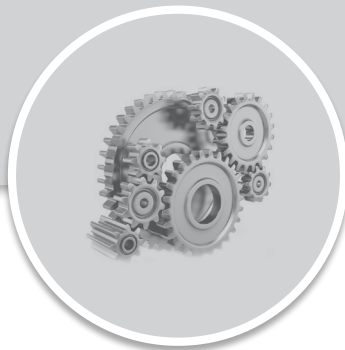


# MECHANICAL ENGINEERING

## Theory of Machines



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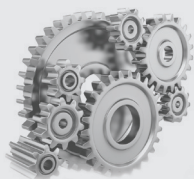


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## Theory of Machines

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## EDITIONS

First Edition : 2015  
Second Edition : 2016  
Third Edition : 2017  
Fourth Edition : 2018  
Fifth Edition : 2019  
Sixth Edition : 2020  
Seventh Edition : 2021  
Eighth Edition : 2022

**Ninth Edition : 2023**

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# Simple Mechanisms

## 1.1 INTRODUCTION

The theory of machines is an applied science used to understand the relationships between the geometry and relative motions of the parts of a machine or mechanism and the forces which produce these motions. It comprises the study of relative motions between the various parts of a machine and the study of the forces that act on these parts.

The major objectives of the theory of machines and mechanisms is to provide engineers with the necessary tools to systematically synthesize a system which means scientifically arriving at the critical shapes and dimensions of the bodies constituting the system.

## 1.2 ELEMENTS OR LINKS

A link or an element is a rigid body which possesses at least two nodes which are points of attachments to other links.

OR

Each part of a machine which has motion relative to some other part is termed an element or a link.

### 1.2.1 Types of Links

Links can be classified as binary, ternary, or quaternary depending upon the ends on which revolute or turning pairs can be placed as shown in figure below.

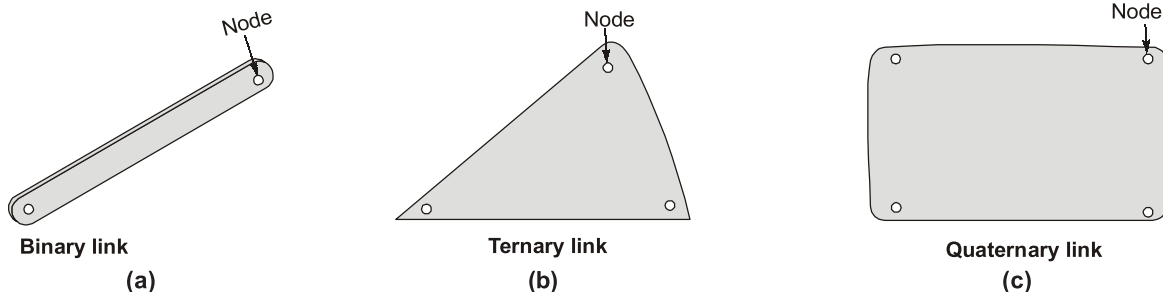


Fig. Types of Links

**Binary Link** : Rigid body with two nodes

**Ternary Link** : Rigid body with three nodes

**Quaternary Link** : Rigid body with four nodes

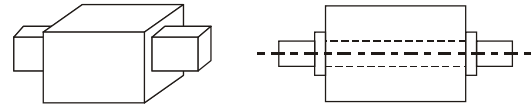
Kinematic links can be divided into three types :

- **Rigid Link** : It does not undergo any deformation while transmitting any motion. Links in general are elastic in nature. Links are considered rigid if they do not undergo any appreciable deformation while transmitting motion, e.g. connecting rod, crank, valve stem of camshaft etc.
- **Flexible Link** : It is one which is partly deformed in a manner not to affect the transmission of motion, e.g. belts, ropes, springs etc.
- **Fluid Link** : It is deformed by having fluid in a closed vessel and the motion is transmitted through the fluid by pressure, e.g. hydraulic jack, hydraulic brake in automobiles etc.

### 1.3 TYPES OF CONSTRAINED MOTION

There are three types of constrained motion:

- **Completely constrained motion** : When the motion between two elements of a pair is in a definite direction irrespective of the direction of the force applied is known as completely constrained motion. Only one independent motion is possible and other motions are restricted by the system itself.



*E.g.*

- Motion of square cross-section shaft in square cross-section hole
- Rotation of circular shaft in circular hole attached with collar.

- **Successfully constrained motion** : When the motion between two elements of a pair is possible in more than one direction but with the help of some external means it is made to have motion in only one direction is known as successfully constrained motion.

*E.g.*

- A shaft in the footstep bearing.
- Piston in the cylinder of an IC engine.

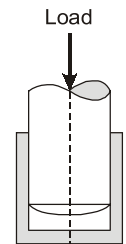
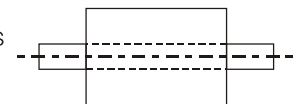


Fig. Footstep bearing

- **Incompletely constrained motion** : When the motion between two elements of a pair is possible in more than one direction.

*E.g.*

- Motion of round bar in a round hole.

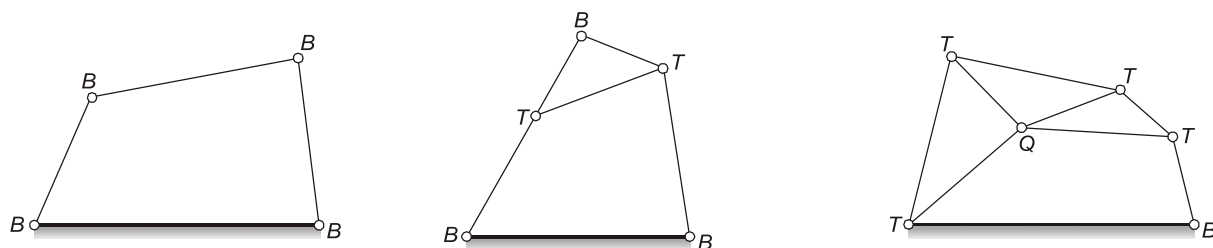


### 1.4 KINEMATIC JOINT

A kinematic joint is the connection between two links by a pin. There is ample clearance between the pin and the hole in the ends of the links being connected to provide free motion of the links.

The usual types of joints in a chain are as shown in given figure.

- **Binary joint** : Two links are connected at the same joint by a pin.
- **Ternary joint** : Three links are connected at the same joint by a pin.
- **Quaternary joint** : Four links are connected at the same joint by a pin.



**Fig.** Types of Kinematic joints

$B \rightarrow$  Binary joints;  $T \rightarrow$  Ternary joints;  $Q \rightarrow$  Quaternary joints

**NOTE :** If  $n$  number of links are connected at a joint, it is equivalent to  $(n - 1)$  binary joints.

## 1.5 KINEMATIC PAIR

The two links of a machine, when in contact with one another, are said to form a pair. A kinematic pair consists of two links that have relative motion between them. The links of a mechanism must be connected together in such a manner that they transmit motion from the driver or input link to the follower or output link.

Kinematic pairs can be classified according to :

- Nature of contact
- Nature of mechanical constraint
- Nature of relative motion

### 1.5.1 Kinematic Pairs according to Nature of Contact

- **Lower Pair :** When the two elements have surface (or area) contact while in motion and the relative motion is purely turning or sliding, they are called as lower pair. All sliding pairs, turning pairs and screw pairs are lower pairs (Table).

*E.g.*

- Nut turning on a screw, shaft rotating in a bearing, all pairs of a slider-crank mechanism, universal joint etc.

**Table**

Name	Relative Motion	Degree of Freedom
Rigid joint	0 rotation 0 translation	0
Revolute	1 rotation 0 translation	1
Prismatic	0 rotation 1 translation	1
Helical	1 rotation 1 translation	1
Cylindrical	1 rotation 1 translation	2
Spherical	3 rotation 0 translation	3
Planar	1 rotation 2 translation	3



- **Higher Pair** : A pair of links having a point or line contact between the members is called a higher pair (Table). The contact surfaces of the two links are dissimilar.

*E.g.*

- Contact between cam and follower, contact between two mating gears, a wheel rolling on a rail, ball rolling on a flat surface, ball and roller bearings.

**Table**

Description	Degree of Freedom
Cylindrical surface on a plane without slipping	1
Cylindrical surface on a plane with slipping	2
Ball on a plane without slipping	3
Point on a plane with slipping	4

### 1.5.2 Kinematic Pairs according to the Relative Motion

- **Sliding pair** : If two links have a sliding motion relative to one another, they form a sliding pair.  
*Example* : Rectangular rod in a rectangular hole in a prism, piston and cylinder of an engine, cross-head and guides of a steam engine, ram and its guide in shaper etc.
- **Turning (Revolute pair)** : When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair.  
*Example* : Four bar chain, crankshaft turning in a bearing etc.
- **Rolling pair** : When two links of a pair have a rolling motion relative to one another, they form a rolling pair.  
*Example* : Ball and roller bearings, wheel rolling on a flat surface etc.
- **Screw pair (Helical pair)** : If two mating links have a turning as well as sliding motion between them, they form a screw pair.  
*Example* : Bolt with a nut, lead screw and nut of a lathe etc.
- **Spherical pair** : When one element in the form of a sphere turns about the other fixed element, it forms a spherical pair.  
*Example* : Ball and socket joint.

### 1.5.3 Kinematic Pairs according to Nature of Mechanical Constraint

- **Closed pair** : When two elements of a pair are held together mechanically, it forms a closed pair. All the lower pairs and some of the higher pairs are closed pairs.  
*Example* : Sliding pairs, turning pairs, spherical pairs, screw pairs.
- **Open pair (Unclosed pair)** : When two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be forced-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

**EXAMPLE : 1.1**

Which of the following statement(s) is/are correct?

- (a) Round bar in a round hole forms a turning pair.
- (b) A square bar in a round hole also forms a turning pair.
- (c) A square bar in a square hole forms a prismatic pair.
- (d) A vertical shaft in a foot-step bearing forms a completely constrained motion.

[MSQ]

**Solution : (a, c)**

Option (b) is wrong because square bar do not match with the circular profile of round hole, so it can not form a turning pair.

Option (d) is wrong because a vertical shaft in a foot-step bearing is successfully constrained motion.



- Screw a helical pair has only one degree of freedom because the sliding and rotational motions are related by the helix angle of the thread. If the helix angle is made zero, the screw pair becomes a turning pair and if it is made  $90^\circ$ , the screw pair becomes a sliding pair.
- Rotating pin joint and translating slider joint are also referred as full joints (i.e. full = 1 DOF) and they are lower pairs.
- Two-freedom joint (2 DOF) is sometimes referred to as a “half joint”. The half joint is also called a roll-slide joint because it allows both rolling and sliding.

## 1.6 DEGREES OF FREEDOM (DOF)

An unconstrained rigid body moving in space possesses six degrees of freedom, as shown in given figure.

**Definition of DOF :** One of the first concerns in either the design or the analysis of a mechanism is the number of degrees of freedom, also called the mobility of the device. The mobility or the degree of freedom is the number of input parameters (usually joint variables) that must be controlled independently to bring the device into a particular posture.

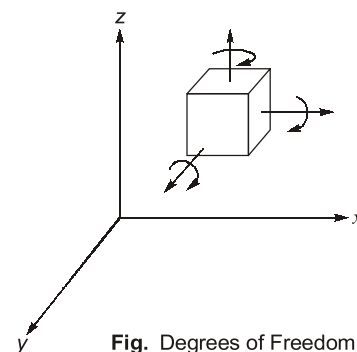


Fig. Degrees of Freedom

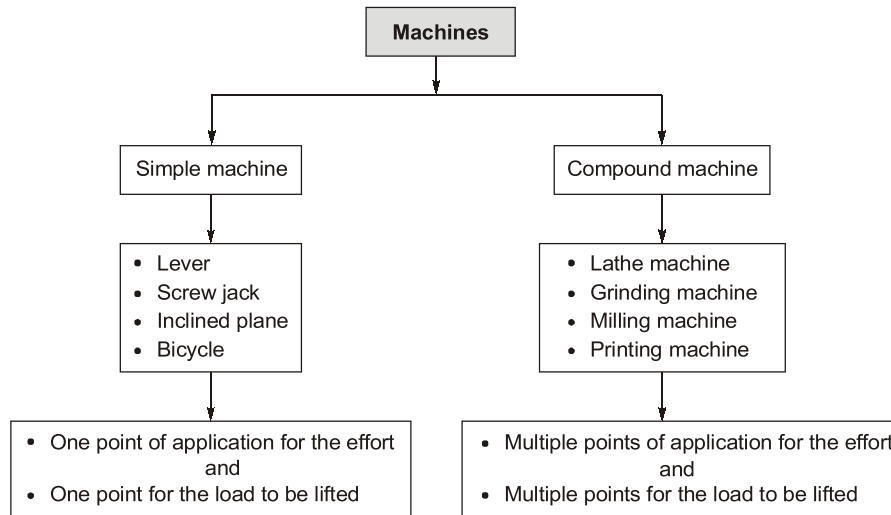
## 1.7 MACHINE, MECHANISM AND KINEMATIC CHAIN

**Machine:** Combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions

**Mechanism:** A mechanism is defined as an assemblage of resistant bodies, connected by movable joints, to form a closed kinematic chain with one link fixed and having the purpose of transforming motion.

**Kinematic chain:** It is defined as “an assemblage of links and joints, interconnected in a way so as to provide a controlled output motion in response to a supplied input motion”.

**1.7.1 Machines:** The layout of machines classification is shown in figure.



**Fig. Machines Classification**

### 1.7.2 Kinematic Chain

A kinematic chain may be defined as an assembly of links in which the relative motion of the links is possible and the motion of each link relative to the others is definite. The last link of the kinematic chain is attached to the first link.

The following relationship holds for a kinematic chain having lower pair only :

$$L = 2P - 4$$

$$J = \frac{3}{2}L - 2$$

where,

$L$  = number of binary links

$P$  = number of lower pairs

$J$  = number of binary joints

If LHS > RHS, then chain is called locked or redundant chain

LHS = RHS, then chain is constrained

LHS < RHS, then chain is unconstrained

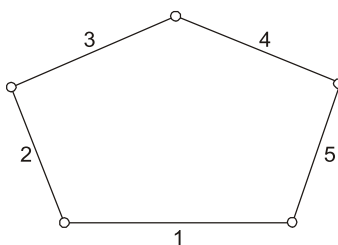
For a kinematic chain having higher pairs, each higher pair is taken equivalent to two lower pairs and an additional link.

$$J + \frac{H}{2} = \frac{3}{2}L - 2$$

where,  $H$  = number of higher pairs

**EXAMPLE : 1.2**

What is the type of following five-bar chain?



- |                         |                       |
|-------------------------|-----------------------|
| (a) Locked chain        | (b) Constrained chain |
| (c) Unconstrained chain | (d) Redundant chain   |

**Solution : (c)**

$$\begin{aligned}
 J &= 5 \\
 L &= 5 \\
 P &= 5 \\
 L &= 2P - 4 \\
 5 &= 2 \times 5 - 4 = 6 \\
 \Rightarrow \quad \text{LHS} &< \text{RHS} \\
 \text{Now,} \quad J &= \frac{3}{2}L - 2 \\
 5 &= \frac{3}{2} \times 5 - 2 = 5.5 \\
 \therefore \quad \text{LHS} &< \text{RHS} \\
 \text{Hence, it is an unconstrained chain.}
 \end{aligned}$$

**EXAMPLE : 1.3**

Open chain mechanism finds application in

- |                            |                           |
|----------------------------|---------------------------|
| (a) Slider crank mechanism | (b) Manipulator of robot  |
| (c) Hart's mechanism       | (d) Scotch-Yoke mechanism |

**Solution : (b)**

### 1.7.3 Mechanisms

When one of the links of a kinematic chain is fixed, the chain is known as mechanism. It is used for transmitting or transforming the input motion.

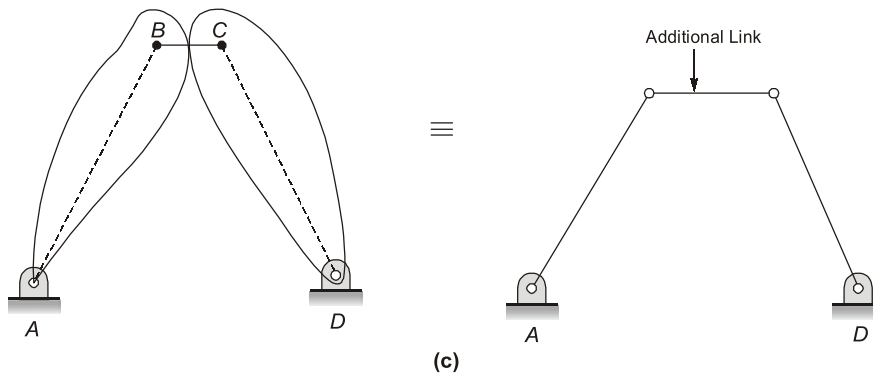
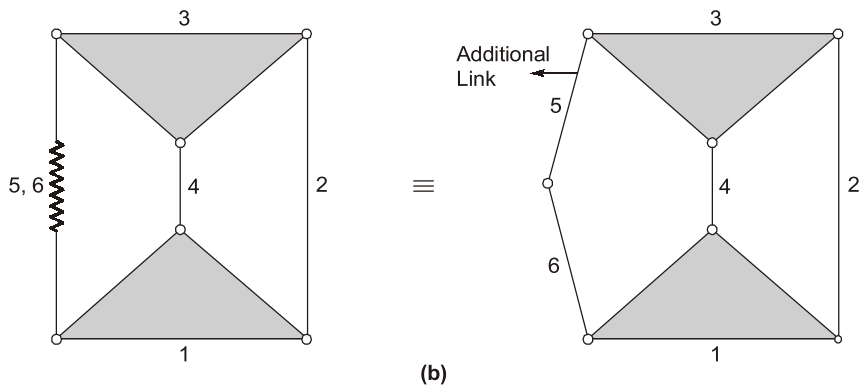
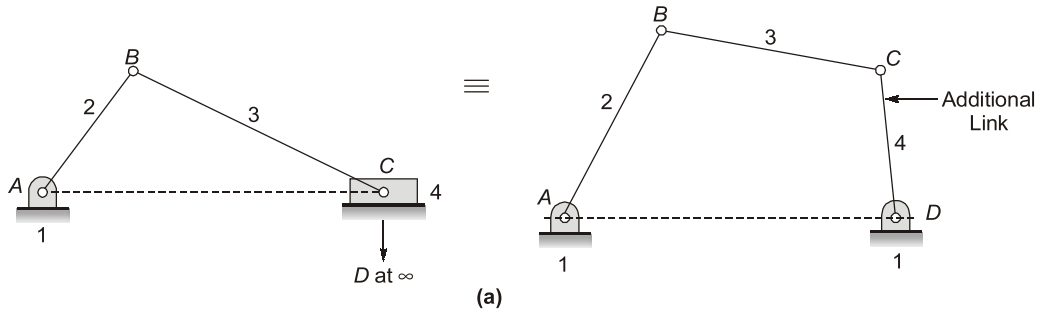
The mechanisms are of following types:

- **Simple mechanism** : A mechanism having four links.
- **Compound mechanism** : A mechanism having more than four links.
- **Complex mechanism** : It is formed by the inclusion of ternary or higher order floating link to a simple mechanism.

- **Planar mechanism** : When all the links of a mechanism lie in the same plane.
- **Spatial mechanism** : When the links of a mechanism lie in different planes.

**Equivalent Mechanisms**

- A sliding pair is equivalent to a turning pair [Fig. (a), (c)].
- A spring can be replaced by two binary links [Fig. (b)].
- A cam pair can be replaced by one binary link together with two turning pairs at each ends Fig. (d).





### OBJECTIVE BRAIN TEASERS

- The purpose of a link is NOT to
  - transmit motion
  - take account of small deflections
  - guide other links
  - act as a support
- Piston reciprocating inside a cylinder in an internal combustion engine undergoes:
  - Incompletely constrained motion
  - Completely constrained motion
  - Successfully constrained motion
  - None of these
- Which of the following is not a lower pair?
  - ball and roller bearings
  - a universal joint
  - all pairs of a slider crank mechanism
  - a shaft rotating in a bearing
- Consider the following type of joints (pairs):
 

1. Revolute	2. Plane
3. Gear pair	4. Prism
5. Helix	

 Which of the above have two degrees of freedom?
  - 2 only
  - 3 and 5
  - 2, 3 and 4
  - 3 only
- In robotics, manipulators designed to simulate human arm and hand motion are an example of
  - closed kinematic chains
  - open kinematic chains
  - closed pairs
  - unclosed pairs
- Which of the following is an example of unclosed pair?
  - All lower pairs
  - All closed pairs
  - The enclosed cam and follower
  - The cam and spring loaded follower pair
- In kinematic chain, a quaternary joint is equivalent to
 

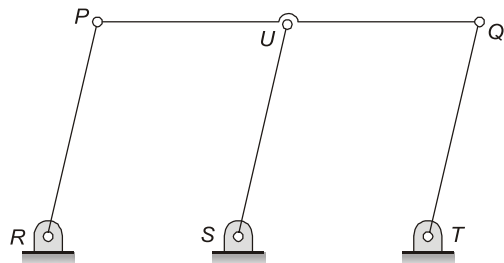
(a) six binary joints	(b) One binary joints
(c) two binary joints	(d) three binary joints
- Match the following with respect to spatial mechanism:
 

Type of Joint	Degree of constraints
P. Cylindrical	1. Three
Q. Spherical	2. Five
	3. Four
	4. Two

  - P - 3, Q - 3
  - P - 4, Q - 3
  - P - 3, Q - 1
  - P - 5, Q - 3
- Mobility of a statically indeterminate structure is
  - zero
  - one
  - negative
  - $\geq 2$
- Select the CORRECT statements.
  - Grashof's rule states that for a planer crank-rocker four bar mechanism, the sum of the shortest and longest link lengths is always less than the sum of the remaining two link lengths.
  - Geneva mechanism is a continuous motion device.
  - Gruebler's criterion assumes mobility of a planer mechanism to be one.
  - Inversions of a mechanism are created by fixing different links one at a time.

[MSQ]

- The  $PQ$  is a single link in a double-parallelgram mechanism as shown below. The correct statement pertaining to this mechanism is:

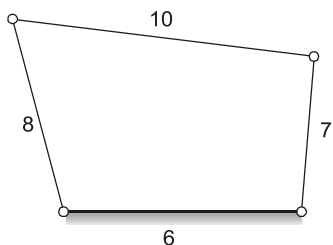


- It is five link, four turning pair mechanism.
- It is a structure with zero degree of freedom.
- Function of the mechanism is greatly affected if link  $QT$  is removed.
- It is a Gruebler paradox.

12. A planar mechanism has 6 links and 7 rotary joints. The number of degrees of freedom of the mechanism, using Gruebler's criterion, is \_\_\_\_.

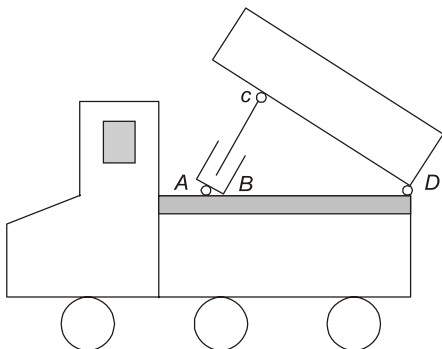
13. Grashof's law approach for planar mechanism is to analyse  
(a) Continuous relative motion  
(b) Velocity and acceleration  
(c) Mobility  
(d) Dynamic-static analysis

14. What is the type of mechanism as shown below?



(a) It is a double-crank mechanism.  
(b) It is a class-III Grashof mechanism.  
(c) It is a double-rocker mechanism.  
(d) It is a crank-rocker mechanism.

15. What is the degrees of freedom of the mechanism for the dump truck as shown below?



(a) 0  
(b) 1  
(c) 2  
(d) Data insufficient

16. Passive or redundant degree of freedom means that  
(a) it transforms a planar mechanism into structure.  
(b) it increase mobility of the mechanism.  
(c) it influences the relationship of the input and output motion of the mechanism.

(d) it does not influences the relationship of the input and output motion of the mechanism.

17. The function of linkages to achieve primary goal of the mechanism includes:

P. Function generation  
Q. Path generation  
R. Motion generation  
(a) P, Q, R  
(b) P, Q  
(c) Q, R  
(d) R only

18. For a crank and slotted lever mechanism, the length of the stroke is 340 mm. Line of stroke passes through the extreme positions of the free end of the lever. Length of slotted lever is 430 mm. What is the ratio of time of cutting stroke to the time of return stroke?

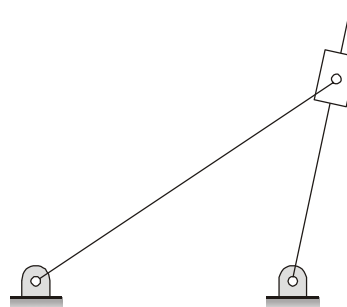
(a) 1.5  
(b) 1.6  
(c) 1.7  
(d) 1.8

19. The distance between two parallel shafts connected by Oldham's coupling is 30 mm. The driving shaft revolves at 240 rpm. The maximum speed of sliding of the tongue of the intermediate piece along its groove is \_\_\_\_ m/s. (Correct upto two decimal places)

20. The number of equivalent lower pairs of a higher pairs are

(a) One  
(b) Two  
(c) Three  
(d) Four

21. What is mobility of the following planar linkage?



(a) 0  
(b) 1  
(c) 2  
(d) -1

**ANSWER KEY**

1. (b) 2. (c) 3. (a) 4. (d) 5. (b)  
6. (d) 7. (d) 8. (c) 9. (c)  
10. (c,d) 11. (d) 12. (1) 13. (a) 14. (c)  
15. (b) 16. (d) 17. (a) 18. (c)  
19. (0.75) 20. (b) 21. (b) 22. (d) 23. (0)  
24. (a) 25. (c) 26. (b,c) 27. (a) 28. (b)  
29. (a)

**HINTS & EXPLANATIONS**

2. (c)

A kinematic pair is said to be partially or successfully constrained if the relative motion between its links occurs in a definite direction, not by itself, but by some other means. Normally, when a piston is placed in a cylinder, it may undergo reciprocating motion (upward and downward motion) and turning motion, depending on the external forces applied. It is incompletely constrained. However, if piston is connected to a connecting rod, its motion is successfully constrained i.e it can only undergo only reciprocating motion inside the cylinder. Here, some other means i.e connecting rod is used for successfully constraining the motion of piston.

3. (a)

Belt, rope and chain drives, gears, the cam and follower, ball and roller bearing etc. all form higher pairs.

4. (d)

Revolute  $\rightarrow$  1 - DOF ( $\theta$ )  
Plane  $\rightarrow$  3 - DOF ( $x, y, \theta$ )  
Gear pair  $\rightarrow$  2 - DOF (rolling and sliding)  
Prism  $\rightarrow$  1- DOF ( $x$ )  
Helix  $\rightarrow$  1- DOF ( $x$  or  $\theta$ )

6. (d)

All lower pairs and some higher pairs (for example, the enclosed cam and follower) are closed pairs.

When the two elements of a pair are not held mechanically and are held in contact by the action of external forces, the pair is called an unclosed pair, for example, the cam and spring loaded follower pair.

8. (c)

**Spherical joint :**

3 restraints on translatory motion.

**Cylindrical joint :**

2 restraints on translatory motion.

2 restraints on rotary motion.

9. (c)

Mobility or DOF of a statically indeterminate structure is always less than zero.

As per Gruebler's criterion:

$$\text{DOF} = 3(n - 1) - 2j - h$$

where  $n$  is number of movable links.

for structure,  $n$  is 0.

So, DOF is -ve value ( $\because j > 0, h \geq 0$ )

10. (c)

(i) Geneva mechanism is an intermittent motion device.

(ii) Grashof's rule states that for a planar crank-rocker four bar mechanism,  
 $(L + S) \leq (P + Q)$ .

12. 1 (1 to 1)

$$F = 3(n - 1) - 2j$$

Given,  $n = 6, j = 7$

$$\begin{aligned} F &= 3(6 - 1) - 2 \times 7 \\ &= 15 - 14 = 1 \end{aligned}$$

14. (c)

Length of the longest link = 10 ( $L$ )

Length of the shortest link = 6 ( $S$ )

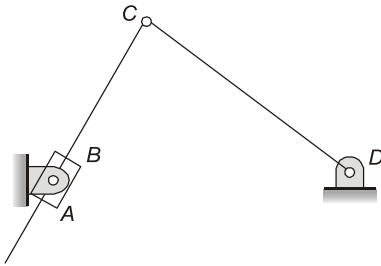
Length of the other links = 8 and 7 ( $P$  and  $Q$ )

Since  $(L + S) > (P + Q)$ , it does not belong to the class-I mechanism therefore, it is a double rocker mechanism.



15. (b)

The mechanism in the dump truck can also be shown as below:



Number of links = 4

Number of lower pairs = 4 (at A, B, C, D)

Number of higher pairs = 0

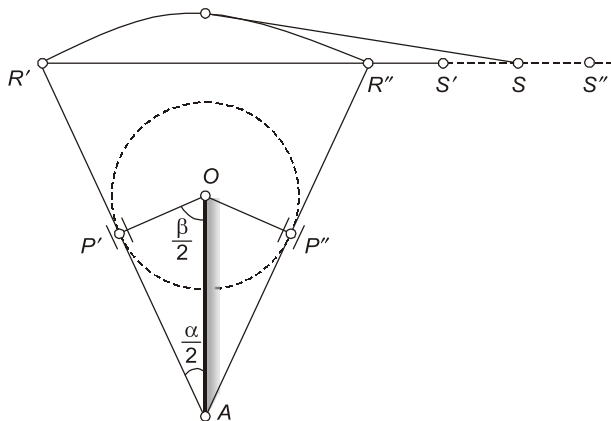
$$F = 3(4 - 1) - 2 \times 4 - 1 \times 0 = 1$$

17. (a)

The function of a link mechanism is to produce rotating, oscillating, or reciprocating motion from the rotation of a crank or vice-versa. Linkages have many different functions, which can be classified according on the primary goal of the mechanism:

- (i) Function generation: the relative motion between the links connected to the frame.
- (ii) Path generation: the path of a tracer point.
- (iii) Motion generation: the motion of the coupler link.

18. (c)



Length of stroke

$$S'S'' = R'R''$$

$$= 2AR' \sin\left(\frac{\alpha}{2}\right)$$

$$\text{or } 340 = 2 \times 430 \times \sin\left(\frac{\alpha}{2}\right)$$

$$\text{or } \frac{\alpha}{2} = 23.287^\circ$$

$$\begin{aligned} \text{or } \alpha &= 46.6^\circ \\ \beta &= 180^\circ - \alpha \\ &= 180^\circ - 46.6^\circ \\ &= 133.4^\circ \end{aligned}$$

$$\begin{aligned} \frac{\text{Time of cutting}}{\text{Time of return}} &= \frac{360^\circ - \beta}{\beta} \\ &= \frac{360^\circ - 133.4^\circ}{133.4^\circ} = \frac{226.6}{133.4} \\ &= 1.698 \end{aligned}$$

19. 0.75 (0.60 to 0.80)

Distance between axis,

$$d = 30 \text{ mm}$$

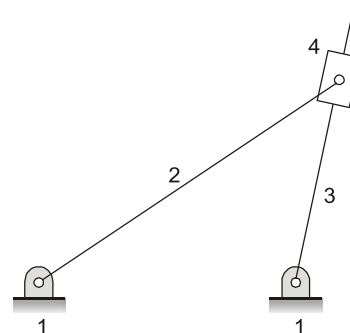
$$N = 240 \text{ rpm}$$

$$\begin{aligned} \omega &= \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 240}{60} \\ &= 25.133 \text{ rad/s} \end{aligned}$$

Maximum velocity of sliding,

$$\begin{aligned} V &= \omega d \\ &= 25.133 \times 0.03 \\ &= 0.754 \text{ m/s} \end{aligned}$$

21. (b)



$$n = 4 \text{ (Binary links)}$$

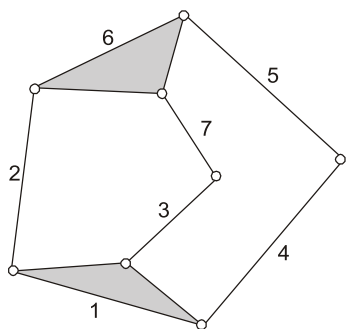
$$j = 4 \text{ (lower pairs)}$$

$$\text{DOF} = 3(n - 1) - 2j - h$$

$$\text{DOF} = 3(4 - 1) - 2 \times 4 - 0$$

$$\text{DOF} = 9 - 8 - 0 = 1$$

22. (d)

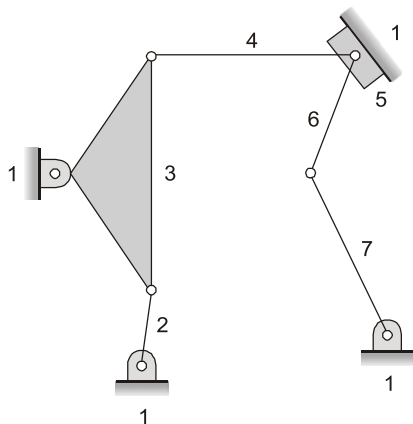


Number of links = 7

Number of joints = 8

$$\begin{aligned} F &= 3(n-1) - 2j - h \\ &= 3(7-1) - 2 \times 8 - 0 \\ &= 3 \times 6 - 2 \times 8 = 2 \end{aligned}$$

23. 0 (0 to 0)



Number of links = 7

Number of lower pairs = 9

Note that,

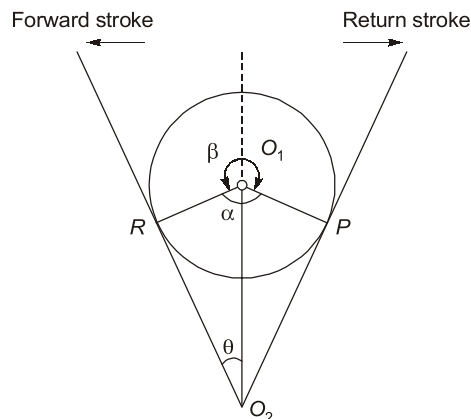
lower pair between link 4 and 5 = 1

lower pair between link 6 and 5 = 1

lower pair between link 5 and 1 = 1

$$\begin{aligned} F &= 3(n-1) - 2j - h \\ &= 3(7-1) - 2 \times 9 - 0 \\ &= 3 \times 6 - 2 \times 9 \\ &= 18 - 18 = 0 \end{aligned}$$

24. (a)



Given  $O_1P = r = 140$  mm

forward to return ratio = 5 : 2

Time of cutting (forward) stroke

Time of return stroke

$$= \frac{\beta}{\alpha} = \frac{360^\circ - \alpha}{\alpha}$$

$$\angle RO_1O_2 = \frac{5}{2}$$

$$\text{or } 720^\circ - 2\alpha = 5\alpha$$

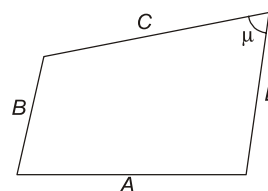
$$\text{or } \frac{720^\circ}{7} = \alpha = 102.86^\circ$$

$$\frac{\alpha}{2} = 51.43^\circ$$

$$\sin\left(90^\circ - \frac{\alpha}{2}\right) = \frac{O_1R}{O_1O_2}$$

$$\begin{aligned} O_1O_2 &= \frac{O_1R}{\cos \frac{\alpha}{2}} = \frac{140}{\cos 51.43^\circ} \\ &= 224.55 \text{ mm} \end{aligned}$$

25. (c)



A : ground link

B : input link

C : coupler

D : output link

So,  $\mu$  is the angle between coupler and output link.

29. (a)

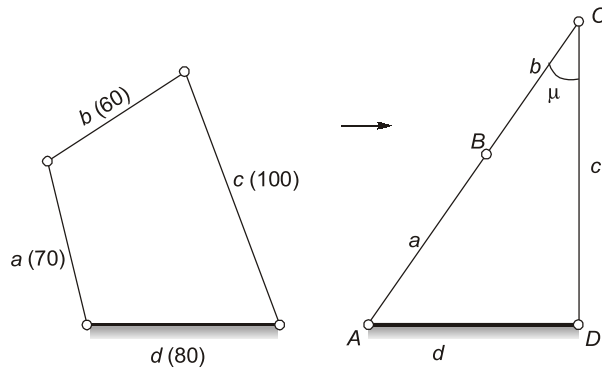
Length of the longest link = 100 mm

Length of the shortest link = 60 mm

Length of other links = 80 and 70

Since  $(L + S = 160 \text{ mm}) > (P + Q = 150 \text{ mm})$ ,

it belongs to the class II mechanism and this is a double-rocker mechanism.



$$d^2 = (a + b)^2 + c^2 - 2(a + b)c \cos \mu$$

$$\text{or } 80^2 = (70 + 60)^2 + 100^2 - 2(70 + 60) \times 100 \cos \mu$$

$$\text{or } 6400 = 16900 + 10000 - 26000 \cos \mu$$

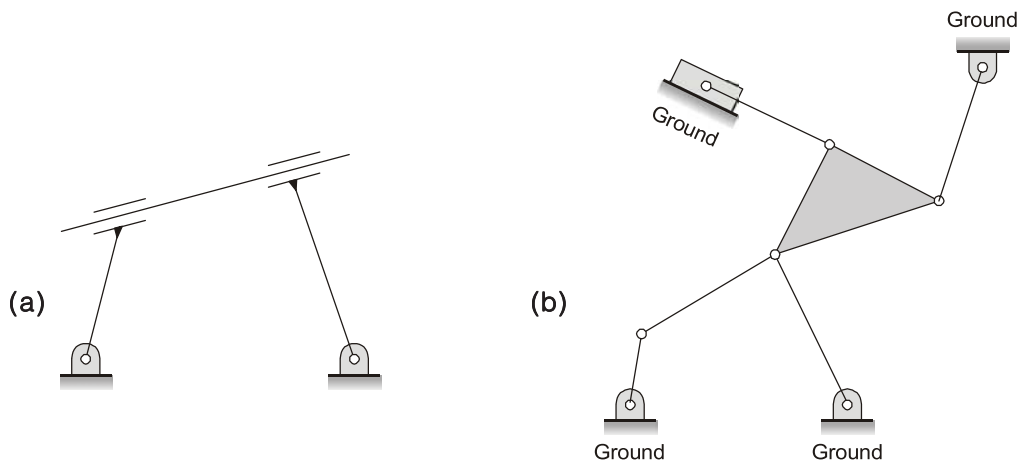
$$\text{or } \cos \mu = \frac{205}{260} = 0.788$$

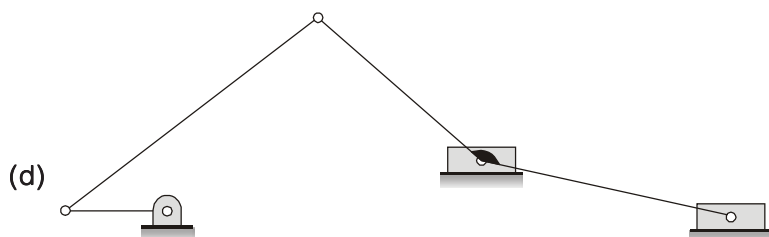
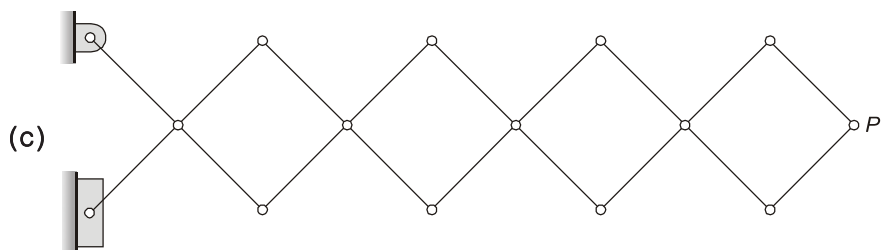
$$\mu = 37.958^\circ \approx 38^\circ$$



## CONVENTIONAL BRAIN TEASERS

1. Determine the degree of freedom of the following mechanism:





**Solution :**

(a) In this mechanism, rod 3 can slide without causing any movement in the rest of the mechanism.

Number of links,  $n = 4$

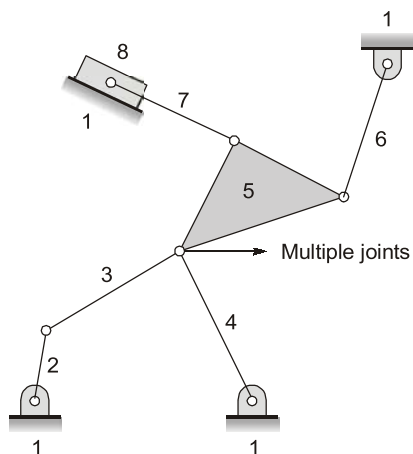
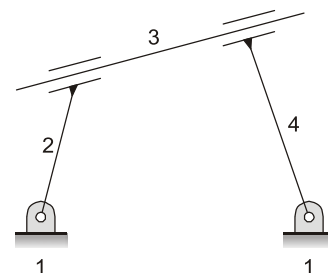
Number of joints,  $j = 4$

Redundant degree of freedom = 1

$$\begin{aligned} \therefore F_e &= 3(n-1) - 2j - F_r \quad (\because \text{There is no higher pair}) \\ &= 3(4-1) - 2 \times 4 - 1 \\ &= 3 \times 3 - 8 - 1 = 0 \end{aligned}$$

As the effective degree of freedom is zero, it is a locked system. In this case, Gruebler's criterion is NOT APPLICABLE.

(b)



Ground link is link no 1

Number of links,  $n = 8$

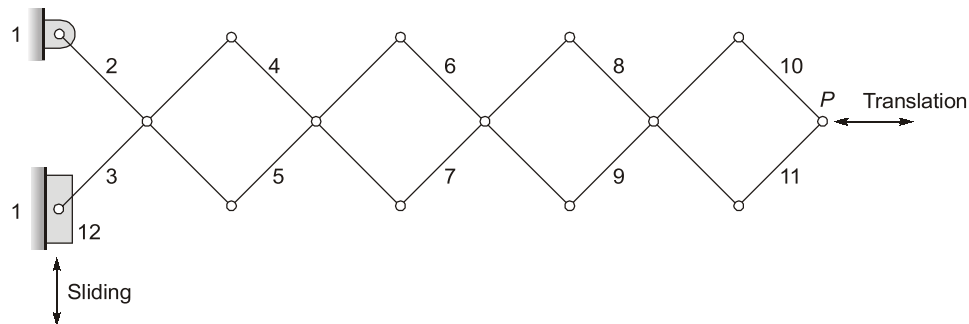
Number of joints or lower pairs,  $j = 10$

$$F = 3(n - 1) - 2j - h$$

$$= 3(8 - 1) - 2 \times 10 = 1$$

There is no higher pair in this mechanism and this mechanism is constrained with  $\text{DOF} = 1$ .

(c)



Link 1 is grounded link

Number of links,

$$n = 12$$

Number of lower pairs,

$$j = 16$$

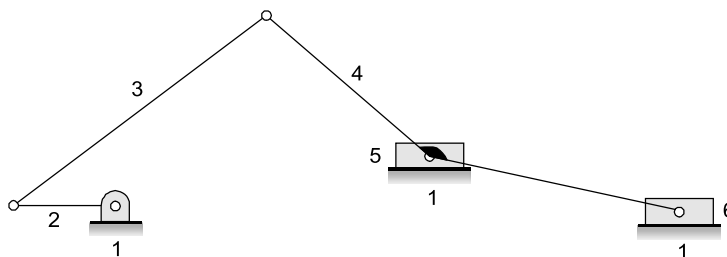
$$F = 3(n - 1) - 2j - h$$

$$F = 3(12 - 1) - 2 \times 16 = 1$$

$$F = 33 - 32 = 1$$

As the degree of freedom is one, this is constrained mechanism.

(d)



Link 4 is the link connecting link no 5 and 6

Link 4 is only one link (it is not two)

Number of link,

$$n = 6$$

Number of lower pairs,

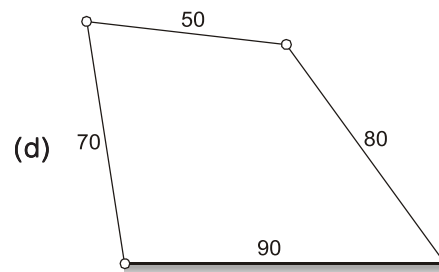
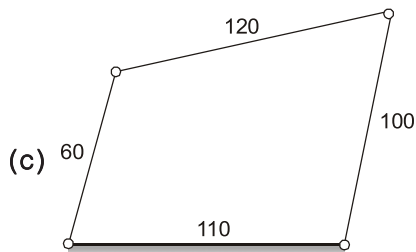
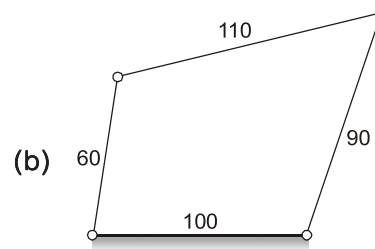
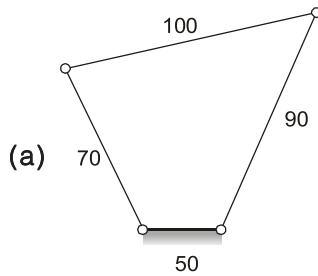
$$j = 7$$

$$F = 3(n - 1) - 2j - h$$

$$= 3(6 - 1) - 2 \times 7 - 0$$

$$= 15 - 14 = 1$$

2. Identify the nature of each mechanism shown in Fig. below :



(All dimensions in mm)

**Solution :**

(a)  $L_1 = 50$  mm,  $L_2 = 70$  mm,  $L_3 = 100$  mm,  $L_4 = 90$  mm  
 $L = 100$  mm,  $S = 50$  mm,  $P + Q = 70 + 90 = 160$  mm  
 $(L + S = 150 \text{ mm}) < (P + Q = 160 \text{ mm})$   
Hence Grashof's law is satisfied.  
 $L_1 < L_2$  or  $L_3 < L_4$  i.e.  $50 < 70$  or  $100 < 90$ , here valid  
 $L_3 + L_4 - L_2 = 100 + 90 - 70 = 120$  mm  
 $\therefore (L_1 = 50 \text{ mm}) < 120$  mm, Hence valid  
 $|L_3 - L_4| + L_2 = |100 - 90| + 70 = 80$  mm  
 $\therefore (L_1 = 50 \text{ mm}) < 80$ , Hence valid  
Here the shortest link  $L_1$  is fixed. Here, it is a double crank (or drag link) mechanism.

(b)  $L_1 = 100$  mm,  $L_2 = 60$  mm,  $L_3 = 110$  mm,  $L_4 = 90$  mm  
 $L + S = 110 + 60 = 170$  mm  
 $P + Q = 100 + 90 = 190$  mm  
As  $(L + S) < (P + Q)$ , Grashof's law is satisfied.  
 $L_2 < L_1$  or  $L_3$  or  $L_4$ . Hence valid  
 $L_3 + L_4 - L_2 = 110 + 90 - 60 = 140$  mm  
 $(L_1 = 100 \text{ mm}) < (L_3 + L_4 - L_2 = 140 \text{ mm})$ . Hence valid.  
 $|L_3 - L_4| + L_2 = |110 - 90| + 60 = 80$  mm  
 $(L_1 = 100 \text{ mm}) > (L_3 + L_4 - L_2 = 80 \text{ mm})$ . Hence valid.  
The link  $L_1$  adjacent to the shortest link  $L_2$  is fixed. Therefore, it is a crank-rocker mechanism.

- (c)  $L_1 = 110 \text{ mm}$ ,  $L_2 = 60 \text{ mm}$ ,  $L_3 = 120 \text{ mm}$ ,  $L_4 = 100 \text{ mm}$

$$L + S = 120 + 60 = 180 \text{ mm}$$

$$P + Q = 110 + 100 = 210 \text{ mm}$$

As  $(L + S) < (P + Q)$ , Grashof's law is satisfied.

$L_2 < L_1$  or  $L_3$  or  $L_4$ . Hence valid

$$L_3 + L_4 - L_2 = 120 + 100 - 60 = 160 \text{ mm}$$

$\therefore L_1 < (L_3 + L_4 - L_2)$ , Hence valid.

$$|L_3 - L_4| + L_2 = |120 - 100| + 60 = 80 \text{ mm}$$

$\therefore L_1 > (L_3 - L_4 + L_2)$ , Hence valid.

The link  $L_1$  adjacent to the shortest link  $L_2$  is fixed. Therefore, it is a crank-rocker mechanism.

- (d)  $L_1 = 90 \text{ mm}$ ,  $L_2 = 70 \text{ mm}$ ,  $L_3 = 50$ ,  $L_4 = 80 \text{ mm}$

$$L + S = 90 + 50 = 140 \text{ mm}$$

$$P + Q = 70 + 80 = 150 \text{ mm}$$

As  $(L + S) < (P + Q)$ , Grashof's law is satisfied.

$L_3 < L_1$  or  $L_2$  or  $L_4$ . Hence valid

$$L_3 + L_4 - L_2 = 120 + 100 - 60 = 160 \text{ mm}$$

$\therefore L_1 < (L_3 + L_4 - L_2)$ , Hence valid.

The link  $L_1$  opposite to the shortest link  $L_3$  is fixed. Therefore, it is a rocker-rocker mechanism.

3. In a quick return mechanism used in shaping machine, the ratio of maximum velocity is  $3/2$ . If the length of the stroke be  $200 \text{ mm}$ , find:

(i) the length of the slotted lever

(ii) quick return ratio

(iii) maximum cutting velocity per second, if the crank rotates at  $30 \text{ rpm}$ .

**Solution :**

Ratio of velocities:

$$\begin{aligned} \frac{V_{R\max}}{V_{F\max}} &= \frac{\text{Maximum velocity of return}}{\text{Maximum velocity during forward stroke}} \\ &= \frac{V_{R\max}}{V_{F\max}} = \frac{l_1 + r}{l_1 - r} = \frac{3}{2} \quad (\text{From Equation 1.10}) \end{aligned}$$

$$\text{or} \quad 3(l_1 - r) = 2(l_1 + r) \quad (l = \text{length of the fixed link})$$

$$\text{or} \quad l_1 = 5r$$

(i) Referring to Figure, we get

$$\sin \theta = \frac{OB_2}{AO} = \frac{r}{5r} = \frac{1}{5}$$

$$\theta = 11.54^\circ$$

