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MECHANICAL ENGINEERING

CONVENTIONAL Practice Sets

CONTENTS

FLUID MECHANICS AND FLUID MACHINERY

1
CHAPTER

Fluid Properties

Q.1 The velocity distribution for flow over a flat plate is given by $u = \frac{3}{4}y - y^2$ in which u is the velocity in metre per second at a distance *y* metre above the plate. Determine the shear stress at *y* = 0.15 m. Take dynamic viscosity of fluid as 8.5 poise.

Solution:

Given, $u = \frac{3}{4}y - y^2$

Viscosity, $\mu = 8.5\text{poise} = \frac{8.5}{10}\text{Ns/m}^2$ $\left(\because 10 \text{ poise} = 1 \text{ Ns/m}^2\right)$

$\therefore$ $\frac{du}{dy} = \frac{3}{4} - 2y$

At y = 0.15, $\frac{du}{dy} = \frac{3}{4} - 2\times 0.15 = 0.75 - 0.30 = 0.45$

$$\boxed{\tau = \mu\frac{du}{dy}}$$

$$= \frac{8.5}{10}\times 0.45 \text{ N/m}^2 = 0.3825 \text{ N/m}^2$$

Q.2 The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a forces of 9.18 N to maintain the speed. Determine:
(i) the dynamic viscosity of the oil in poise, and
(ii) the kinematic viscosity of the oil in stoke if the specific gravity of the oil is 0.95.

Solution:

Given: Each side of a square plate = 60 cm = 0.60 m

$\therefore$ Area, $A = 0.6 \times 0.6 = 0.36 \text{ m}^2$

Thickness of oil film, $\Delta y = 12.5 \text{ mm} = 12.5 \times 10^{-3} \text{ m}$

Velocity of upper plate, $u = 2.5$ m/sec

$\therefore$ Change of velocity between plates,

$\Delta u = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

$\therefore$ Shear stress, $\boxed{\tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}}$

$$= \frac{98.1\text{N}}{0.36\text{m}^2} = 27.25 \text{ N/m}^2$$

(i) Let μ = Dynamic viscosity of oil

$$\boxed{\tau = \mu\frac{du}{dy}}$$

or $$27.25 = \mu \times \frac{2.5}{12.5\times10^{-3}}$$

$\therefore$ $$\mu = 27.25\times\frac{12.5\times10^{-3}}{2.5} = 0.13625\ \text{Ns/m}^2 \qquad (\because 1\ \text{Ns/m}^2 = 10\ \text{poise})$$

$$= 0.13625 \times 10 = 1.3625\ \text{Poise}$$

(ii) Specific gravity of oil, $S = 0.95$

Let ν = kinematic viscosity of oil

Mass density of oil, $\rho = S\times 1000 = 0.95 \times 1000 = 950\ \text{kg/m}^3$

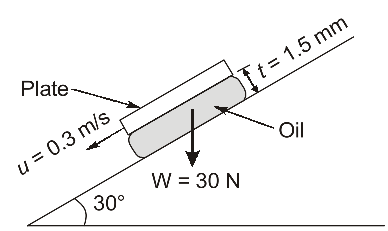
Using the relation, $$\nu = \frac{\mu}{\rho}$$

We get, $$\nu = \frac{0.13625\ \text{Ns/m}^2}{950\ \text{kg/m}^3} = 1.434\times10^{-4}\ \text{m}^2/\text{sec}$$

$$= 1.434 \times 10^4\ \text{cm}^2/\text{s}$$
$$= 1.434\ \text{stokes}$$

Q.3 **Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m × 0.8 m and an inclined plane with angle of inclination 30° as shown in figure. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.**

Solution:



Given: Area of plate, $A = 0.8 \times 0.8 = 0.64\ \text{m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300\ \text{N}$

Velocity of plate, $u = 0.3\ \text{m/s}$

Thickness of oil film, $t = dy = 1.5\ \text{mm} = 1.5 \times 10^{-3}\ \text{m}$

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane = $W\cos 60° = 300\cos 60° = 150$ N

Thus the shear force, F, on the bottom surface of the plate = 150 N

and shear stress, $$\boxed{\tau = \frac{F}{\text{Area}}} = \frac{150}{0.64}\ \text{N/m}^2$$

Now, $$\boxed{\tau = \mu\frac{du}{dy}}$$

Assuming linear velocity profile,

du = change of velocity = $u - 0 = 0.3$ m/sec

$dy = t = 1.5 \times 10^{-3}$ m

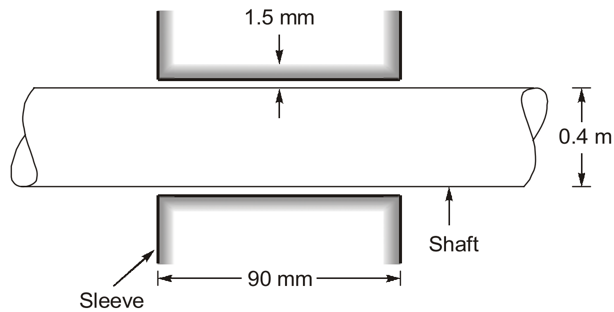
$\therefore$ $$\frac{150}{0.64} = \mu\times\frac{0.3}{1.5\times10^{-3}}$$

$$\mu = \frac{150\times1.5\times10^{-3}}{0.64\times0.3} = 1.17\ \text{Ns/m}^2 = 11.7\ \text{Poise}$$

Q4 The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in teh bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.

Solution:

Given:



Viscosity, $\mu = 6$ Poise

$$= \frac{6}{10}\frac{\text{Ns}}{\text{m}^2} = 0.6\,\text{Ns/m}^2$$

Diameter of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ rpm

Sleeve length, $L = 90\text{ mm} = 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5\text{ mm} = 1.5 \times 10^{-3}$ m

Tangential velocity of shaft, $\boxed{u = \frac{\pi DN}{60}}$

$$u = \frac{\pi \times 0.4 \times 190}{60} = 3.98 \text{ m/s}$$

Using the relation and assuming linear velocity profile,

$$\boxed{\tau = \mu \frac{du}{dy}}$$

where, du = Change of velocity $= u - 0 = u = 3.98$ m/s

dy = Change of distance $= t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{15 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft,

$\therefore$ Shear force on the shaft, $\boxed{F = \text{Shear stress} \times \text{Area}}$

$$= 1592 \times \pi D \times L = 1592 \times \pi \times 0.4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft, $\boxed{T = \text{Force} \times \frac{D}{2}}$

$$= 180.5 \times \frac{0.4}{2} = 36.01\,\text{Nm}$$

$\therefore$ $\boxed{\text{Power lost} = \frac{2\pi NT}{60}}$

$$= \frac{2\pi \times 190 \times 36.01}{60} = 716.48\,\text{W}$$

Q.5 A vertical gap 23.5 mm wide of infinite extent contains oil of specific gravity 0.9 and viscosity 2.5 N-s/m^2. A metal plate 1.5 m × 1.5 m × 1.5 mm weighing 50 N is to be lifted through the gap at a constant speed of 0.1 m/sec. Estimate the force required to lift the plate.

Solution:

Given: Width of gap = 23.5 mm

Viscosity, μ = 2.5 Ns/m^2

Specific gravity oil = 0.9

$\therefore$ Weight density of oil = $0.9 \times 1000 = 900$ kgf/m^3

= 900×9.81 N/m^3 ($\because$ 1 kgf = 9.81 N)

Assuming that the plate lies in the middle of the gap

Volume of plate = 1.5 m × 1.5 m × 1.5 mm

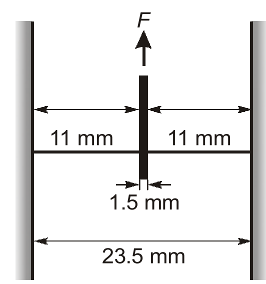
= $1.5 \times 1.5 \times 0.0015$ m^3

= 0.003375 m^3

Thickness of plate = 1.5 mm

Velocity of plate = 0.1 m/sec

Weight of plate = 50 N



When the plate is in the middle of the gap, the distance of plate from

$$\boxed{\text{Vertical surface of the gap} = \left(\frac{\text{Width of gap} - \text{Thickness of plate}}{2}\right)}$$

$$= \left(\frac{23.5 - 1.5}{2}\right) = 11 \text{ mm} = 0.011 \text{ m}$$

Now, shear force on left side of the metallic plate

$$\boxed{F_1 = \text{Shear stress} \times \text{Area}}$$

$$= \mu\left(\frac{du}{dy}\right)_1 \times (1.5 \times 1.5) = 2.5 \times \left(\frac{0.1}{0.011}\right) \times 1.5 \times 1.5 = 51.136 \text{ N}$$

Similarly, the shear force on the right side of the metallic plate,

$$\boxed{F_2 = \text{Shear stress} \times \text{Area}}$$

$$= 2.5 \times \left(\frac{0.1}{0.011}\right) \times (1.5 \times 1.5) = 51.136 \text{ N}$$

$\therefore$ **Total shear force,** $F = F_1 + F_2 = 51.136 + 51.136 = 102.273$ N

In this case the weight of plate (which is acting downward) and upward thrust is also to be taken into account.

$\therefore$ The upward thrust = Weight of fluid displaced = $\rho v g$

= (unit weight of fluid) × Volume of fluid displaced

= $9.81 \times 900 \times 0.003375$

= 29.80 N

The net force acting in the downward direction due to the weight of the plate and upward thrust

= Weight of plate – Upward thrust = 50 – 29.80 = 20.20 N

$\therefore$ Total force required to lift the plate up

= Total shear force + 20.20 = 102.273 + 20.20 = 122.473 N

Q.6 Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.

Solution:

Given: Diameter of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3}$ m

Pressure in excess of outside, $p = 2.5$ N/m^2

For a soap bubble, $\boxed{\Delta p = \dfrac{8\sigma}{d}}$

or
$$2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$
$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} = 0.0125 \text{ N/m}$$

Q.7 The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm^2 (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Solution:

Given, diameter of droplet, $d = 0.04 \text{ mm} = 0.04 \times 10^{-3}$ m

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725$ N/m

The **pressure inside the droplet**, in excess of outside pressure is given by

$$\boxed{p = \frac{4\sigma}{d}}$$
$$= \frac{4 \times 0.0725}{0.04 \times 10^{-3}} = 7250 \text{ N/m}^2$$
$$= \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

Pressure inside the droplet $= \Delta p$ + pressure outside the droplet

$= 0.725 + 10.32 = 11.045 \text{ N/cm}^2$

Q.8 Calculate the capillary effect in mm in a glass tube 3 mm in diameter when immersed in (a) water (b) mercury. Both the liquids are at 20°C and the values of the surface tensions for water and mercury at 20°C in contact with air are respectively 0.0736 N/m and 0.51 N/m. Contact angle for water = 0° and for mercury = 130°.

Solution:

The capillary rise (or depression) is given as

$$\boxed{h = \frac{2\sigma\cos\theta}{\rho g r}}$$

(a) For water $\theta = 0$, $\cos\theta = 1$

$\sigma = 0.0736$ N/m

$\rho g = 9810$ N/m^3

$d = 3$ mm

$$r = \frac{3}{2} = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

By substitution, we get
$$h = \frac{2 \times 0.0736 \times 1}{9810 \times 1.5 \times 10^{-3}}$$
$= 1.00 \times 10^{-2} \text{ m} = 10 \text{ mm}$

(b) For mercury $\theta = 130°$, $\cos\theta = -0.6428$

$$\sigma = 0.51 \text{ N/m}$$
$$\rho g = (13.6 \times 9810) \text{ N/m}^3$$
$$r = \frac{3}{2} = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

By substitution, we get
$$h = \frac{2 \times 0.51 \times (-0.6425)}{13.6 \times 9810 \times 1.5 \times 10^{-3}}$$
$$= -3.276 \times 10^{-3} \text{ m}$$
$$= -3.276 \text{ mm}$$

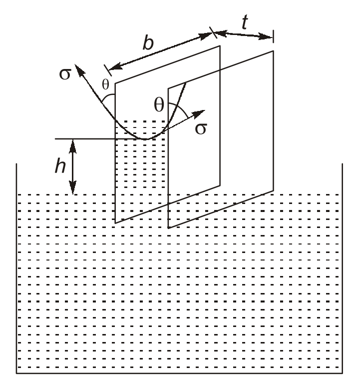
The negative (–) sign in the case of mercury indicates that there is capillary depression.

Q.9 Determine capillarity rise between two thin vertical plates spaced '*t*' distance apart. Calculate the distance between the plates when the capillarity rise is not to exceed 60 mm. Assume surface tension of water at 20°C as 0.075 N/m.

Solution:

For two vertical plates, '*t*' distance apart

Let width of plate be '*b*' and contact angle be 'θ'



Force due to surface tension = Force due to gravity

$$2\sigma\cos\theta\, b = \rho g\,(b \times t)h$$

Height of capillarity rise,
$$\boxed{h = \frac{2\sigma\cos\theta}{\rho g t}}$$

For $\sigma = 0.075$ N/m and $h = 60$ mm

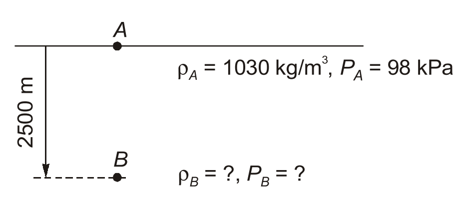
Assuming $\theta = 0°$ i.e., $\cos\theta = 1$

$$0.06 = \frac{2 \times 0.075 \times 1000}{9.81 \times 1000 \times t}$$
$$t = 0.255 \text{ mm}$$

Q.10 The density of sea water at free surface where pressure is 98 kPa is 1030 kg/m^3. Taking bulk modulus of sea water to be 2.34×10^9 N/m^2 (assume constant), determine the density and pressure at a depth of 2500 m. Neglect the effect of temperature

Solution:

Calculation of density:



$$K = 2.34 \times 10^9 \text{ N/m}^2$$
$$K = \rho\frac{dP}{d\rho}$$

Since $dP = \gamma dh$

$\Rightarrow$
$$K = \frac{\rho\gamma dh}{d\rho} = \rho^2 g\frac{dh}{d\rho}$$
$$\int_{\rho_A}^{\rho_B} \frac{d\rho}{\rho^2} = \int_0^H \frac{g}{K}dh$$

$\Rightarrow$
$$\left.\frac{1}{\rho(-1)}\right|_{\rho_A}^{\rho_B} = \frac{g}{K} \times H$$

$\Rightarrow$
$$\frac{1}{\rho_A} - \frac{gH}{K} = \frac{1}{\rho_B}$$

$$\therefore \qquad \rho_B = \frac{1}{\left(\frac{1}{\rho_A} - \frac{gH}{K}\right)}$$

$$\rho_B = \frac{1}{\frac{1}{1030} - \frac{9.81 \times 2500}{2.34 \times 10^9}} = 1041.24 \text{ kg/m}^3$$

Calculation of pressure:

$$K = \frac{dP}{\left(\frac{d\rho}{\rho}\right)}$$

$$\int_{P_A}^{P_B} dP = K.\int_{\rho_A}^{\rho_B} \frac{d\rho}{\rho}$$

$$P_B - P_A = K[\ln \rho]_{\rho_A}^{\rho_B}$$

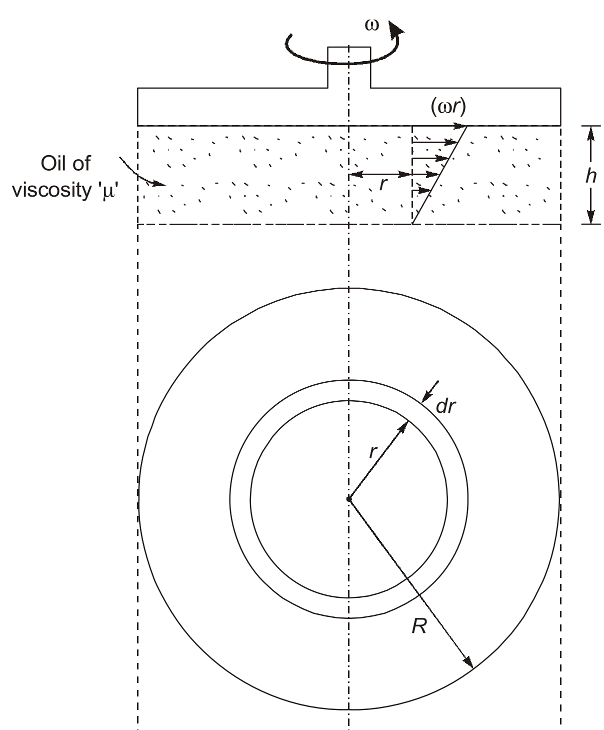
$$P_B = P_A + K\ln\left(\frac{P_B}{P_A}\right)$$

$$P_B = 98 + 2.34 \times 10^6 \ln\left(\frac{1041.24}{1030}\right)$$
$$= 25495.20 \text{ kPa} = 25.5 \text{ MPa}$$

Q.11 Consider a fluid of viscosity μ between two circular parallel plates of radii 'R' separated by a distance 'h'. Upper plate is rotated at an angular velocity ω whereas bottom plate is held stationery. The velocity profile between two plate is linear. Estimate the torque experienced by the bottom plate.

Solution:

Consider an annual ring with width *dr* at radius'*r*'



Shear stress on the ring, $\tau = \mu\left(\frac{du}{dy}\right) = \mu\frac{\omega r}{h}$ ..(i)

Force on the ring, $F = \tau \times \text{area of contact}$

$$= \left(\frac{\mu\omega r}{h}\right)(2\pi r dr)$$

$\therefore$ Torque on the ring, $dT = F.r = \left(\frac{2\pi\mu\omega}{h}\right)r^2.dr.r$

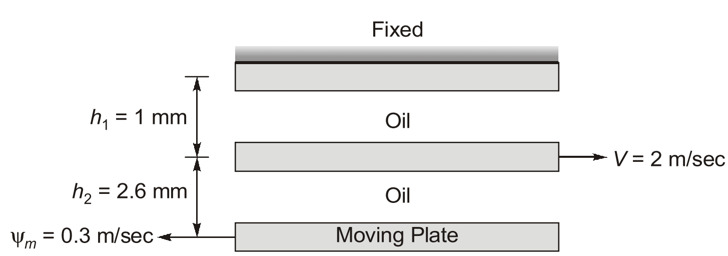
$$= \left(\frac{2\pi\mu\omega}{h}\right)r^3.dr$$

$\therefore$ Total torque on the disc, $T = \int_0^R\left(\frac{2\pi\mu\omega}{h}\right)r^3\,dr$

$$= \left(\frac{2\pi\mu\omega}{h}\right)\left(\frac{R^4}{4}\right)$$

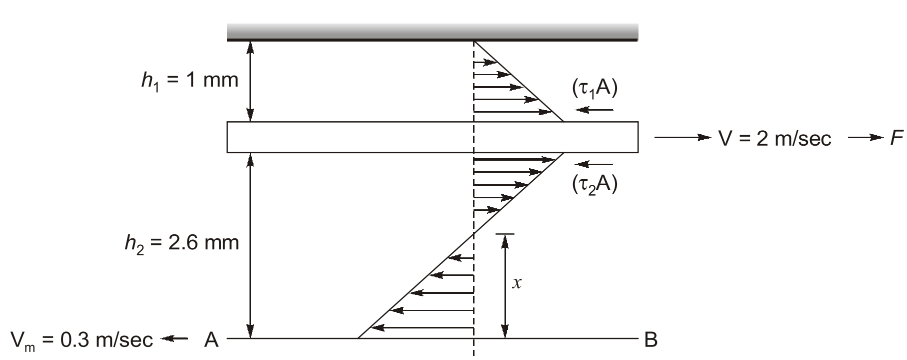
$$\boxed{T = \frac{\pi\mu\omega R^4}{2h}}$$

Q.12 A thin 40 cm × 40 cm flat plate is pulled at 2 m/sec horizontally through a 3.6 mm thick oil layer sandwitched between two plates, one stationary and other moving at a constant speed of 0.3 m/sec as shown in figure. Determine the force that is required to be applied on the plate to maintain this motion. Take (μ_{oil} = 0.027 Pa-s).



Solution:

Given:



Let at a distance x from plate AB, where the velocity of oil will be zero.

From the property of similarly of triangle

$$\frac{x}{2.6 - x} = \frac{0.3}{2}$$

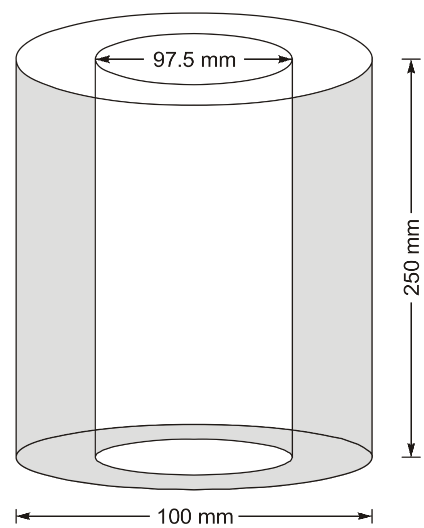
$$2x = 2.6 \times 0.3 - 0.3x$$
$$2.3x = 2.6 \times 0.3$$
$$x = \frac{2.6 \times 0.3}{2.3} = 0.34 \text{ mm}$$

Now, force, F required to maintain this motion

$$\boxed{F = (\tau_1 A + \tau_2 A)}$$
$$= \mu\left[\frac{V}{h_1} + \frac{V-(-V_m)}{h_2}\right]A$$
$$= 0.027\left[\frac{2}{1\times 10^{-3}} + \frac{2+0.3}{2.6\times 10^{-3}}\right]\times 0.4 \times 0.4$$
$$= 12.46 \text{ N}$$

Q.13 Two coaxial cylinders 250 mm high have a liquid in between them, the outer cylinder has internal diameter 100 mm and the internal cylinder has external diameter 97.5 mm. Find the viscosity of liquid which produces a torque of 1 Nm upon the inner cylinder when outer one rotates at 90 rpm.

Solution:



For given assembly

Internal diameter, $d_i = 97.5$ mm

External diameter, $d_o = 100$ mm

Speed of external cylinder, $N = 90$ rpm

Height of cylinder, $H = 250$ mm

Tangential velocity of external cylinder,

$$\boxed{u = \frac{\pi d_o N}{60}}$$
$$= \frac{\pi \times 0.1 \times 90}{60} = 0.47 \text{ m/s}$$

Shear stress at internal cylinder, $\boxed{\tau = \mu\frac{\partial V}{\partial y}}$

$$= \frac{\mu \times 0.47}{\frac{0.1 - 0.0975}{2}} = 376\mu \text{ N/m}^2$$

Force on internal cylinder, $\boxed{F = \tau A} = \tau(\pi d_1 H)$

$$= 376\,\mu \times \pi \times 0.0975 \times 0.25 = 28.79\,\mu$$

Torque on internal cylinder, $\boxed{T = F\frac{d_i}{2}}$

$$1 = \frac{28.79\mu \times 0.0975}{2}$$
$$\mu = 0.712 \text{ Pa-s}$$

Q.14 A three-cylinder car has pistons of 75 mm and cylinders of 75.1 mm. Find the percentage change in force required to drive the piston, when the lubricant warms from 25°C to 100°C. The dynamic viscosity of the lubricant at 25°C is 2 Ns/m^2 and at 100°C, it is 0.4 Ns/m^2.

Solution:

Given, Piston diameter, $d_1 = 75$ mm

Cylinder diameter, $d_2 = 75.1$ mm

$\therefore$ Clearance $= \dfrac{d_1 - d_2}{2} = \dfrac{0.1}{2} = 0.05$ mm

Also, $\mu_{25} = 2\ \text{Ns/m}^2,\ \mu_{100} = 0.4\ \text{NS/m}^2$

As per **Newtonian's law of viscosity**,

$$\boxed{\tau = \mu \frac{du}{dy}}$$

Force required to drive the piston $\boxed{F = \tau A_s}$

$$\therefore \quad \boxed{F = \left(\mu \frac{du}{dy}\right) A_s}$$

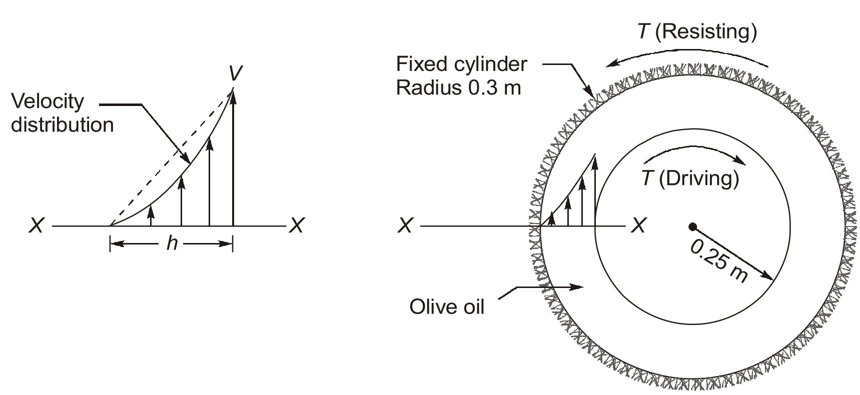
$$\therefore \quad \frac{F_1}{F_2} = \frac{\mu_1}{\mu_2} \text{ (The other factors remains constant)}$$

$$\Rightarrow \quad \frac{F_{25^\circ}}{F_{100^\circ}} = \frac{\mu_{25^\circ}}{\mu_{100^\circ}} = \frac{2.0}{0.4} = 5.0$$

$$\text{\% change in force, } \Delta F = \left(\frac{F_{25} - F_{100}}{F_{25}}\right) \times 100 = 1 - \frac{1}{5} = 80\%$$

Q.15 A cylinder 0.25 m in radius and 2 m in length rotates coaxially inside a fixed cylinder of the same length and 0.30 m radius. Olive oil of viscosity 4.9×10^{-2} Ns/m² fills the annular space between the cylinders. A torque 4.9 N-m is applied to the inner cylinder. After constant velocity is attained, calculate the velocity gradient at the cylinder walls, the resulting rpm, and the power dissipated by fluid resistance ignoring end effect. Assume non-linear velocity profile between the cylinders.

Solution:



The surface area of the outer cylinder is larger than that of the inner one, since the former has a larger radius. Accordingly the shear force and the velocity gradient at the outer cylinder will be less than the respectively quantities on the inner one. The velocity profile through the fluid will be non-linear as indicated in the figure, since the gap between the inner and outer cylinders is comparatively larger.

The torque of 4.9 Nm is transmitted from inner cylinder to the outer one through fluid friction (viscous effect). Let r be the radial distance of any fluid layer in the annular space.

Then $4.9 = \tau \times (2\pi r l) \times r = \tau \cdot (2\pi r)(2) r$ $\quad [\because l = 2 \text{ m given}]$

$$= \tau 4\pi r^2 = \mu \frac{du}{dy} \cdot 4\pi r^2$$

$$\therefore \qquad \frac{du}{dy} = \frac{4.9}{4\pi r^2 \mu} = \frac{4.9}{4\pi r^2 \times 4.9 \times 10^{-2}} = \frac{100}{4\pi r^2} = \frac{7.96}{r^2}$$

The velocity gradients at the inner and outer cylinders are

$$\left(\frac{du}{dy}\right)_i = \frac{7.96}{(0.25)^2} = 127.4 \text{ sec}^{-1}$$

and

$$\left(\frac{du}{dy}\right)_o = \frac{7.96}{(0.30)^2} = 88.4 \text{ sec}^{-1}$$

Substituting $(-dr)$ for dy in the equation for $\frac{du}{dy}$ since velocity decreases as r increases i.e.

$$y = R - r \Rightarrow dy = -dr$$

$$\therefore \qquad V = \int_0^V du = -7.96 \int_{0.30}^{0.25} \frac{dr}{r^2}$$

$\therefore$ Velocity of inner cylinder,

$$V = 7.96\left(\frac{1}{r}\right)_{0.30}^{0.25} = 5.31 \text{ m/s}$$

$\therefore$ Rotational speed of inner cylinder,

$$\boxed{\omega = \frac{V}{r}}$$

$$= \frac{5.31}{0.25} = 21.24 \text{ rad/sec}$$

$\therefore$ RPM,

$$N = \frac{60\omega}{2\pi} = \frac{60 \times 21.24}{2\pi} = 202.8 \text{ rpm}$$

$$\boxed{\text{The power dissipated in fluid friction} = \frac{2\pi NT}{60}}$$

$$= \frac{2\pi \times 202.8 \times 4.9}{60}$$

$$= 104.06 \text{ Nm/s} \simeq 104.0 \text{ W}$$

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