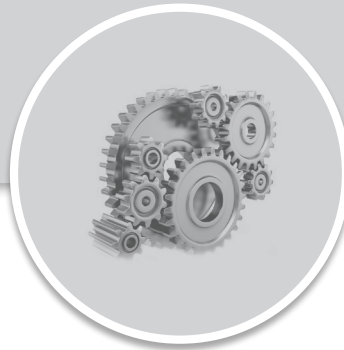# MECHANICAL ENGINEERING

Refrigeration & Air-Conditioning

Comprehensive Theory
with Solved Examples *and* Practice Questions

MADE EASY
Publications

Refrigeration & Air-Conditioning

CONTENTS

Refrigeration & Air-Conditioning

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Introduction & Basic Concepts of Refrigeration

1.1 INTRODUCTION

Refrigeration is a process of removing heat from a confined space so that its temperature is first lowered and then maintained at lower temperature compared to that of the surroundings, i.e. it is a phenomenon by virtue of which one reduces and maintains the temperature of a confined space below that of the surroundings.

In earlier times, refrigeration was achieved by natural means such as evaporative cooling or by use of ice. Ice was generally transported from colder regions, made at night using nocturnal cooling or harvested in winter and stored appropriately in ice houses for use in summer. At present, refrigeration is mostly produced by artificial means.

The device used to produce cold or refrigeration effect is called a refrigeration system or refrigerator. There can be different kinds of refrigeration systems based on working principle. The basic components of a refrigeration system are evaporator, compressor, condenser and expansion valve. In addition to these, there may be a refrigerant accumulator, temperature controller, etc. The refrigerant is a working substance, which circulates through the refrigeration system during its operation.

For example, R134a is the refrigerant used in domestic refrigerators and low-temperature refrigeration systems. R22 is the refrigerant used in window air-conditioning units while ammonia is preferred in large air-conditioning systems. Air is the refrigerant in air-refrigeration cycles which is predominantly used in aircrafts.

Refrigeration cycles are classified mainly into the following:
- Vapour Compression Refrigeration Cycle.
- Vapour Absorption Refrigeration Cycle.
- Air Refrigeration Cycle and Steam-jet Refrigeration Cycle.

Unit of Refrigeration or Rating for Refrigeration

The definition of refrigeration indicates that refrigeration is nothing but the rate of removal of heat. The SI unit of heat is Joule, the time rate of which is Watt. The unit of refrigeration effect is Watt (W) or kilo Watt (kW).

The standard unit of refrigeration is **ton of refrigeration** or simply ton denoted by the symbol TR. One TR is defined as the amount of heat to be removed from 1000 kg of water at 0° to convert it into ice at 0°C in 24 hours.

Thus,
$$1 \text{ TR} = \frac{1 \times 2000 \text{ lb} \times 144 \text{ Btu/lb}}{24 \text{ h}} = 12{,}000 \text{ Btu/h} = 200 \text{ Btu /min}$$

But $1 \text{ Btu} = 1.055 \text{ kJ}$ $\therefore$ $1 \text{ TR} = 211 \text{ kJ/min} = 3.516 \text{ kW}$

NOTE: In the definition of 1 TR, one ton equals 2000 lb instead of 2240 lb as per the conversion factor. In US, one ton is equal to 2000 lb.

1.2 APPLICATIONS OF REFRIGERATION

In earlier days, refrigeration was mainly used for the production of ice, which was used for food preservation, cooling beverages and refrigerated transportation, etc. Now-a-days, with the advancement in technology and expansion of human needs refrigeration finds a large number of applications in diverse fields. Also the temperature range of interest in refrigeration extends down to –100°C.

The applications of refrigeration can be grouped into following major areas:

1. **Food processing, preservation and distribution**

   • Refrigeration finds widespread applications in food processing and in storage units.

   (a) **Milk and milk products:** Whole milk for human consumption is pasteurized at 75°C for a short time, and then re-cooled to 4°C immediately. This cooling is done in a counterflow plate heat exchanger with the use of chilled water at 2°C. The pasteurization of milk is carried out in the following stages:

   (i) Raw milk at 4°C is heated by the outgoing milk up to about 25°C.

   (ii) This milk is finally heated by hot water up to the pasteurizing temperature of 75°C and held for a few seconds.

   (iii) Then milk is cooled by the incoming milk down to about 10°C.

   (iv) The final stage of cooling from 10°C to 4°C is by water at 2°C.

   (b) **Soft drinks:** Most of the soft drinks are carbonated, i.e. they have a proportion of dissolved carbon dioxide, which causes the bubbles and also the typical effervescent taste. Each litre of water will have dissolved in it 3.5 to 5 litre of carbon dioxide. The solubility of CO_2 in water depends on the pressure and temperature.

   (c) **Storage of vegetables and food products:** Table below shows the storage temperature of a few vegetables and food products and their approximate storage life.

Table: Storage conditions for food products

Product	Temperature, °C	Relative humidity, %	Approximate storage life
Mangoes	10	85–90	2–3 weeks
Potatoes	3 to 4.5	85–90	5–8 months
Tomatoes	14 to 21	85–90	2–4 weeks
Bananas	14.5	95	8–10 days for ripening
Fish fresh	0.5 to 1.5	90–96	5–15 days
Butter	– 18 to – 23	80–85	2 months
Pasteurised milk	0.5	–	7 days
Eggs	– 1.5 to – 0.5	80–85	6–9 months
Grapes	– 0.5	85–90	3–6 months

   Domestic refrigeration includes household refrigerators and home freezers. Domestic units are usually small in size, producing refrigerating effect in the range 300–500 W at about – 5°C evaporator temperature.

2. **Chemical and Process industries:** The industries like petroleum refineries, petrochemical plants and paper pulp industries etc. require very large cooling capacities. The main applications of refrigeration in process industries involve the following:

(a) In petrochemical plant, temperature as low as –150°C with refrigeration capacities as high as 10000 tons of refrigeration (TR) are used for separation of gases by fractional distillation.

(b) Some gases that are produced synthetically, are condensed to liquid state by cooling, so that these can be easily stored and transported in liquid state.

(c) One of the processes of separation of a substance or pollutant or impurity from a liquid mixture is by its solidification at low temperature. Lubricating oil is dewaxed in petroleum industry by cooling it below –25°C. Wax solidifies at about –25°C.

(d) Storage as liquid at low pressure: Liquid occupies less space than gases. Most of the refrigerants are stored at high pressure. This pressure is usually their saturation pressure at atmospheric temperature. For some gases, saturation pressure at room temperature is very high hence these are stored at relatively low pressure and low temperature. For example, natural gas is stored at 0.7 bar gauge pressure and –130°C.

(e) In many chemical reactions, efficiency is better if the reaction occurs below room temperature. This requires refrigeration. Production of viscose rayon, cellular acetate and synthetic rubber are some of the examples. Fermentation is also one of the examples of such a reaction.

(f) Many compounds decompose at room temperature. Certain drugs, explosives and natural rubber can be stored for long periods at lower temperatures.

3. **Air-conditioning applications:** Refrigeration processes are used in various air-conditioning applications. Air-conditioning applications can by broadly categorized into comfort and industrial air conditioning applications. Residences, offices, auditorium, aircrafts, large buildings, etc. all have different design and capacity requirements. Besides this, there are various industrial air-conditioning applications. The main purpose of industrial air conditioning application is to provide conducive conditions so that required processes can be carried out. For example, manufacturing of precision parts like VLSI, chips, textile industry, pharmaceutical industry, etc.

4. **Special applications:** Some special applications of refrigeration are there in manufacturing processes, medicine and construction unit, etc. Some listed examples are below:
   - Cold treatments of metals for desired properties and stabilizing precision parts.
   - Blood plasma and antibiotics are manufactured by freeze-drying process.
   - Desalination of water
   - Manufacturing of ice

1.3 REFRIGERATION AND HEATING LOAD

The refrigeration effect or cooling effect is produced in a refrigeration cycle by the refrigerating equipment. The average rate at which heat is removed from the cold space by the equipment is known as the **cooling load**. It is expressed in kW or TR.

The cooling load on refrigerating equipment results from several different sources. Some of the common sources of heat that contribute to the cooling load on the refrigerating equipments are as follows:

1. Heat entering into the refrigerated space from outside by conduction through the insulated walls.

2 Solar radiations that enter the refrigerated space through transparent glass or other transparent material.

3. Heat on account of warm outside air entering the refrigerated space through open doors or through cracks around windows and doors.

4. Heat emitted by warm products whose temperature is to be lowered to the refrigerated space temperature.

5. Heat emitted by people occupying the refrigerated space. For example, people present in an air-conditioned space or the people working in the cold storages during loading and unloading the goods.

6. Heat emitted by any heat generating equipment installed in the refrigerated space such as lamps, motors, electronic devices, etc.

It is to be noted here that all these sources of heat are not present in every application. The significance of any one heat source with relation to the total cooling load will vary considerably with each application.

In those regions where the atmospheric temperature falls considerably (below 10°C), especially during winters, heating is needed to keep the rooms warm. The rate of heat to be supplied to such a conditioned space is known as **heating load**. In Western countries, the houses are facilitated with solar heating systems.

1.4   CONCEPT OF HEAT ENGINE, REFRIGERATOR AND HEAT PUMP

It is a well-known fact that heat flows in the direction of decreasing temperature, i.e. from a high temperature body to a low temperature body. Such heat transfer occurs in nature without any external aid or device.

(i) Heat Engine

It is a prime mover that generates heat from the fossil fuel. It works according to the second law of thermodynamics stated by Kelvin and Planck—It is impossible to construct a device that operates continuously in a cycle and produces no effect other than the withdrawal of heat energy from a single reservoir and converts all the heat into useful work. This means that the heat engine (HE) rejects part of the heat available from the heat source while converting it to useful work. As shown in adjacent figure, it takes heat at the rate of Q_1 from the heat source and generates the work at the rate of W while rejecting heat at the rate of Q_2 to the heat sink. The performance of a heat engine is measured with the help of its 'thermal efficiency' or 'Carnot efficiency' and is mathematically represented as

Figure: Heat engine

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{Q_2}{Q_1}$$

In heat engine, how much work is extracted from a given amount of heat is an important quantity.

(ii) Refrigerator

Reverse heat transfer, i.e. from a body at low temperature to a body at high temperature, is possible only with a special device called refrigerator.

It works according to the second law of thermodynamics stated by Clausius. It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a body at low temperature to a body at high temperature without any external aid.
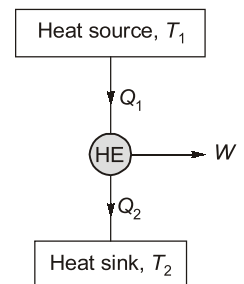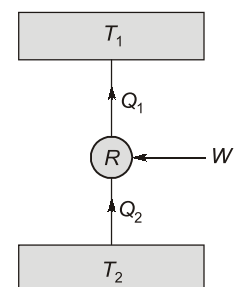

Figure: Refrigerator

The working principle of the refrigerator (R) is shown in figure. Here Q_2 is the amount of heat removed from the cold space at temperature T_2. Q_1 is the amount of heat rejected to the environment at temperature T_1 and W is the net work input to the refrigerator.

In refrigerator, heat extracted from lower temperature is an important quantity.

Coefficient of Performance (COP)

The performance of a device is given by its ability to carry out the assigned task. The performance of a refrigerator is given by its COP. The objective of the refrigerator is to remove heat (Q_2) from a closed space. To accomplish this, it needs W as work input. Therefore, COP of a refrigerator is :

$$\text{COP}_R = \frac{\text{Desired effect}}{\text{Required input}} = \frac{Q_2}{W} \qquad \text{. . . (i)}$$

But $$W = Q_1 - Q_2$$

$\therefore$ $$\text{COP}_R = \frac{Q_2}{Q_1 - Q_2} \qquad \text{. . . (ii)}$$

The COP may be greater than one in many cases, but not necessarily.

If the refrigerator operates on a reversible cycle then,

$$\text{COP}_R = \frac{T_2}{T_1 - T_2}$$

(iii) **Heat Pump**

A heat pump (HP) transfers heat from a low temperature space to a higher temperature space. The objective of a heat pump is to supply heat Q_1 to warm a space as shown in figure. The COP of a heat pump is expressed as the ratio of heat supplied (Q_1) to the work input (W). Mathematically,

$$\text{COP}_{HP} = \frac{\text{Desired effect}}{\text{Work input}} = \frac{\text{Heating effect}}{\text{Work input}} = \frac{Q_1}{W} \qquad \text{. . . (iii)}$$

But, $$W = Q_1 - Q_2$$

Figure: Heat pump

$\therefore$ $$(\text{COP})_{HP} = \frac{Q_1}{Q_2 - Q_2} \qquad \text{... (iv)}$$

If the heat pump operates on a reversible cycle then,

$$(\text{COP})_{HP} = \frac{T_1}{T_1 - T_2}$$

Comparing equations (ii) and (iv),

$$\text{COP}_{HP} = \text{COP}_R + 1 \qquad \text{. . . (v)}$$

For fixed rates of Q_2 and Q_1, equation (iv) shows that the COP_{HP} is always greater than one since the COP_R is a positive value. In reality, however, part of Q_1 is lost to the outside air through piping and other devices, and COP_{HP} may drop below one when the outside air temperature is too low.

In heat pump, how much heat is rejected to higher temperature is an important quantity.
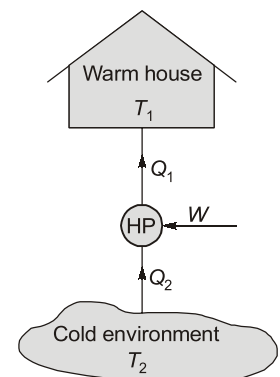
> **NOTE**
> - A heat engine is a work producing device whereas, refrigerator and heat pump are work consuming devices.
> - A refrigerating machine, that is used for cooling in summer, can be used as a heat pump for heating in winter. This can be done either
>   (i) By rotating the machine to interchange positions of two heat exchangers between space and surroundings, or
>   (ii) By exchanging the functions of the two heat exchangers by the operation of valves. Example, a four way valve used in a window AC.

1.5 REVERSED CARNOT CYCLE

It is only theoretical in its conception but serves as an **ideal refrigeration cycle** ever to be achieved in practice. The performance of reversed Carnot cycle is independent of the physical properties of the refrigerant used, so it is valuable as an indication of the highest COP attainable for given conditions.

The *p-v* and *T-s* diagrams of Carnot cycle using air as refrigerant are shown in figure below.

Figure: Reversed Carnot Cycle

Here, T_1 = Temperature to be maintained in refrigerator
T_2 = Temperature of the atmosphere to which heat is rejected
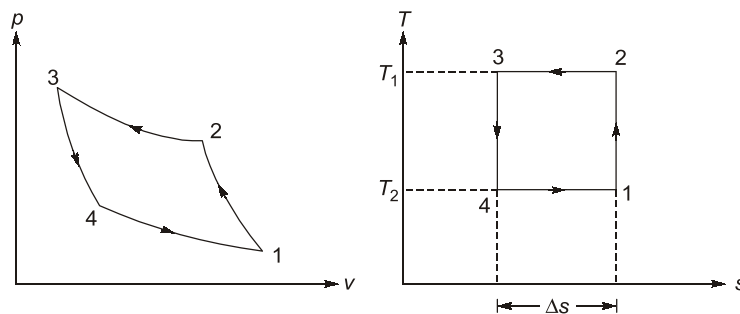
In reversed Carnot cycle;

Process 1-2: Isentropic compression
Process 2-3: Isothermal reversible heat rejection
Process 3-4: Isentropic expansion
Process 4-1: Isothermal reversible heat addition

From first law for cyclic processes,

$$\oint \delta Q = \oint \delta W$$

If W_{net} denotes net work input to the refrigerator then

$$W_{net} = \text{Heat rejected} - \text{Heat absorbed}$$
$$W_{net} = Q_r - Q_a = T_2(s_2 - s_3) - T_1(s_1 - s_4)$$
$$= T_2 \Delta s - T_1 \Delta s = (T_2 - T_1) \Delta s$$

Thus, $$(COP)_{\text{rev. Carnot}} = \frac{Q_a}{W} = \frac{T_1(s_1 - s_4)}{(T_2 - T_1)\Delta s} = \frac{T_1 \Delta s}{(T_2 - T_1)\Delta s} = \frac{T_1}{T_2 - T_1}$$

A high COP is desirable because it indicates that for a required amount of refrigeration, small amount of work is required.

Low value of T_2 will make COP high. High value of T_1 increases the numerator and decreases the denominator and therefore increases the coefficient of performance. The value of T_1 has more pronounced effect upon COP than T_2. The effects of T_1 and T_2 on COP are shown in figure.

The curve '*a*' shows the effect of T_1 on COP when T_2 is constant and the curve '*b*' shows the effect of T_2 on COP when T_1 is constant.

It can be observed from the figure that, if

$$\Delta T_1 = \Delta T_2$$

then,

$$\Delta COP_1 > \Delta COP_2$$

Although, the reversed Carnot cycle is most efficient between the fixed temperature limits and is therefore useful as a criterion of perfection, it possesses undesirable characteristics because isentropic process of the cycle requires high speed while the isothermal process requires an extremely low speed. This variation in flow of air (or any other refrigerant) is not practicable.
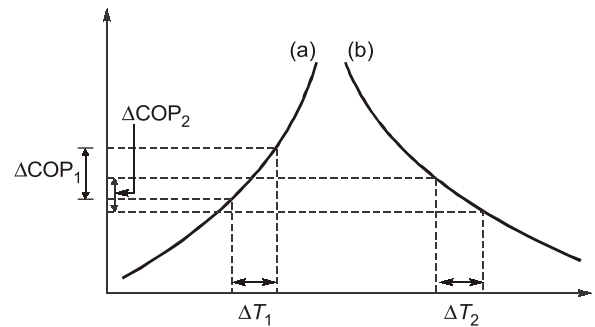
EXAMPLE : 1.1	A refrigeration system operates on reversed Carnot cycle between the temperature limits – 23°C and 45°C. For a refrigerating capacity of 10 TR,

(a)   the COP of the system is 3.67

(b)   the COP of the system is 4.67.

(c)   the heat rejected from the system per hour is 161 MJ nearly.

(d)   the power required by the system is 7.5 kW.   [MSQ]

Solution: (a, c)

(i)

$$COP_R = \frac{T_1}{T_2 - T_1} = \frac{273 - 23}{(273 + 45) - (273 - 23)}$$

$$= \frac{250}{318 - 250} = 3.67$$

(ii) Also,

$$COP_R = \frac{\text{Refrigerating effect}}{\text{Work input}}$$

∴   Work input $= \dfrac{10 \times 211 \times 60}{3.67} = 34332$ kJ/h

Heat rejected = Refrigerating effect per hour + Work input per hour

$$= (10 \times 211 \times 60) + 34332$$

$$= 160932 \text{ kJ/h} \simeq 161 \text{ MJ/h}$$

(iii)   Power required $= \dfrac{34332}{3600} = 9.536$ kW

EXAMPLE : 1.2	A refrigerator works on a reversed Carnot cycle. This unit requires 1.5 kW power for every one TR of refrigeration at –23°C. If this device is now used as a heat pump, then the corresponding COP will be_____. [Correct upto 1 decimal place]

Solution:

$$T_1 = -23°C = 250 \text{ K}$$

(i) For refrigerator: $\quad \text{COP}_R = \dfrac{\text{Refrigerating effect}}{\text{Work done}} = \dfrac{1 \times 3.516}{1.5} = \textbf{2.34}$

(ii) Also, $\quad \text{COP}_R = \dfrac{T_1}{T_2 - T_1} = \dfrac{250}{T_2 - 250}$

$$2.34 = \dfrac{250}{T_2 - 250}$$

or $\quad T_2 = \textbf{356.6 K}$

For heat pump :

Heat rejected or supplied to a space at T_2, = Heat absorbed + Work done

$$= 3.516 + 1.5 = 5.01 \text{ kW}$$

∴ $\quad \text{COP}_{HP} = \dfrac{\text{Heat rejected}}{\text{Work done}} = \dfrac{5.01}{1.5} = 3.3$

Alternatively, $\quad \text{COP}_{HP} = 1 + \text{COP}_R = 1 + 2.34 = 3.34 \simeq 3.3$

NOTE

For the purpose of heating, it is far more economical to use a heat pump rather than an electrical resistance heater. Suppose, if "W" be the work input in electric resistance heater, then its heat output will be "W" only whereas, in case of heat pump, if W be the work input then heat output will be more than W as described below.

$$\text{COP}_{HP} = \dfrac{Q_H}{W}$$

⇒ $\quad Q_H = W + Q_L$

Though, the initial cost of heat pump will be high, but in the long run it is more economical compared to electric resistance heater.
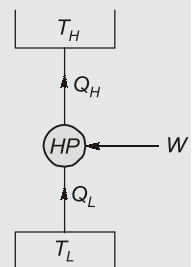
Figure: Heat pump

EXAMPLE : 1.3   A heat engine of efficiency 'η' runs a refrigerator of COP 'C'. The COP of the combined refrigerating machine is

(a) ηC

(b) $\dfrac{1}{\eta C}$

(c) $C + \dfrac{1}{\eta}$

(d) $\eta + \dfrac{1}{C}$

Solution: (a)

(a) Seebeck effect   (b) Peltier effect

Figure: Principle of thermoelectric refrigeration.

The heat transfer rate at each junction is given by

$$Q = \Phi I$$

where Φ is the Peltier coefficient in volts and I is the current in amperes.

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OBJECTIVE BRAIN TEASERS

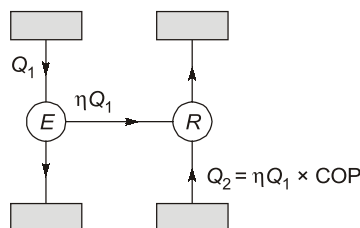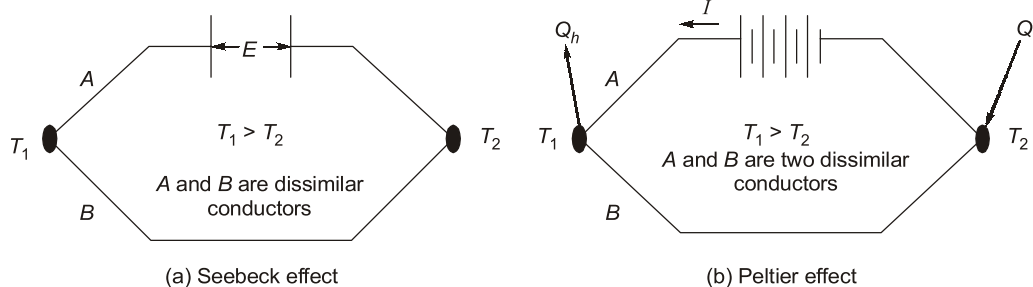**Q.1** A refrigerator has a second law efficiency of 30%, and heat is removed from the refrigerated space at a rate of 850 kJ/min. If the space is maintained at 0°C while the surrounding air temperature is 36°C. The power required by the refrigerator is_____. [Correct upto 1 decimal place]

Q.2 The Joule-Thomson coefficient for a refrigerant which is to be used in vapour compression refrigeration system should be
(a) positive   (b) negative
(c) zero   (d) infinite

Q.3 A reversible heat engine operates between the temperature limits T_1 and T_2 and has an efficiency η. If a reversible refrigeration cycle operates between same temperature limits then its COP is

(a) $\dfrac{1}{\eta} - 1$ (b) $\dfrac{1}{\eta} + 1$

(c) $\dfrac{1}{\eta}$ (d) $1 - \dfrac{1}{\eta}$

Q.4 The required horsepower per ton refrigeration for a Carnot refrigerator operating between temperature limits 37°C and –23°C is
(a) 0.94   (b) 1.13
(c) 0.785   (d) 1.32

Q.5 A room is kept warm with a system consisting of an electric heater and a heat pump operating on reversed carnot cycle, both supplying equal amount of heat. Heat pump operates between temperature limits of –23°C and 27°C. The COP of the combined heating system is
(a) 1.32   (b) 1.43
(c) 1.61   (d) 1.71

Q.6 A heat pump working on a reversed carnot cycle has a COP of 5 for a work input of 1 kW. If it works as a refrigerator then, the refrigerating effect will be
(a) 1 kW   (b) 2 kW
(c) 3 kW   (d) 4 kW

Q.7 A carnot refrigerator operates between temperatures –13°C and 27°C. It is desired to make the COP equal to 5 by changing temperatures. If the increase (or decrease) in upper temperature is to be equal to the decrease (or increase) in lower temperature, then the new value of lower temperature will be
(a) 254.5 K   (b) 245.7 K
(c) 265.8 K   (d) 270.6 K

Q.8 A reversible refrigerator working between two fixed temperatures
(a) Has the same COP whatever be the working substance
(b) Has its COP increased for working substance with higher enthalpy of evaporation
(c) Has its COP increased for working substance with specific heats
(d) None of the above

Q.9 In a certain ideal refrigeration cycle, the COP of heat pump is 5. The cycle under identical conditions running as heat engine will have efficiency as
(a) 0   (b) 0.2
(c) 0.5   (d) 1

Q.10 The power (kW) per ton of refrigeration is N/COP, then N is equal to _____. [Corect upto 1 decimal place]

Q.11 A Carnot refrigerator works between the temperatures of 200 K and 300 K, if the refrigerator receives 1 kW of heat, the work requirement will be
(a) 0.5 kW   (b) 0.67 kW
(c) 1.5 kW   (d) 3 kW

Q.12 The efficiency of a Carnot engine is 0.75, if the direction of cycle is reversed, what will be the value of COP of carnot refrigerator.
(a) 0.27   (b) 0.33
(c) 1.27   (d) 2.33

Q.13 When the lower temperature is fixed, how can the COP of a refrigerating machine be improved?
(a) By raising the higher temperature
(b) By lowering the higher temperature
(c) By operating the machine at high speeds
(d) By operating the machine at low speeds

Q.14 Which one of the following is correct relation between $(COP)_{HP}$ and $(COP)_R$
(a) $(COP)_{HP} - (COP)_R = 1$
(b) $(COP)_R - (COP)_{HP} = 1$
(c) $(COP)_{HP} + (COP)_R = 1$
(d) $(COP)_{HP} + (COP)_R = 0$

Q.15 The COP of a heat pump can be increased either by decreasing T_H by ΔT or by increasing T_L by ΔT. The new COP of the heat pump is
(a) same in both the cases
(b) highest if T_H is decreased
(c) highest if T_L in increased
(d) independent of change in T_H and T_L

Q.16 COP of a Carnot heat pump operating between –3°C and 27°C is
(a) 10   (b) 9
(c) 0.111   (d) 0.10

Q.17 If COP of a refrigerator is 5 and efficiency of a heat engine of the same temperature limit is 50%. Then what is the ratio of heat supplied to engine to heat absorbed by the refrigerator from the space
(a) 2.5   (b) 0.4
(c) 4   (d) 0.25

Q.18 If an engine of 40% thermal efficiency drives a refrigerator having a coefficient of performance of 5, then the heat input to engine for each kJ of heat removed from the cold body of the refrigerator is
(a) 0.50 kJ   (b) 0.75 kJ
(c) 1 kJ   (d) 1.25 kJ

Q.19 A refrigerator works on reversed Carnot cycle producing a temperature of –40°C. Work done per TR is 700 kJ per ten minutes. What is the value of its COP?
(a) 3   (b) 4.5
(c) 5.8   (d) 7.0

■■■■

ANSWER KEY

1. (5.5)	2. (a)	3. (a)	4. (b)	5. (d)
6. (d)	7. (a)	8. (a)	9. (b)	10. (3.5)
11. (a)	12. (b)	13. (b)	14. (a)	15. (c)
16. (a)	17. (b)	18. (a)	19. (a)	

HINTS & EXPLANATIONS

1. **(5.5) (5.0 to 6.0)**

    Given : $T_H = 36°C = 309$ K, $T_L = 0°C = 273$ K,
    $Q_2 = 850$ kJ/min

    $$COP_{rev} = \frac{T_H}{T_H - T_L} = \frac{309}{36} = 8.58$$

    Since, $\quad \eta_{II} = \frac{COP_{act}}{COP_{rev}}$

    $\therefore \quad COP_{act} = COP_{rev} \cdot \eta_{II}$
    $$= 8.58 \times 0.3$$
    $$= 2.574$$

    Thus, power required,

    $$P = \frac{850}{2.574} = 330.2 \text{ kJ/min} = 5.5 \text{ kW}$$

3. **(a)**

    $$\eta = 1 - \frac{T_2}{T_1}$$

    $$\Rightarrow \quad \frac{T_2}{T_1} = 1 - \eta$$

    $\therefore \quad COP = \frac{T_2}{T_1 - T_2} = \frac{1}{\frac{T_1}{T_2} - 1}$

    $$COP = \frac{1}{\frac{1}{1-\eta} - 1} = \frac{1-\eta}{1 - 1 + \eta}$$

    $$COP = \frac{1-\eta}{\eta} = \frac{1}{\eta} - 1$$

4. **(b)**

    For a Carnot refrigerator,

    $$COP = \frac{T_L}{T_H - T_L} = \frac{250}{60} = \frac{25}{6}$$

    $$\frac{\dot{Q}}{\dot{W}} = \frac{25}{6}$$

    $$\Rightarrow \quad \frac{\dot{W}}{\dot{Q}} = \frac{6}{25}$$

    1 TR = 3.5 kW

    1 HP = 0.746 kW

    $$\frac{HP}{TR} = \frac{3.5}{0.746} \times \frac{6}{25} = 1.126$$

5. **(d)**

    For heat pump :

    $$(COP)_{HP} = \frac{T_H}{T_H - T_L} = \frac{300}{300 - 250} = 6$$

    $$\dot{W}_{HP} = \frac{\dot{Q}}{(COP)_{HP}}$$

    For electric heater:

    $$\dot{Q} = W$$

    $$(COP)_{EH} = 1$$

    $$(COP)_{system} = \frac{\dot{Q} + \dot{Q}}{\dot{Q} + \frac{\dot{Q}}{(COP)_{HP}}} = \frac{2(COP)_{HP}}{(COP)_{HP} + 1}$$

    $$= \frac{2 \times 6}{6 + 1} = \frac{12}{7} = 1.71$$

6. **(d)**

    $$(COP)_{ref} = (COP)_{HP} - 1$$
    $$= 5 - 1 = 4$$

    or $\quad \dfrac{R.E.}{w_{in}} = 4$

    $\therefore \quad R.E. = 4 \times 1 = 4$ kW

7. **(a)**

    For carnot refrigerator,

    $$COP = \frac{T_2}{T_1 - T_2}$$

    Current COP $= \dfrac{260}{40} = 6.5$

    So decrease in COP is required.
    Let the change of temperature be 'x' then new COP would be

    $$5 = \frac{T_2 - x}{(T_1 + x) - (T_2 - x)}$$

    $\because \quad T_1 = 27 + 273 = 300$ K
    $\quad\quad T_2 = 273 - 13 = 260$ K

$$\therefore \quad 5 = \frac{260 - x}{(300 + x) - (260 - x)}$$

$$\text{or} \quad 5 = \frac{260 - x}{40 + 2x}$$

$$200 + 10x = 260 - x$$

$$11x = 60$$

$$x = \frac{60}{11} = 5.45$$

$$\therefore \quad T_2' = T_2 - x$$

$$T_2' = 260 - 5.45$$

$$T_2' = 254.55 \text{K}$$

9. (b)

$$\eta_{HE} = \frac{1}{COP_{HP}} = \frac{1}{5} = 0.2$$

10. (3.5)

$$\because \quad COP = \frac{1TR}{W} = \frac{Q_A}{W}$$

$$W = \frac{1TR}{COP} = \frac{N}{COP}$$

$$\Rightarrow \quad N = 1TR = 3.5 \text{ kW}$$

11. (a)

$$\because \quad COP = \frac{T_L}{T_H - T_L} = \frac{200}{300 - 200} = 2$$

$$\text{Also,} \quad COP = \frac{Q_A}{W}$$

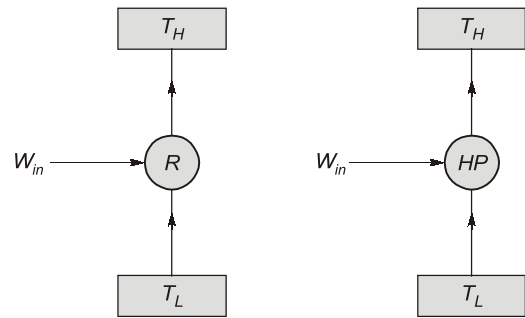$$\Rightarrow \quad W = \frac{Q_A}{COP} = \frac{1}{2} = 0.5 \text{ kW}$$

12. (b)

$$\eta = 0.75 = \frac{3}{4}$$

As we know,

$$COP_R = \frac{1}{\eta} - 1 = \frac{4}{3} - 1 = \frac{1}{3}$$

$$= 0.33$$

14. (a)

Relation between coefficient of performance of heat pump and refrigerator:

$$(COP)_R = \frac{T_L}{T_H - T_L}$$

$$(COP)_{HP} = \frac{T_H}{T_H - T_L}$$

To have relation among them, subtracting 1 on the $(COP)_{HP}$, we get

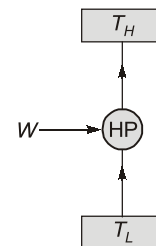$$= \frac{T_H}{T_H - T_L} - 1 = \frac{T_L}{T_H - T_L}$$

$$= (COP)_R$$

Hence,

$$(COP)_R = (COP)_{HP} - 1$$

15. (c)

Assuming initially,

$$T_H = 30°C = 303 \text{ K} \; ; \; T_L = 10°C = 283 \text{ K}$$

$$(COP)_{HP} = \frac{T_H}{T_H - T_L} = \frac{303}{20} = 15.15$$

If higher temperature is decreased by 5°C.
New conditions

$$T_H = 25°C = 298 \text{ K}$$

$$T_L = 10°C = 283 \text{ K}$$

Then the COP of heat pump is

$$(COP)_{HP} = \frac{298}{15} = 19.86$$

If lower temperature is increased by 5°C.
New conditions

$$T_H = 30°C = 303 \text{ K}$$

T_L = 15°C = 288 K
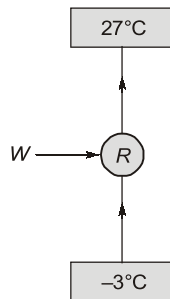
Then the COP of heat pump is

$$(COP)_{HP} = \frac{303}{15} = 20.2$$

Hence COP will be highest when lower temperature is increased rather than decreasing higher temperature by same amount.

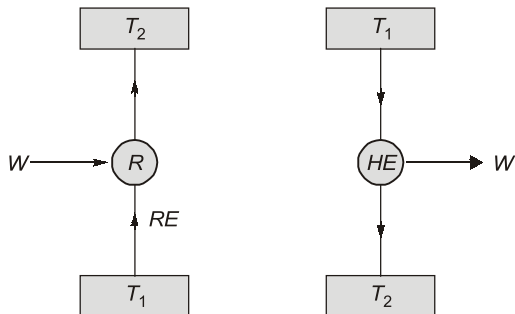**16.** **(a)**

$$COP = \frac{27+273}{30} = \frac{300}{30} = 10$$

17. **(b)**

$(COP)_R = 5,$ $\eta_{HE} = 50\% = 0.5$

$$\frac{T_1}{T_2 - T_1} = 5 = \frac{RE}{W_{in}}$$

$$\Rightarrow \qquad W_{in} = \frac{RE}{5}$$

$$\frac{T_1 - T_2}{T_1} = 0.5 = \frac{W}{Q_1}$$

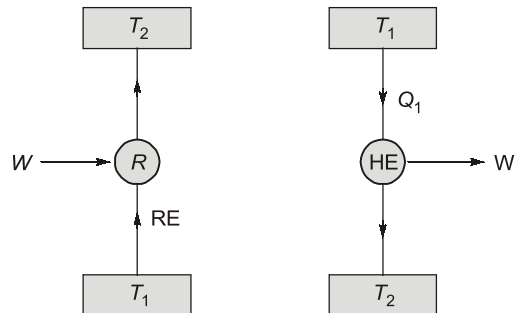$$\Rightarrow \qquad W = 0.5\, Q_1$$

By equating, we get

$$\frac{RE}{5} = \frac{Q_1}{2}$$

$$\Rightarrow \qquad \frac{Q_1}{RE} = \frac{2}{5} = 0.4$$

18. **(a)**

$$\eta_{th} = 0.4$$
$$(COP)_{ref} = 5$$

For heat engine

$$0.4 = \frac{W}{Q_1}$$

For refrigerator

$$5 = \frac{RE}{W}$$

If $\qquad RE = 1$ kJ

$$W = \frac{1}{5} \text{ kJ} = 0.2 \text{ kJ}$$

$$Q_1 = \frac{W}{0.4} = \frac{0.2}{0.4} = 0.5 \text{ kJ}$$

19. **(a)**

$$W = 700 \text{ kJ/ten minutes}$$

$$W = \frac{700}{10 \times 60} \text{kJ/s}$$

Now, $(COP)_R = \dfrac{RE}{W} = \dfrac{3.5}{700/10 \times 60}$

$$= \frac{3.5 \times 10 \times 60}{700}$$

$$= 3$$

■■■■

CONVENTIONAL BRAIN TEASERS

Q.1 A reversed Carnot cycle working as heat pump is delivering 4×10^4 kJ/min to heat the conditioned space and maintaining it at 25°C when the outside atmospheric air temperature is 15°C.

(i) Determine the heat drawn in or pumped into conditioned space from atmospheric air and power required to operate cycle.

(ii) If the same conditioned space is heated by electric coil heaters, determine consumption of electricity in terms of power consumed.

Solution:
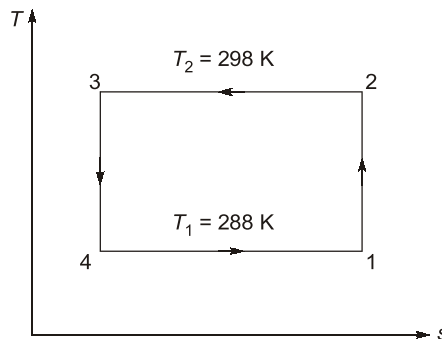
Given: $Q_R = 4 \times 10^4$ kJ/min, $T_1 = T_4 = 15°C = 288$ K, $T_2 = T_3 = 25°C = 298$ K

(i) Heat delivered per minute at 25°C is given by

$$Q_R = m(s_2 - s_3) \times T_2$$

$$\Rightarrow \qquad m(s_2 - s_3) = \frac{4 \times 10^4}{298} = 134.228 \text{ kJ/}°\text{K}$$

Therefore heat absorbed from atmosphere at 15°C

$$Q_A = m(s_1 - s_4)T_1$$

But we know that $(s_1 - s_4) = (s_2 - s_3)$

So, $\qquad\qquad\qquad Q_A = 288 \times 134.228$

$$= 38657.72 \text{ kJ/min}$$

Now, Work done per minute $= 4 \times 10^4 - 38657.72$

$$= 1342.28 \text{ kJ/min}$$

So, Power required $= \dfrac{1342.28}{60} = 22.37 \text{ kW}$

(ii) With electric heating coils, for same heat to be delivered,

Consumption of electricity $= \dfrac{4 \times 10^4}{60} = 666.67 \text{ kW}$

Q.2 "Refrigeration can be produced either by expansion of a gas or throttling of gas". Discuss the above statement.

Solution:

Refrigeration by expansion of air

According to the first law of thermodynamics applied to closed system,

$Q = W + \Delta U$

For reversible adiabatic process, $Q = 0$.

Therefore, $W = -\Delta U$

This means that there is reduction in temperature. The temperature of the gas can be reduced by an adiabatic expansion of the gas. This principle was used in the Bell-Coleman air-refrigeration system.

The effect of expansion for producing colds can be explained by the following example.

Assuming the temperature of the atmosphere is 27°C and if air is compressed isentropically with the pressure ratio 5 then the final temperature of the compressed air T_2 is given by

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{0.286} = 300(5)^{0.286} = 475\,\text{K}$$

Cold water out

Heat exchanger

Q

Water in

Expander

Compressor

Refrigerator

Air Refrigerator

This air with high temperature and pressure is cooled in the heat exchanger, then the final temperature will be 27°C under ideal condition and pressure will be 5 bar considering the pressure loss in heat exchanger is zero and if this air expands isentropically until the pressure falls to atmospheric pressure then the final temperature,

$$T_3 = \frac{300}{5^{0.286}} = 189.5\,\text{K}$$

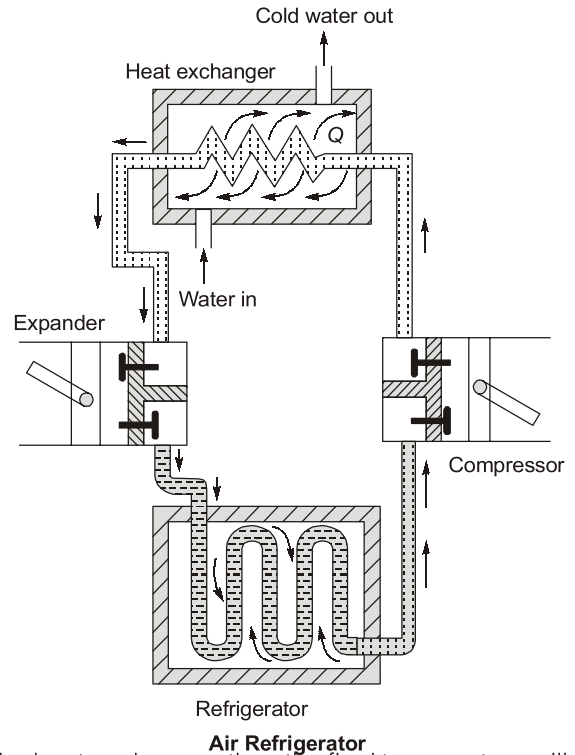which is below atmospheric temperature (300 K).

This principle is universally used for producing low temperature in all air-refrigeration systems.

Refrigeration by Throttling of the gas

As we know that adiabatic throttling process is a constant enthalpy process, and as the enthalpy is only a function of temperature and the temperature of perfect gas remains constant after and before throttling. However with actual gases the temperature of the gas after throttling may either increase, decrease or remain constant.

The term which indicates the magnitude and sign of the change in temperature is called the Joule Thomson Coefficient and is given by

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h$$

which is the change in the temperature with respect to the pressure during constant enthalpy process.

The pressure versus temperature lines are drawn as shown in figure, taking enthalpy as a parameter.


Constant enthalpy lines for actual gas
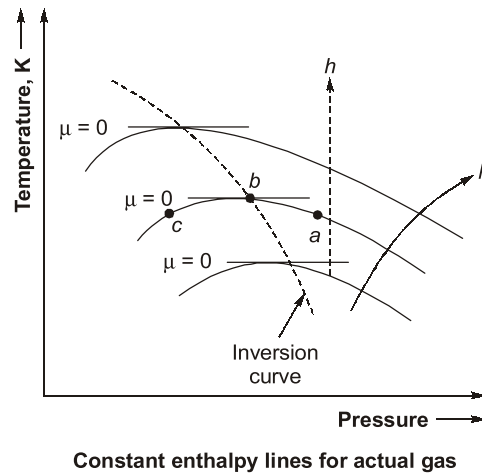
In throttling process, the pressure of the gas decreases so that motion along the curve is towards the left. The graph shows that, if throttling occurs from the point *a* to *b*, the temperature of the gas increases, and at the point *b*; the temperature is maximum, and the Joule Thomson coefficient is zero. This point is known as "Inversion point" of the gas. If throttling occurs from the point *b* to *c*, then the temperature of the gas drops.

This is obvious from figure, that for reducing the temperature of the gas after throttling, the value of μ must be positive. The amount of the temperature drop depends upon Joule Thomson coefficient, the pressure drop, and the original state of the gas. In any event, the resulting temperature will be too high for refrigerating purposes unless the original temperature is relatively low.

Use of positive values of Joule Thomson coefficient are made in the liquification of gases such as air, nitrogen and oxygen.

Q.3 A heat pump is driven by the work output of a heat engine as shown in figure. If we assume ideal devices then show that,

$$\frac{\dot{Q}_{L1} + \dot{Q}_{H2}}{\dot{Q}_{H1}} = \frac{T_{room}}{T_H} \left[\frac{T_H - T_{amb}}{T_{room} - T_{amb}} \right]$$
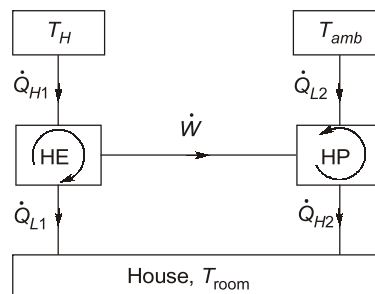
Solution:

We know,

$$(COP)_{HP} = \frac{\dot{Q}_{H2}}{\dot{W}} = \frac{\dot{Q}_{H2}}{\dot{Q}_{H2} - \dot{Q}_{L2}} = \frac{T_{room}}{T_{room} - T_{amb}}$$