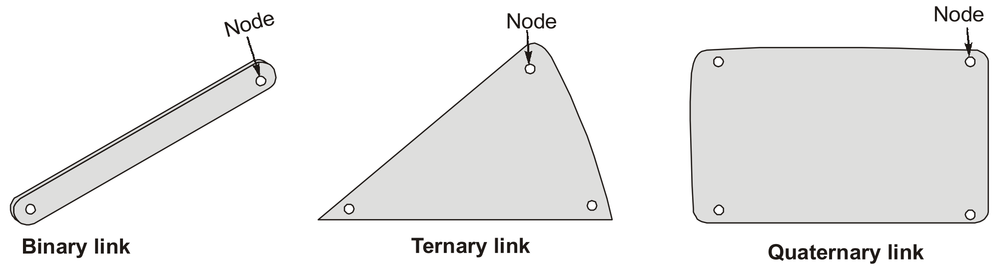


Figure 1.4: Types of Links

1.4 Kinematic Pair and Their Classification

A kinematic pair or simply a pair is a joint of two links having a constrained relative motion between them.

(i) According to nature of contact

- Lower Pair: A pair of links having surface or area contact between the members is known as lower pair. The contact surfaces of the two links are similar. Such as, Nut turning on a screw, shaft rotating in a bearing, universal Joint etc.
- Higher Pair: When a pair has a point or line contact between the members is known as a higher Pair. The contact surfaces of two links are dissimilar. Such as, wheel rolling on a surface, cam and follower pair, tooth gears, ball and roller bearings etc.

(ii) According to nature of Mechanical constraint

- Closed Pair: When the elements are geometrically identical, one is solid and full and other is hollow or open. The contact between the two can be broken only by destruction of at least one of the members. This means that elements of a pair are held together mechanically and there is a permanent contact. All the lower pairs and some of the higher pairs are closed pairs. These are also called self closed pair.

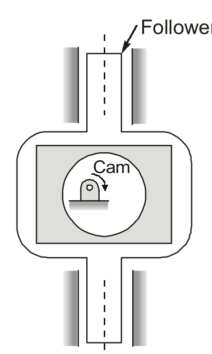


Figure 1.5

- Unclosed Pair: When two links of a pair are in contact either due to forces of gravity or some spring action, is called an unclosed pair. Two elements may or may not be geometrically identical. Example : Cam and follower pair, door lock mechanism, automatic clutch operating system. These are also called open or forced closed pairs.

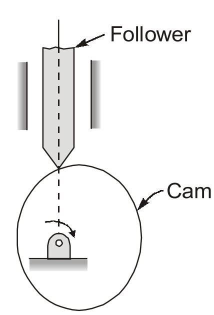


Figure 1.6

(iii) According to Nature of Relative Motion

- **Sliding Pair:** If two links have a sliding motion relative to each other, they form a sliding pair. Such as a rectangular rod in a rectangular hole in a prism is a sliding pair.

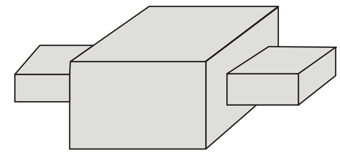


Figure 1.7

- **Turning Pair:** When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair.

  In slider-crank mechanism, all pairs except the slider and guide pair are turning pairs.

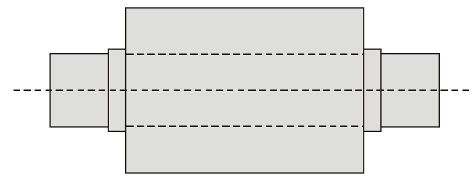


Figure 1.8

- **Rolling Pair:** When the links of a pair have a rolling motion relative to each other they form a rolling pair e.g. rolling wheel on a flat surface, ball and roller bearings etc.
- **Screw Pair (Helical Pair):** If two mating links have a turning as well as sliding motion between them they form a screw pair. The lead screw and the nut of a lathe is a screw pair.
- **Spherical Pair:** When one link in the form of a sphere turn inside a fixed link, it is a spherical pair.

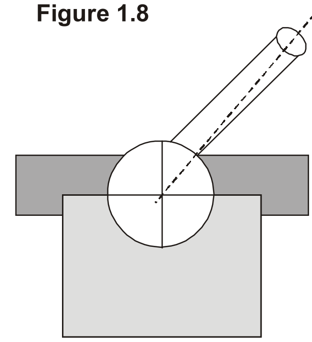


Figure 1.9

NOTE

- Sliding pair → Lower Pair
  Turning / Revolute / Pin Joint → Lower Pair
  Rolling Pair → Higher Pair
  Screw Pair → Lower Pair
  Spherical Pair → Lower Pair

Kinematic Chain and Mechanism

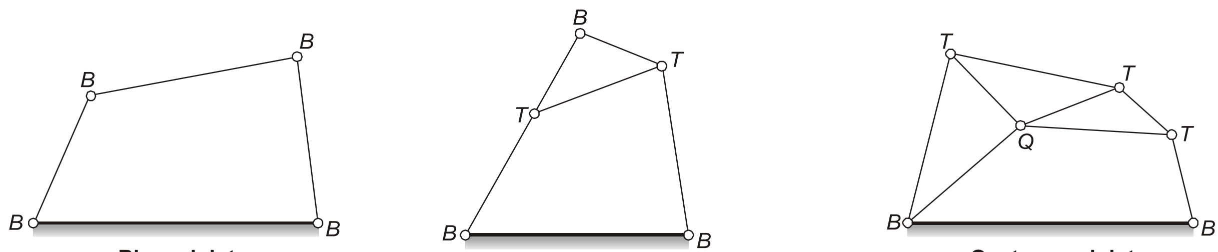
- A kinematic chain is an assembly of links in which the relative motions of the links is possible and the motions are definite and constrained.
- In case the motion of a link results in indefinite motion of other links, it is a non-kinematic chain.
- If there is no relative motion of the links, it is a redundant chain.
- Kinematic chain $\xrightarrow{\text{One link fixed}}$ Linkage $\xrightarrow{\text{if motion if constrained and definite}}$ Mechanism

Kinematic Joint

A kinematic joint is the connection between two links by a pin. There is ample clearance between the pin and the hole in the ends of the links being connected to provide free motion of the links.

The usual types of joints in a chain are as shown in figure

- **Binary joint :** Two links are connected at the same joint by a pin.
- **Ternary joint :** Three links are connected at the same joint by a pin.
- **Quaternary joint :** Four links are connected at the same joint by a pin.



Binary joints **Ternary joints** **Quaternary joints**

Figure 1.10: Type of Kinematic joints

NOTE : If n number of links are connected at a joint, it is equivalent to $(n - 1)$ binary joint.

1.5 Degree of Freedom (DoF)

- An unconstrained rigid body can have following independent motions :
  - (i) Translation along x, y and z axis.
  - (ii) Rotation about these axes.
- So, a rigid body in space can have 6 independent motions or degrees of freedom.
- So, for a pair, DoF is the number of independent relative motions (both translational and rotational) a pair can have.
- The connection of a link with another link imposes certain constraints on their relative motion. Constraints can never be zero (joint is disconnected) or six (rigid joint).

$$\text{DoF} = 6 - \text{Number of restraints}$$

NOTE

(i) DoF of a system or mechanism can also be defined as number of independent variables required to define a position or motion of the system.

(ii) If a link of redundant chain is fixed $\rightarrow$ structure or locked system

$\therefore$ DoF = 0

If DoF is negative $\rightarrow$ Superstructure

(iii) If DoF = 1 $\rightarrow$ Constrained chain

DoF > 1 $\rightarrow$ Unconstrained chain

1.5.1 Degree of Freedom of Mechanism

- Let,

F = Degree of freedom (DoF)

L = Total number of links in mechanism

P_1 = Number of pair having one DoF

P_2 = Number of pair having two DoF

P_3 = Number of pair having three DoF

P_4 = Number of pair having four DoF

P_5 = Number of pair having five DoF

- If In a mechanism one link is fixed then number of movable links = $N - 1$.
- Number of degrees of freedom of $(N - 1)$ movable links = $6(N - 1)$.
- Each pair having one degree of freedom imposes 5 restraints on the mechanism reducing its degree of freedom by $5P_1$.

- Each pair having two degrees of freedom will imposes 4 restraints reducing the degrees of freedom of the mechanism by $4P_2$.
- Similarly other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of the mechanism
- Degree of freedom $6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5$.

1.5.2 Degree of Freedom of Plane (2D) Mechanism (Grubler Criterion)

- Most of the mechanism are two dimensional such as four link or a slider crank mechanism in which displacement is possible along two axes (one restraint) and rotation about only one axis (two restraints). Thus there are three general restraints. Hence.

$$F = 3(N-1) - 2P_1 - 1P_2.$$

Here,
L = Number of link in a mechanism
P_1 = Number of pair having one degree of freedom
P_2 = Number of pair having two degree of freedom

- Kutzback's equation

$$F = 3(L-1) - 2j - h$$

Here,
L = Number of link
j = Number of **binary joint**
h = Number of **higher pair**

- Grubler's Equation : It is for those mechanism which have **single** degree of freedom and **zero** higher pair.

$$3L - 2j - 4 = 0$$

Here,
l = Number of links
j = Number of binary joints

- The following relationship holds for a kinematic chain having lower pairs :

$$L = 2P - 4$$

and

$$j = \frac{3}{2}L - 2$$

where
L = Number of binary links
j = Number of binary joints
P = Number of lower pairs

If
LHS > RHS $\rightarrow$ Locked or redundant chain
LHS = RHS $\rightarrow$ Constrained chain
LHS < RHS $\rightarrow$ Unconstrained chain

- We know that

$$3L - 2j - 4 = 0$$

$$\Rightarrow \quad L = \frac{2j+4}{3}$$

$\therefore$ to satisfy this condition minimum value of $j = 4$

So, $L_{minimum} = 4$

Therefore, minimum number of links to have a mechanism (1 DoF) with only lower pairs is four. But minimum number of links to have a mechanism (1 DoF) with both lower and higher pairs is three.

1.5.3 Redundant Links and Redundant Degree of Freedom :

- Links of a system which do not introduce any extra constraint are called redundant links. These should not be counted to find DOF.

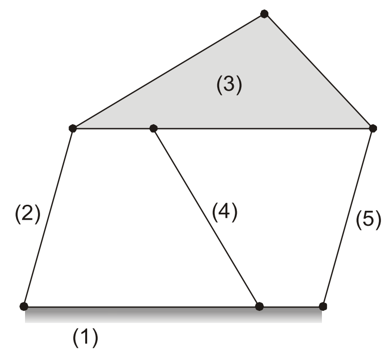


Figure 1.11

$$F = 3(5-1) - 2 \times 6 = 0$$

But no effect is there if (2) or (4) or (5) are removed, i.e.,

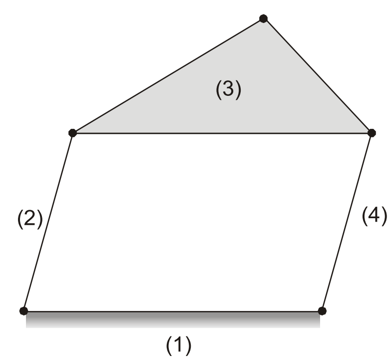


Figure 1.12

$$F = 3(4-1) - 2 \times 4 = 1$$

- Sometimes one or more links of a mechanism can be moved without causing any movement to the rest of the links. Such a link is said to have redundant degree of freedom F_r. e.g.,

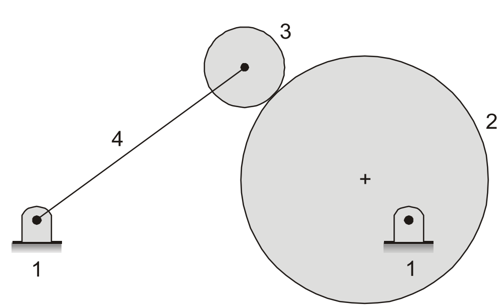


Figure 1.13

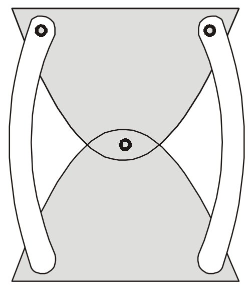
(3) can rotate about its axis without causing any movement to other links.

$$\therefore \quad F = 3(N-1) - 2P_1 - P_2 - F_r$$

$$N = 4,\ P_1 = 3,\ P_2 = 1$$

$$\therefore \quad F = 3 \times 3 - 6 - 2 - 1 = 1$$

Example 1.1 The degrees of freedom of the above chain in planar motion is



(a) –1 (b) 0 (c) 2 (d) 3

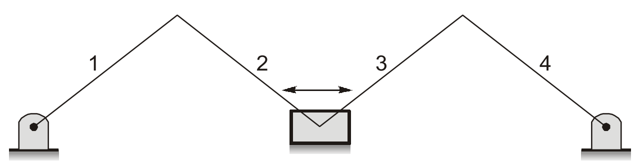
(UPSC JWM 2008)

Solution **(c):**

None of the link is fixed

$$F = 3N - 2P_1$$
$$F = 3 \times 4 - 2 \times 5$$
$$F = 2$$

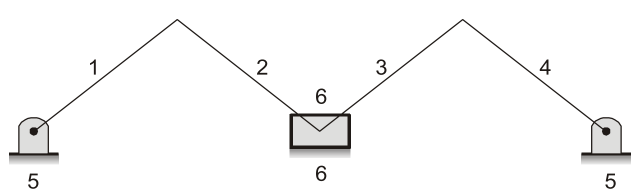
Example 1.2 The number of degree of freedom for the following mechanism is



(a) 0 (b) 1 (c) 2 (d) 4

(ISRO 2012)

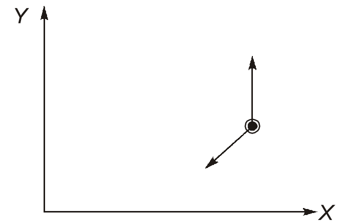
Solution **(b):**



At slider 3 links are connected ≡ 2 binary joints

$$P_1 = 7$$
$$F = 3(6 - 1) - 2 \times 7$$
$$= 15 - 14 = 1$$

Example 1.3 What is the degree of freedom of the system shown in the figure below :



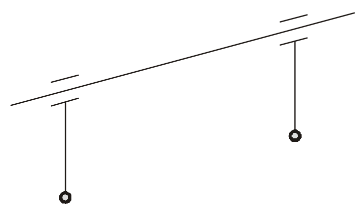
(a) 3 (b) 2 (c) 4 (d) 6

(SSC JE 2018)

***Solution* (c):**

$$F = 3N - 2P_1 \quad (\because \text{No link is fixed})$$
$$= 3 \times 2 - 2 \times 1 = 4$$

Example 1.4 How many degrees of freedom does the mechanism shown below has?

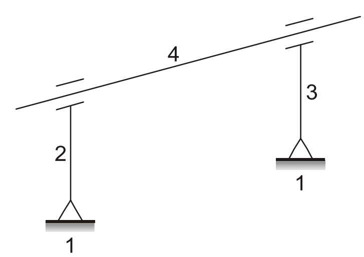


(a) 0 (b) 2 (c) 1 (d) 3

(SSC JE 2018)

***Solution* (a):**

Mechanism is given, so one link is fixed.

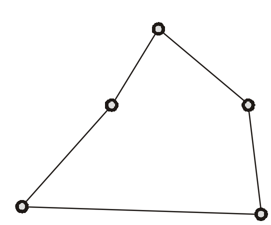


$$F = 3(N-1) - 2P_1 - F_r$$

($\because$ rod can be moved without causing any movement to other links)

$$= 3(4-1) - 2 \times 8 - 1$$
$$= 0$$

Example 1.5 The given figure shows a/an



(a) locked chain
(b) constrained kinematic chain
(c) unconstrained kinematic chain
(d) mechanism

(CSE 2000, SSC JE 2018)

***Solution* (c):**

$$F = 3N - 2P_1 \text{ (no fixed link)}$$
$$= 3 \times 5 - 2 \times 5 = 15 - 10 = 5 > 1$$

$\Rightarrow$ Unconstrained chain