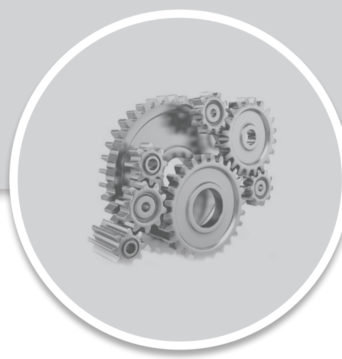
MECHANICAL ENGINEERING

Heat Transfer



Comprehensive Theory
with Solved Examples *and* Practice Questions

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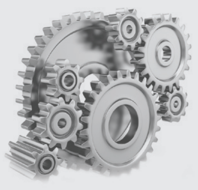

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CHAPTER

1

Introduction and Basic Concepts

1.1 INTRODUCTION

- The definition of **'heat'** is provided by classical thermodynamics. It is defined as an energy that flows due to difference in temperature .
- Heat flows in a direction from higher temperature to lower temperature.
- Heat energy can neither be observed nor be measured directly. However, the effects produced by the transfer of this energy are amenable to observations and measurements.

1.1.1 Difference between Heat and Temperature

Temperature is a measure of the amount of energy possessed by the molecules of a substance. It manifests itself as a degree of hotness, and can be used to predict the direction of heat transfer. Heat, on the other hand, is energy in transit. Spontaneously, heat flows from a hotter body to a cooler one. The usual symbol for heat is Q.

1.1.2 Difference between Thermodynamics and Heat Transfer

Thermodynamics deals with the amount of heat and work transfer along with final state of the system. It gives no indication about how long process will take whereas heat transfer tells us that how the heat is transferred (i.e. modes of heat transfer), the rate at which heat is transferred and thus, the time of cooling or heating and the temperature distribution inside the body.

NOTE: Heat flux is a quantitative, vectorial representation of heat flow through a surface.

1.2 MODES OF HEAT TRANSFER

Heat transfer can be achieved by three distinct modes: Conduction, Convection and Radiation.

1.2.1 Conduction

It is transfer of energy from more energetic particles of a substance to adjacent less energetic particles due to interaction of particles. Conduction can take place in solids liquids and gases. In solids, it occurs due to combination of vibrations of molecules in lattice and energy transport by free electrons while in liquids and gases, it occus due to collision and diffusion of molecules during their random motion. Examples of conduction include that end of a metal rod placed in a fire heating up from one end to the other, cooling of ice-cream in the bowl it is placed in.

The law which describes the rate of heat transfer in conduction is known as **Fourier's law**.
According to Fourier's law,

$$q_x = -k\frac{dT}{dx} \qquad ...(1.1)$$

- In circular coordinates, it may be convenient to work in the radial direction.

$$q_r = -kA_r\left(\frac{dT}{dr}\right);$$

$\frac{dT}{dr}$ is temperature gradient in radial direction.

- The minus sign in Equation (1.1) indicates that heat flows in the direction of decreasing temperature.
- The constant 'k' is known as thermal conductivity.

When the temperature becomes a function of three space coordinates, say, x, y, z in a rectangular Cartesian frame, heat flows along the three coordinate directions. Equation (1.1) under this situation, is written in vector form as

$$q = -k\nabla T \qquad ...(1.2)$$

where, $q = iq_x + jq_y + kq_z$

and, $\nabla T = i\frac{\partial T}{\partial x} + j\frac{\partial T}{\partial y} + k\frac{\partial T}{\partial z}$

NOTE

- Thermal conductivity is a transport property of the medium through which heat is conducted.
- For an isotropic medium, the thermal conductivity k is a scalar quantity which depends upon temperature only

EXAMPLE : 1.1

The rate of heat transfer from a hot surface to a cold surface is directly proportional to the difference in temperature between the two surfaces and the surface area normal to the direction of heat flow. This is

(a) Newton's law of cooling (b) Kirchhoff's law
(c) Fourier's law (d) Wien's law

Solution: (c)

EXAMPLE : 1.2

Heat transfer takes place according to

(a) Fick's law (b) Zeroth law of thermodynamics
(c) First law of thermodynamics (d) Second law of thermodynamics

Solution: (d)

1.2.2 Convection

The mode by which heat is transferred between a solid surface and the adjacent fluid in motion when there is a temperature difference between the two is known as convection heat transfer.

- The mode of convective heat transfer comprises of two mechanisms:
  (i) Conduction at the solid surface and

(ii) Advection by the bulk or macroscopic motion of the fluid a little away from the solid surface.

- Examples of convection include the effect of hot air rising and falling (convection currents) or the large-scale convection currents of the atmosphere and oceans.
- The convection is of two types: Forced convection and Free convection.
- In Forced convection, the fluid is forced to flow over a solid surface by external means such as fan, pump or atmospheric wind.
- When the fluid motion is caused by the buoyancy forces that are induced by density differences due to the variation in temperature in the fluid, the convection is called Natural (or Free) convection.

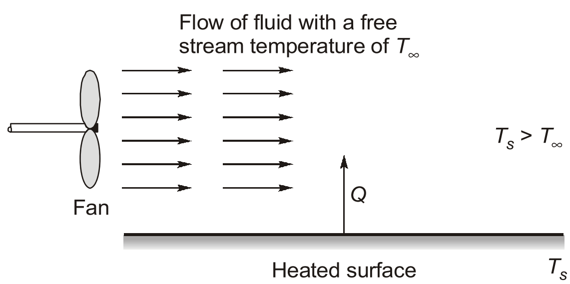

Figure: Forced convective heat transfer from a horizontal surface

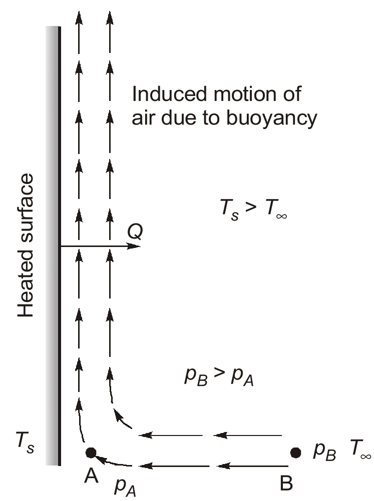

Figure: Free convective heat transfer from a heated vertical surface

- Irrespective of the details of the mechanism, the rate of heat transfer by convection (both forced and free) between a solid surface and a fluid is calculated from the relation,

$$Q = \bar{h} A \Delta T \qquad ...(1.3)$$

This relation is known as **Newton's law of cooling**.

where, Q = Rate of heat transfer by convection

A = Heat transfer area

$\Delta T = (T_s - T_f)$, is the difference between the surface temperature T_s and the temperature of the fluid T_f at some reference location.

$\bar{h}$ = Average convective heat transfer coefficient over the area A.

NOTE

The convection heat transfer coefficient h is not a property of the fluid. It is an experimentally determined parameter whose value depends on all the variables influencing convection such as the surface geometry, the nature of fluid motion, the properties of the fluid, and the bulk fluid velocity.

EXAMPLE : 1.3

The average forced convective heat transfer coefficient for a hot fluid flowing over a cold surface is 200 W/(m²°C). The fluid temperature upstream of the cold surface is 100°C and the surface is held at 20°C. The heat transfer rate per unit surface area from the fluid to the surface is _____ kW/m². [Round off to nearest integer]

Solution: (16) (16 to 16)

The rate of heat transfer per unit area, q

$$q = \frac{Q}{A} = \bar{h}(T_\infty - T_s)$$
$$= 200\,(100 - 20) = 16{,}000 \text{ W/m}^2 = 16 \text{ kW/m}^2$$

1.2.3 Radiation

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as result of the changes in the electronic configurations of the atoms or molecules. Radiation does not need material medium to propagate.

The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s (in K) is given by the **Stefan-Boltzmann law** as

$$\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4 \text{ Watt} \quad \text{...(1.4)}$$

where $\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4 is the Stefan-Boltzmann constant.

The idealized surface that emits radiation at this maximum rate is called as black body, and the radiation emitted by a black body is called as black body radiation.

The radiation emitted by all real surfaces is less than the radiation emitted by a black body at the same temperature, and is expressed as

$$\dot{Q}_{\text{emit}} = \varepsilon \sigma A_s T_s^4 \text{ W} \quad \text{...(1.5)}$$

where, T_s is Absolute temperature of the surface, and ε is the emissivity of the surface, (whose value is in the range $0 \le \varepsilon \le 1$), which is a measure of how closely a surface approximates a black body. ($\varepsilon = 1$ for black body),

Examples include an in can descent light bulb emitting visible light, the infrared radiation emitted by a common household raditior or electric heater, as well as the sun heating the earth.

NOTE

- The heat transfer by conduction or convection requires the presence of a medium. But the radiation heat transfer does not necessarily require a medium, rather it occurs most efficiently in a vacuum.
- Radiation is a volumetric phenomenon, and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees. However, radiation is usually considered to be a surface phenomenon for solids that are opaque to thermal radiation such as metal, wood and rock.

EXAMPLE : 1.4

After sunset, radiant energy can be sensed by a person standing near a brick wall. Such walls frequently have a surface temperature around 50°C, and the typical brick emissivity value is approximately 0.9. The radiant heat flux per square metre from a brick wall at this temperature is____ W/m^2. [Correct upto 2 decimal places]

Solution: (555.44) (554 to 557)

Applying Equation (1.5), we have

$$\frac{E}{A} = \epsilon \sigma T^4 = 0.9 \times 5.67 \times 10^{-8} \times (50 + 273)^4$$
$$= 555.44 \text{ W/m}^2$$

1.3 THERMAL CONDUCTIVITY

Thermal conductivity of a material can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference. The thermal conductivity of a material is a measure of the ability of the material to conduct heat. A high value of thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or insulator. The thermal conductivities of some common materials at room temperature are given in the table.

Table: Thermal conductivity of some materials at room temperature (300 K)

Material	Thermal conductivity, [W/(m°C)]	Material	Thermal conductivity, [W/(m°C)]
Diamond	2300	Brick	0.72
Silver	429	Water (*l*)	0.613
Copper	401	Human skin	0.37
Gold	317	Wood (oak)	0.17
Aluminium	237	Helium (g)	0.152
Nickel	1	Soft rubber	0.13
Iron	80.2	Refrigerant-12	0.072
Steel	60.5	Glass fibre	0.043
Mercury (*l*)	8.54	Air (g)	0.026
Glass	0.78	Urethane, rigid foam	0.026

1.3.1 Thermal Conductivity of Solids

In solids, heat conduction is due to two effects - **flow of free electrons** and **propagation of lattice vibrational waves**. The thermal conductivity is therefore determined as the addition of these two components. In pure metals, the electronic component is more prominent than the component of lattice vibration and gives rise to a very high value of thermal conductivity. The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. Highly ordered crystalline non-metallic solids like diamond, silicon, quartz exhibit very high thermal conductivities (more than that of pure metals) due to lattice vibration only, but are poor conductors of electricity.

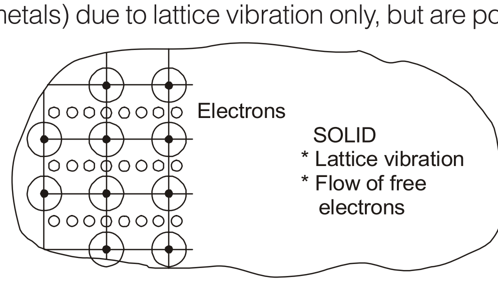

Figure: Mechanism of heat conduction in solids

The temperature dependence of thermal conductivity causes considerable complexity in conduction analysis. Therefore, it is a common practice to evaluate the thermal conductivity (k) at the average temperature and treat it as a constant in calculations.

NOTE

- Thermal conductivity of an alloy of two metals is usually much lower than that of either metals. (Refer to Table)
- Thermal conductivity of pure metals decreases with increase in temperature. (Refer to figure below)
- Thermal conductivity of alloys increases with increase in temperature. (Refer to figure below).

Table: The comparison of thermal conductivities of metallic alloys with those of constituting pure metals

Pure metal or alloy	*k* (W/(m°C))
Copper	401
Aluminium	237
Nickel	91
Constantan (55% Cu, 45% Ni)	23
Commercial bronze (90% Cu, 10% Al)	52

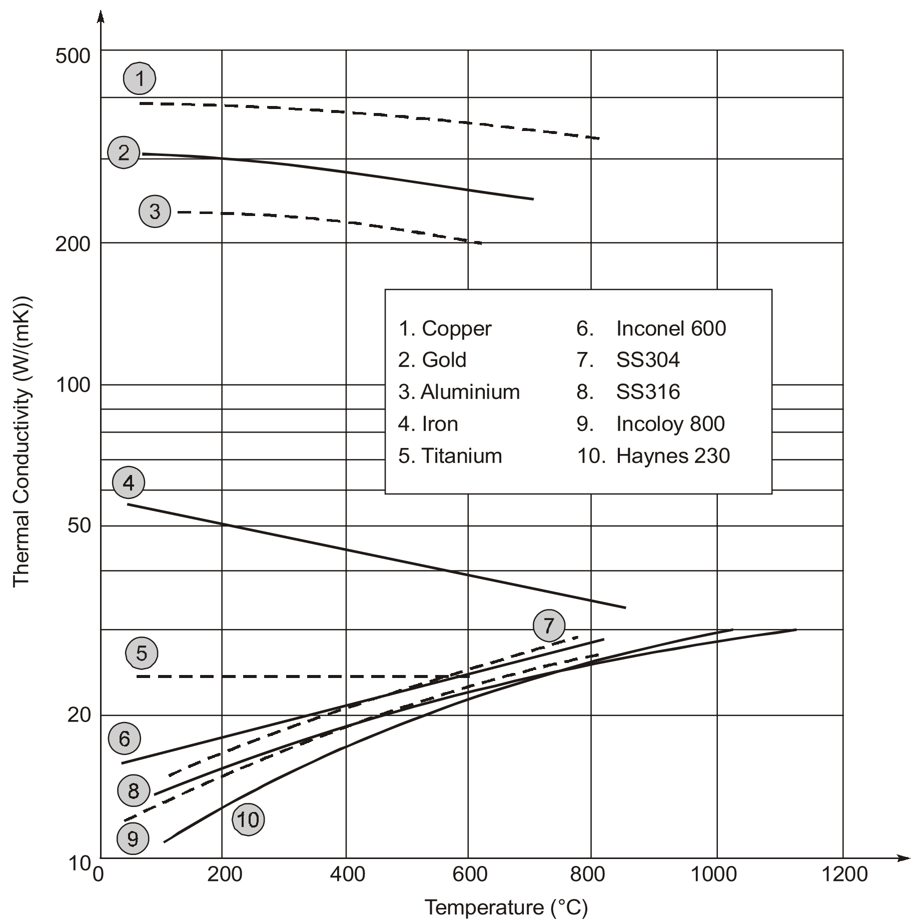

Figure: The variation of thermal conductivity with temperature for typical metals and their alloys

1.3.2 Thermal Conductivity of Liquids and Gases

The thermal conductivity of liquids and gases is attributed to the transfer of kinetic energy between the randomly moving molecules due to their collisions. In a liquid or gas, the kinetic energy of the molecules is due to their random translational motion as well as their vibrational and rotational motions. When two molecules possessing different kinetic energies collide, part of the kinetic energy of the more energetic (higher temperature) molecule is transferred to the less energetic (lower- temperature) molecule.

The kinetic theory of gases predicts and the experiments confirm that the thermal conductivity of gases is proportional to the square root of the thermodynamic temperature T, and is inversely proportional to the square root of the molar mass M. Therefore, the thermal conductivity of a gas increases with increasing temperature and decreasing molar mass. So, it is not surprising that the thermal conductivity of helium ($M = 4$) is much higher than those of air ($M = 29$) and argon ($M = 40$).

Unlike gases, the thermal conductivities of most liquids decrease with increasing temperature, with water being a notable exception. Like gases, the conductivity of liquids decreases with increasing molar mass.

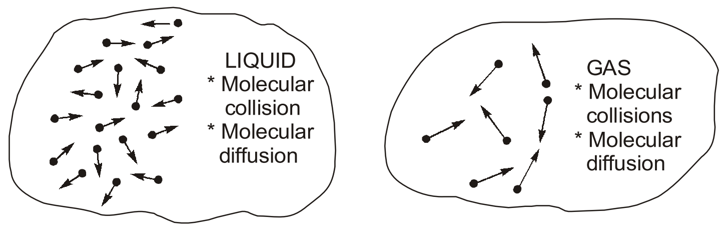

Figure: Mechanisms of heat conduction in liquids and gases

NOTE

- The thermal conductivity of gases is independent of pressure in a wide range of pressures encountered in practice.
- Because of large intermolecular spaces and hence a smaller number of molecular collisions, the thermal conductivities exhibited by gases are lower than those of the liquids.

EXAMPLE : 1.5

In general, the thermal conductivity of a substance is

(a) independent of temperature (b) a strong function of pressure
(c) strongly temperature dependent (d) independent of pressure

Solution: (c)

EXAMPLE : 1.6

With increase in temperature, the thermal conductivity of gases

(a) decreases (b) increases
(c) remains constant (d) first increases and then decreases

Solution: (b)

EXAMPLE : 1.7

Consider a cube of 2 cm side at 1000 K. If the emissivity of the cube surface is 0.75, what is the total amount of radiation emitted by five surfaces of the cube in 20 minutes?

(a) 122 kJ (b) 136 kJ
(c) 98 kJ (d) 102 kJ

Solution: (d)

$$\dot{Q} = A_{\text{Total}} \times \sigma \times \varepsilon \times T^4$$

$$Q = \dot{Q} \times \Delta t = A_{\text{total}} \times \sigma \times \varepsilon \times T^4 \times 20 \times 60$$

$$= 5 \times (0.02)^2 \times 5.67 \times 10^{-8} \times 0.75 \times (1000)^4 \times 20 \times 60$$

$$= 102060 \text{ J}$$

$$Q = 102 \text{ kJ}$$

EXAMPLE : 1.8

Choose the correct statement.

(a) The thermal conductivity of insulating solids increases with temperature

(b) The thermal conductivity of good electrical conductors is generally low

(c) The thermal conductivity of gases decreases with temperature

(d) The thermal conductivity of liquids is a strong function of temperature

Solution: (a)

EXAMPLE : 1.9

Which of the following has the lowest thermal conductivity?

(a) Air (b) Water
(c) Brick (d) Copper

Solution: (a)

1.4 THERMAL DIFFUSIVITY

It is the ratio of thermal conductivity of material to the heat capacity of the material per unit volume. It represents how fast the heat is diffused through the material.

$$\alpha = \frac{\text{Heat conducted}}{\text{Heat stored}} = \frac{k}{\rho c} \quad ...(1.6)$$

Thermal conductivity (k) represents how well a material conducts heat, and the heat capacity ρc represents how much energy a material stores per unit volume.

The thermal diffusivity of a material is the measure of its ability to conduct thermal energy relative to its ability to store thermal energy. Materials having large values of α will respond quickly to a change in the thermal environment in establishing a steady-state temperature field within the material in transporting heat, while materials having small values of α will do it sluggishly.

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IMPORTANT POINTS TO REMEMBER

- The science of thermodynamics deals with the amount of heat transfer a system undergoes during a process from one equilibrium state to another, whereas the science of heat transfer deals with the rate of heat transfer, which is the main quantity of interest in the design and evaluation of heat transfer equipment.
- Heat can be transferred by three different modes: **Conduction, Convection,** and **Radiation**.
- **Conduction** is the transfer of heat from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles, and is expressed by Fourier' law of heat conduction as

$$\dot{Q}_{cond} = -kA\frac{dT}{dx}$$

- **Convection** is the mode of heat transfer between a solid surface and the adjacent liquid or gas that is in motion and involves the combined effects of conduction and bulk fluid motion. The rate of convection heat transfer is expressed by Newton's law of cooling as

$$\dot{Q}_{convection} = hA_s(T_s - T_\infty)$$

- **Radiation** is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s is given by the Stefan-Boltzmann law as $\dot{Q}_{emit,\ max} = \sigma A_s T_s^4$.

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OBJECTIVE BRAIN TEASERS

1. Eggs with a mass of 0.15 kg per egg and a specific heat of 3.32 kJ/kg°C are cooled from 32°C to 10°C at a rate of 300 eggs per minute. The rate of heat removal from the eggs is_____ kW. [Round off to nearest integer]

2. Which equation below is used to determine the heat flux for conduction?

   (a) $-kA\frac{dT}{dx}$ (b) $-k$ grad T
   (c) $h(T_2 - T_1)$ (d) $\varepsilon\sigma T^4$

3. A 2 kW electric resistance heater submerged in 30 kg water is turned on and kept on for 10 minutes. During the process, 500 kJ of heat is lost from the water. The temperature rise of water is___ °C. [Correct upto 1 decimal place]

4. A 1 kW electric resistance heater in a room is turned on and kept on for 50 minutes. The amount of energy transferred to the room by the heater is

   (a) 1 kJ (b) 50 kJ
   (c) 3000 kJ (d) 3600 kJ

5. Which equation below is used to determine the heat flux for convection?

   (a) $-kA\frac{dT}{dx}$ (b) $-k$ grad T
   (c) $h(T_1 - T_2)$ (d) $\varepsilon\sigma T^4$

6. A hot 16 cm × 16 cm × 16 cm cubical iron block is cooled at an average rate of 80 W. The heat flux is _____ W/m^2. [Round off to nearest integer]

ANSWER KEY

1. (55)	2. (b)	3. (5.6)	4. (c)	5. (c)
6. (521)				

HINTS & EXPLANATIONS

1. (55)(55 to 55)

$$m = 0.15 \text{ kg/egg}$$
$$c = 3.32 \text{ kJ/kg°C}$$
$$T_{initial} = 32°C$$
$$T_{final} = 10°C$$

No. of eggs cooled = 300 per minute

The rate of heat removal

= mass of 1 egg × No. of eggs cooled per minute × specific heat × $[T_{initial} - T_{final}]$

= 0.15 × 300 × 3.32 × [32 – 10] = 3286.8 kJ/min

= 54.78 kW ≈ 55 kW

2. (b)

$$\text{Heat flux} = \frac{Q}{A}$$

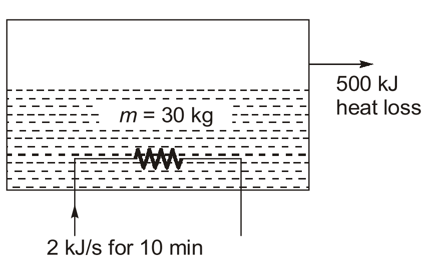
From Fourier's law

$$\frac{Q}{A} = \text{Heat flux} = -k\frac{dt}{dx}$$

$$\frac{dT}{dx} = \text{grad or slope}$$

∴ Heat flux = $-k$ grad.

3. (5.6)(5.4 to 5.8)

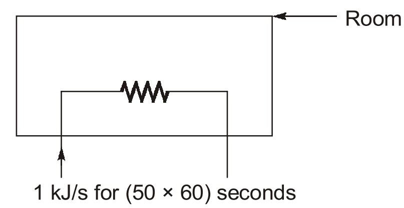


$$Q_{input} = \frac{2 \text{ kJ}}{s} \times (10 \times 60)s = 1200 \text{ kJ}$$
$$Q_{out} = 500 \text{ kJ}$$
$$Q_{stored} = 1200 - 500 = 700 \text{ kJ}$$

Heat stored is utilized in rise of temperature of water.

$$\text{Heat stored} = mcdT$$
$$700 = 30 \times 4.18 \times dT$$
$$dT = \frac{700}{30 \times 4.18} = 5.58°C \approx 5.6°C$$

4. (c)



Amount of energy transferred to the room by the heater

= Rate of energy × Time input

= 1 kJ/s × (50 × 60) second = 3000 kJ

5. (c)

$$Q = hA\Delta T \quad \text{(convection heat transfer)}$$
$$Q/A = \text{heat flux for convection}$$
$$\text{Heat flux} = h\,\Delta T = h[T_1 - T_2]$$

6. (521) (521 to 521)

Dimension of cube = 16 × 16 × 16 cm^3

Area of cube = $6a^2$

Heat flux is Q/A

$$= \frac{80}{6 \times 16 \times 16} = \frac{0.3125}{6} \text{ W/cm}^2$$

$$= \frac{0.3125}{(10^{-2})^2} = \frac{3125}{6} \text{ W/m}^2$$

$$= 520.83 = 521 \text{ W/m}^2$$

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CONVENTIONAL BRAIN TEASERS

Q.1 Heat is transferred along the axis of a truncated conical cylinder of length l, radius r_1 at the shorter end and radius r_2 at the bigger end. The circumference of the cylinder is completely insulated. Develop an expression to calculate heat transfer along the axis of cylinder. Assume no variation of conductivity with temperature.

Solution:

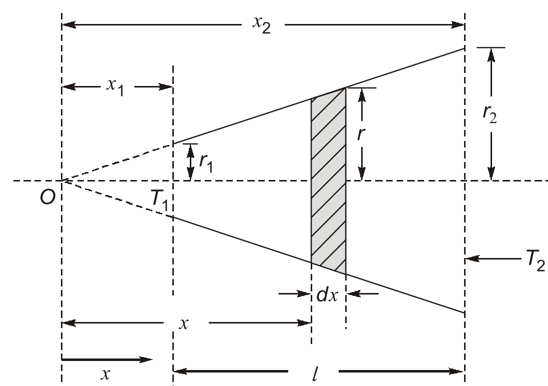
Consider a frustum of constant thermal conductivity (k) and apex at O. An element of thickness dx at a distance x from the apex is taken.

From similar triangle property,

$$r = \left(\frac{r_2 - r_1}{l}\right)x = Cx$$

$\therefore$ Area of element, $A = \pi C^2 x^2$

Heat transfer rate through this element



$$Q = -kA\frac{dT}{dx}$$

$$\frac{dx}{A} = -\frac{k}{Q}\cdot dT$$

Integrating both sides we get

$$\int_{x_1}^{x_2} \frac{dx}{\pi C^2 x^2} = -\frac{k}{Q}\int_{T_1}^{T_2} dT$$

or, $$\frac{1}{\pi C^2}\left[\frac{1}{x_1} - \frac{1}{x_2}\right] = \frac{k}{Q}(T_1 - T_2)$$

Also, $$r_1 = Cx_1 \text{ and } r_2 = Cx_2$$

$\therefore$ $$\frac{1}{\pi C^2}\left[\frac{C}{r_1} - \frac{C}{r_2}\right] = \frac{k}{Q}(T_1 - T_2)$$

$\therefore$ Heat transfer rate, $$Q = \frac{T_1 - T_2}{\frac{1}{\pi k C}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \frac{T_1 - T_2}{\frac{l}{\pi k (r_2 - r_1)}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \qquad \left[\because C = \frac{r_2 - r_1}{l}\right]$$

or $$Q = \frac{T_1 - T_2}{\left(\frac{l}{\pi k r_1 r_2}\right)}$$

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