INSTRUMENTATION ENGINEERING

CONTROL SYSTEMS AND PROCESS CONTROL

Comprehensive Theory
with Solved Examples *and* Practice Questions

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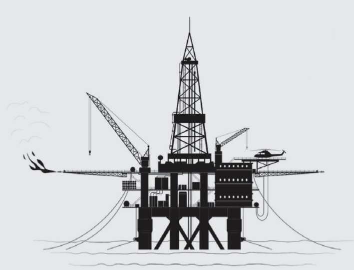
Control Systems and Process Control



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CONTENTS

Control Systems and Process Control

Control Systems : Section-A

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CHAPTER

2

Transfer Function

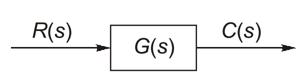
2.1 TRANSFER FUNCTION AND IMPULSE RESPONSE FUNCTION

In control theory, transfer functions are commonly used to characterise the input-output relationships of components or systems that can be described by linear, time-invariant differential equations.

Transfer Function

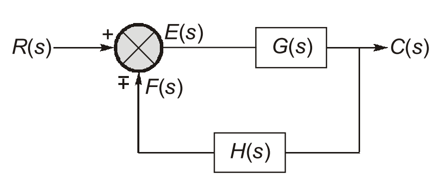
The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

Transfer Function of Open Loop System :



$$G(s) = \frac{C(s)}{R(s)}$$

Transfer Function of Closed Loop System :



Transfer function of closed loop system

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

$R(s)$ = Reference input

$C(s)$ = Controlled output

$E(s)$ = Actuating error signal

$G(s)$ = Forward path transfer function

$H(s)$ = Feedback path transfer function

$C(s) = G(s)E(s)$

$$= G(s)[R(s) \pm C(s)H(s)]$$

$$= G(s)[R(s) \pm G(s)C(s)H(s)$$

$$C(s) \pm G(s)H(s)C(s) = G(s)R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

Shortcut Method

1. To find close loop transfer function from open loop transfer function

   If $\quad \text{O.L.T.F.} = \dfrac{\text{Numerator}}{\text{Denominator}}$

   Then, $\quad \text{C.L.T.F.} = \dfrac{\text{Numerator}}{\text{Denominator} + \text{Numerator}}$

2. To find open loop transfer function from close loop transfer function

   If $\quad \text{C.L.T.F.} = \dfrac{\text{Numerator}}{\text{Denominator}}$

   Then, $\quad \text{O.L.T.F.} = \dfrac{\text{Numerator}}{\text{Denominator} - \text{Numerator}}$

Linear Systems

A system is called linear if the principle of superposition and principle of homogeneity apply. The principle of superposition states that the response produced by the simultaneous application of two different forcing functions is the sum of the two individual responses. Hence, for the linear system, the response to several inputs can be calculated by transferring one input at a time and adding the results. It is the principle that allows one to build up complicated solutions to the linear differential equations from simple solutions.

In an experimental investigation of a dynamic system, if cause and effect are proportional, thus implying that the principle of superposition holds, then the system can be considered as linear.

Linear Time-Invariant Systems and Linear-Time Varying Systems

A differential equation is linear if the coefficients are constants or functions only of the independent variable. Dynamic systems that are composed of linear time-invariant lumped-parameter components may be described by linear time-invariant differential equations i.e. constant-coefficient differential equations. Such systems are called linear time-invariant (or linear constant-coefficient) systems. Systems that are represented by differential equations whose coefficients are function of time are called linear time varying systems. An example of a time-varying control system is a space craft control system (the mass of a space craft changes due to fuel consumption).

The definition of transfer function is easily extended to a system with multiple inputs and outputs (i.e. a multivariable system). In a multivariable system, a linear differential equation may be used to describe the relationship between a pair of input and output variables, when all other inputs are set to zero. Since the principle of superposition is valid for linear systems, the total effect (on any output) due to all the inputs acting simultaneously is obtained by adding up the outputs due to each input acting alone.

EXAMPLE : 2.1

When deriving the transfer function of a linear element
(a) both initial conditions and loading are taken into account.
(b) initial conditions are taken into account but the element is assumed to be not loaded.
(c) initial conditions are assumed to be zero but loading is taken into account.
(d) initial conditions are assumed to be zero and the element is assumed to be not loaded.

Solution : (c)

While deriving the transfer function of a linear element only initial conditions are assumed to be zero, loading (or input) can't assume to be zero.

EXAMPLE : 2.2

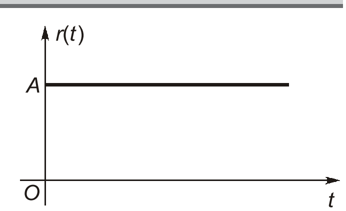
If the initial conditions for a system are inherently zero, what does it physically mean?
(a) The system is at rest but stores energy
(b) The system is working but does not store energy
(c) The system is at rest or no energy is stored in any of its part
(d) The system is working with zero reference input

Solution : (c)

A system with zero initial conditions is said to be at rest since there is no stored energy.

2.2 STANDARD TEST SIGNALS

1. Step Signal

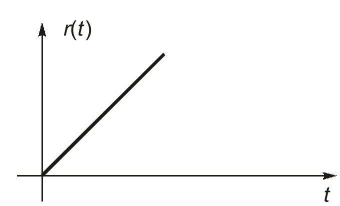


$$r(t) = A\,u(t)$$

where, unit step signal $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

Laplace transform, $R(s) = A/s$

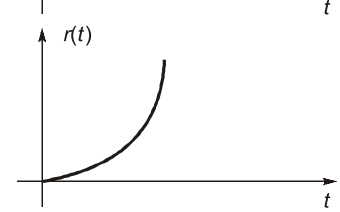
2. Ramp Signal



$$r(t) = \begin{cases} A\,t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Laplace transform, $R(s) = A/s^2$

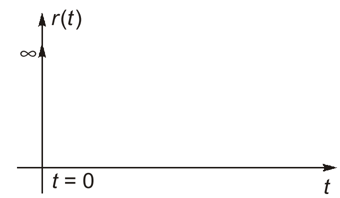
3. Parabolic Signal



$$r(t) = \begin{cases} A\,t^2/2, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Laplace transform, $R(s) = A/s^3$

4. Impulse Signal



$$r(t) = \begin{cases} \infty \ , & t = 0 \\ 0 \ , & t \neq 0 \end{cases} \quad ; \quad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

Laplace transform, $\quad R(s) = 1$

Transfer function, $\quad G(s) = \dfrac{C(s)}{R(s)}$

$$C(s) = F(s)\, R(s)$$

Let, $\quad R(s) = \text{Impulse signal} = 1$

$$C(s) = \text{Impulse response} = G(s) \times 1 = \text{Transfer Function}$$

$$\mathcal{L}\{\text{Impulse Response}\} = \text{Transfer function} = \left[\frac{C(s)}{R(s)}\right]$$

NOTE

- Td/dt (Parabolic Response) = Ramp Response
- d/dt (Ramp Response) = Step Response
- d/dt (Step Response) = Impulse Response

Consider, a linear time-invariant system has the input $u(t)$ and output $y(t)$. The system can be characterized by its impulse response $g(t)$, which is defined as the output when the input is a unit-impulse function $\delta(t)$. Once the impulse response of a linear system is known, the output of the system $y(t)$, with any input $u(t)$, can be found by using the transfer function.

Let $G(s)$ denotes the transfer function of a system with input $u(t)$, output $y(t)$, and impulse response $g(t)$. The transfer function $G(s)$ is defined as

$$G(s) = \mathcal{L}[g(t)] = \left.\frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]}\right|_{\text{initial conditions} \to 0} = \frac{Y(s)}{U(s)}$$

REMEMBER

Sometimes, students do a common mistake, they first find $y(t)/u(t)$ and then take its Laplace transform to determine the transfer function which is absolutely wrong. Because,

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]} \neq \mathcal{L}\left[\frac{y(t)}{u(t)}\right]$$

2.3 POLES AND ZEROS OF A TRANSFER FUNCTION

The transfer function of a linear control system can be expressed as

$$G(s) = \frac{A(s)}{B(s)} = \frac{K(s - s_1)(s - s_2)\ldots(s - s_n)}{(s - s_a)(s - s_b)\ldots(s - s_m)}$$

where K is known as gain factor of the transfer function $G(s)$.

In the transfer function expression, if s is put equal to s_a, s_b ... s_m then it is noted that the value of the transfer function is infinite. These s_a, s_b, ... s_m are called the poles of the transfer function.

In the transfer function expression, if s is put equal to $s_1, s_2 \ldots s_n$ then it is noted that the value of the transfer function is zero. These $s_1, s_2 \ldots s_n$ are called the zeros of the transfer function.

2.3.1 Multiple Poles and Multiple Zeros

The poles $s_a, s_b \ldots s_m$ or the zeros $s_1, s_2 \ldots s_n$ are either real or complex and the complex poles or zeros always appear in conjugate pairs.

It is possible that either poles or zeros may coincide; such poles or zeros are called multiple poles or multiple zeros.

2.3.2 Simple Poles and Simple Zeros

Non-coinciding poles or zeros are called simple poles or simple zeros. From the transfer function expression, it is observed that

- If $n > m$, then the value of transfer function is found to be infinity for $s = \infty$. Hence, it is concluded that there exists a pole of the transfer function at infinity (∞) and the multiplicity (order) of such a pole being $(n - m)$.
- If $n < m$, then the value of transfer function is found to be zero for $s = \infty$. Hence, it is concluded that there exists a zero of the transfer function at infinity (∞) and the multiplicity (order) of such a zero being $(m - n)$.

  Therefore, for a rational transfer function the total number of zeros is equal to the total number of poles. The transfer function of a system is completely specified in terms of its poles, zeros and the gain factor.

Consider the following transfer function:

$$G(s) = \frac{s+3}{(s+2)(s+1+3j)(s+1-3j)}$$

For the above transfer function, the poles are at :

(a) $s_a = -2$; (b) $s_b = -1 - 3j$; and (c) $s_c = -1 + 3j$

The zeros are at $s_1 = -3$.

As the number of zeros should be equal to number of poles, the remaining two zeros are located at $s = \infty$. The pole-zero plot is plotted as shown:

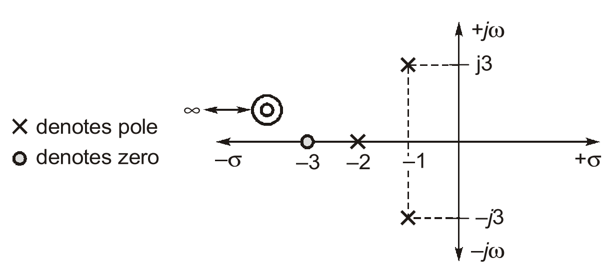


***Fig. :** Pole-zero plot*

Note : Poles and zero are those complex/critical frequencies which make the transfer function infinity or zero.

2.3.3 Proper Transfer Functions

The transfer functions are said to be strictly proper if the order of the denominator polynomial is greater than that of the numerator polynomial (i.e. $m > n$). If $m = n$, the transfer function is called proper. The transfer function is improper if $n > m$.

In the transfer function expression of a control system, the highest power of s in the numerator is generally either equal to or less than that of the denominator.

EXAMPLE : 2.3

A transfer function has two zeros at infinity. Then the relation between the numerator degree (N) and the denominator degree (M) of the transfer function is

(a) $N = M + 2$ (b) $N = M - 2$

(c) $N = M + 1$ (d) $N = M - 1$

Solution : (b)

For a rational transfer function, the total number of zeros are equal to total number of poles.

Therefore, Number of poles $= M$; Number of zeros $= N + 2$

For a rational transfer function : $M = N + 2$ or $N = M - 2$

2.4 PROPERTIES OF TRANSFER FUNCTION

The properties of the transfer function are summarized as follows:

1. The transfer function is defined only for a linear time-invariant system. It is not defined for non-linear or time variant systems.
2. The transfer function between an input variable and an output variable of a system is defined as the Laplace transform of the impulse response. Alternately, the transfer function between a pair of input and output variables is the ratio of the Laplace transform of the output to the Laplace transform of the input.
3. All initial conditions of the system are set to zero.
4. Transfer function is independent of the input of the system.
5. The transfer function of a continuous-data system is expressed only as a function of the complex variables. It is not a function of the real variable, time, or any other variable that is used as the independent variable or discrete-data system modelled by difference equations, the transfer function is a function of Z, when the Z-transform is used.
6. If the system transfer function has no poles or zeros with positive real parts, the system is a minimum phase system.

   Non-minimum phase functions are the functions which have poles or zeros on right hand side of s-plane.
7. The stability of a time-invariant linear system can be determined from its characteristic equation.

   Characteristic equation: The characteristic equation of a linear system is defined as the equation obtained by setting the denominator polynomial of the closed loop transfer function to zero.

OBJECTIVE BRAIN TEASERS

Q.1 A control system with certain excitation is governed by the following mathematical equation $\frac{d^2x}{dt^2}+\frac{1}{2}\frac{dx}{dt}+\frac{1}{18}x=10+15e^{-4t}+2e^{-5t}$. The natural time constants of the response of the system are

(a) $2s$ and $5s$ (b) $3s$ and $6s$

(c) $4s$ and $5s$ (d) $\frac{1}{3}s$ and $\frac{1}{6}s$

Q.2 The response $g(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relaxed condition is $g(t) = e^{-t} + e^{-2t}$. The response of this system for a unit step input $u(t)$ is

(a) $(1 + e^{-t} + e^{-2t})u(t)$

(b) $(e^{-t} + e^{-2t})u(t)$

(c) $(1.5 - e^{-t} - 0.5e^{-2t})u(t)$

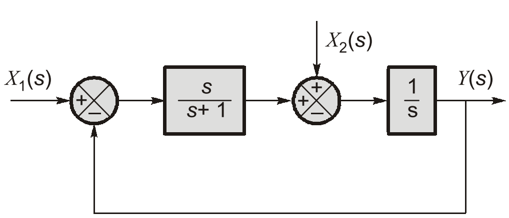
(d) $e^{-t}\delta(t) + e^{-2t}u(t)$

Q.3 The frequency response of a linear time-invariant system is given by $H(f)=\frac{5}{1+j10\pi f}$. The step response of the system is

(a) $5(1 - e^{-5t})u(t)$ (b) $5\left(1-e^{-t/5}\right)u(t)$

(c) $\frac{1}{5}\left(1-e^{-5t}\right)u(t)$ (d) $\frac{1}{(s+5)(s+1)}$

Q.4 For the following system :

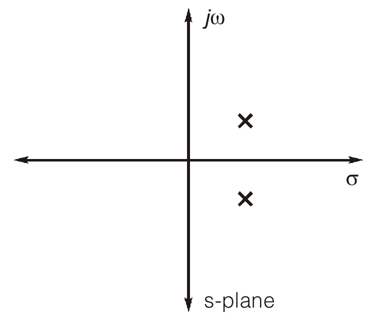


when $X_1(s) = 0$, the transfer function $\frac{Y(s)}{X_2(s)}$ is

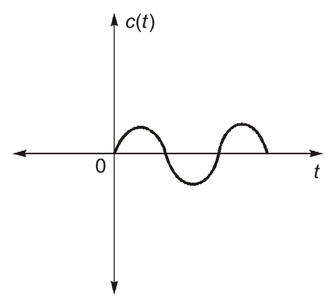
(a) $\frac{s+1}{s^2}$ (b) $\frac{1}{s+1}$

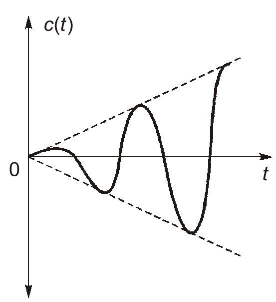
(c) $\frac{s+2}{s(s+1)}$ (d) $\frac{s+1}{s(s+2)}$

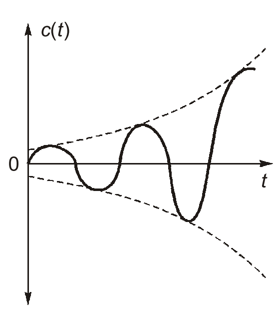
Q.5 If closed-loop transfer function poles shown below:

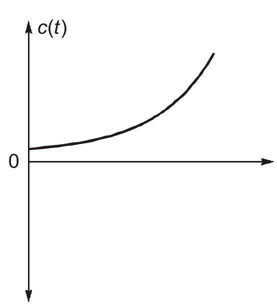


Impulse response is

(a)


(b)


(c)


(d)


Q.6 Ramp response of the transfer function $F(s) = \frac{s+1}{s+2}$ is

(a) $\frac{1}{4} - \frac{1}{4}e^{-2t} + \frac{1}{2}t$ (b) $\frac{1}{4}e^{-2t} + \frac{1}{4} + \frac{1}{2}t$

(c) $\frac{1}{2} - \frac{1}{2}e^{-2t} + t$ (d) $\frac{1}{2}e^{-2t} + \frac{1}{2} - t$

Q.7 Which of the following statements are correct?

1. Transfer function can be obtained from the signal flow graph of the system.
2. Transfer function typically characterizes to linear time invariant systems.
3. Transfer function gives the ratio of output to input in frequency domain of the system.

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

Q.8 Which of the following is not a desirable feature of a modern control system?

(a) Quick response
(b) Accuracy
(c) Correct power level
(d) Oscillations

Q.9 In regenerating feedback, the transfer function is given by

(a) $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

(b) $\frac{C(s)}{R(s)} = \frac{G(s)\,H(s)}{1-G(s)\,H(s)}$

(c) $\frac{C(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}$

(d) $\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$

Q.10 The principle of homogeneity and superposition are applied to

(a) linear time variant systems
(b) non-linear time variant systems
(c) linear time invariant systems
(d) non-linear time invariant systems

Q.11 If a system is represented by the differential equation, is of the form $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = r(t)$

(a) $k_1e^{-t} + k_2e^{-9t}$ (b) $(k_1 + k_2)e^{-3t}$
(c) $ke^{-3t}\sin(t+\phi)$ (d) $te^{-3t}u(t)$

Q.12 A linear system initially at rest, is subject to an input signal $r(t) = 1 - e^{-t}$ $(t \geq 0)$. The response of the system for $t > 0$ is given by $c(t) = 1 - e^{-2t}$. The transfer function of the system is

(a) $\frac{(s+2)}{(s+1)}$ (b) $\frac{(s+1)}{(s+2)}$

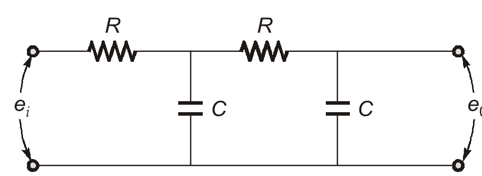
(c) $\frac{2(s+1)}{(s+2)}$ (d) $\frac{(s+1)}{2(s+2)}$

Q.13 The unit step response of a system is given by, $c(t) = \frac{5}{2} + 5t - \frac{5}{2}e^{-2t}$. The corresponding transfer function of the system will be

(a) $\frac{10(s+1)}{(s+2)}$ (b) $\frac{10(s+2)}{(s+3)}$

(c) $\frac{10(s+1)}{s(s+2)}$ (d) $\frac{10(s+2)}{s(s+3)}$

Q.14 Consider the RC circuit shown in figure below:

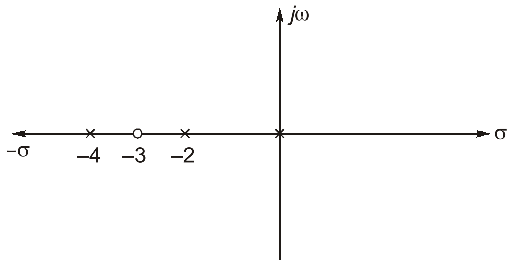


The transfer function $\frac{E_0(s)}{E_i(s)}$ will be $(\tau = RC)$

(a) $\frac{1}{\tau^2s^2 + 2\tau s + 1}$ (b) $\frac{1}{\tau^2s^2 + 3\tau s + 1}$

(c) $\frac{\tau_s}{\tau^2s^2 + 2\tau s + 1}$ (d) $\frac{\tau_s}{\tau^2s^2 + 3\tau s + 1}$

Q.15 The pole-zero configuration of a transfer function is shown in figure. The value of transfer function at $s = 1$ is found to be 4. Then the transfer function of system is



(a) $\dfrac{12(s+3)}{s(s+2)(s+4)}$ (b) $\dfrac{15(s+3)}{s(s+2)(s+4)}$

(c) $\dfrac{4(s+3)}{s(s+2)(s+4)}$ (d) $\dfrac{10(s+2)}{s(s+3)(s+4)}$

Q.16 The transfer function of a system is given by $\dfrac{C(s)}{R(s)} = \dfrac{100}{(s+10)(s^2+2s+1)}$, using the concept of dominant pole, the 2nd order approximation of above transfer function is

(a) $\dfrac{100}{s^2+2s+1}$ (b) $\dfrac{10}{s^2+2s+1}$

(c) $\dfrac{10}{s+10}$ (d) $\dfrac{100}{(s+10)}$

Q.17 A differentiator has a transfer function whose

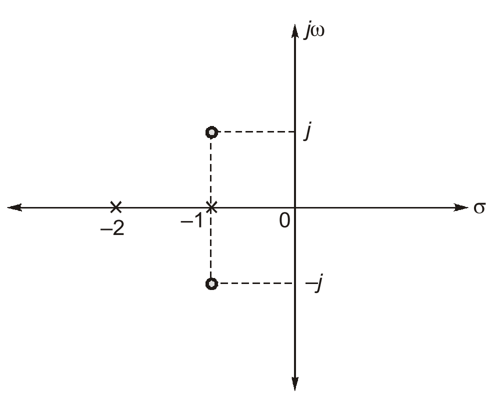
(a) Magnitude decreases linearly with frequency

(b) Magnitude increases linearly with frequency

(c) Phase increases linearly with frequency

(d) Phase is constant

Q.18 The pole-zero plot of a system is given below. If $G(s)$ = 15 for s = 2, then the transfer function of the system is



(a) $\dfrac{12(s^2+2s+5)}{(s+1)(s+2)}$ (b) $\dfrac{18(s^2+2s+2)}{(s+1)(s+2)}$

(c) $\dfrac{12(s^2+2s+3)}{(s+1)(s+2)}$ (d) $\dfrac{6(s^2+2s+3)}{(s+1)(s+2)}$

Q.19 A linear time invariant system initially at rest, when subjected to unit step input gives a response of $2te^{-5t}$, $t > 0$, the corresponding transfer function is

(a) $\dfrac{2}{s(s+5)^2}$ (b) $\dfrac{2s}{(s+5)}$

(c) $\dfrac{2s}{(s+5)^2}$ (d) $\dfrac{2s}{(s-5)^2}$

ANSWER KEY

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(d)	**5.**	(c)
6.	(a)	**7.**	(d)	**8.**	(d)	**9.**	(d)	**10.**	(c)
11.	(d)	**12.**	(c)	**13.**	(c)	**14.**	(b)	**15.**	(b)
16.	(b)	**17.**	(b, d)	**18.**	(b)	**19.**	(c)		

HINTS & EXPLANATIONS

1. (b)

Natural time constants of the response depend only on poles of the system.

$$T(s) = \frac{C(s)}{R(s)}$$

$$= \frac{1}{s^2 + s/2 + 1/18}$$

$$= \frac{18}{18s^2 + 9s + 1}$$

$$= \frac{1}{(6s+1)(3s+1)}$$

This is in the form $\dfrac{1}{(1+sT_1)(1+sT_2)}$

$\therefore$ T_1, T_2 = 6 sec, 3 sec.

2. (c)

Transfer function of system is impulse response of the system with zero initial conditions.

Transfer function = $G(s) = \mathcal{L}\left(e^{-t} + e^{-2t}\right)$

$$= \frac{1}{s+1} + \frac{1}{s+2}$$

$$G(s) = \frac{C(s)}{R(s)} = \left(\frac{1}{s+1} + \frac{1}{s+2}\right)$$

For step input, $R(s) = \frac{1}{s}$

$$C(s) = R(s)\cdot G(s) = \frac{1}{s}\left(\frac{1}{s+1} + \frac{1}{s+2}\right)$$

$$= \frac{1}{s(s+1)} + \frac{1}{s(s+2)}$$

$$C(s) = \left(\frac{1}{s} - \frac{1}{s+1}\right) + \frac{1}{2}\left(\frac{1}{s} - \frac{1}{s+2}\right)$$

$$= \frac{1.5}{s} - \frac{1}{s+1} - \frac{0.5}{s+2}$$

Response = $c(t) = \mathcal{L}^{-1}\left[C(s)\right]$

$$= \mathcal{L}^{-1}\left[\frac{1.5}{s} - \frac{1}{s+1} - \frac{0.5}{s+2}\right]$$

$$c(t) = (1.5 - e^{-t} - 0.5e^{-2t})u(t)$$

3. (b)

$$H(f) = \frac{5}{1 + j10\pi f}$$

$$H(s) = \frac{5}{1+5s} = \frac{5}{5\left(s + \frac{1}{5}\right)} = \frac{1}{s + \frac{1}{5}}$$

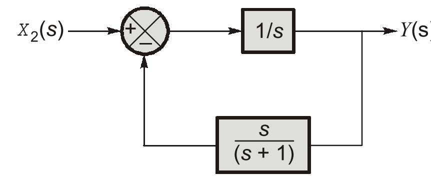
Step response = $\frac{1}{s}\frac{1}{\left(s + \frac{1}{5}\right)}$

$$Y(s) = \frac{5}{s} - \frac{5}{s + \frac{1}{5}}$$

$\Rightarrow$ $$y(t) = 5[1 - e^{-t/5}]\,u(t)$$

4. (d)

Redrawing the block diagram with $X_1(s) = 0$



The transfer function

$$T(s) = \frac{Y(s)}{X_2(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad \text{...(i)}$$

Here, $G(s) = \frac{1}{s}$ and $H(s) = \frac{s}{s+1}$

$$\frac{Y(s)}{X_2(s)} = \frac{1/s}{1 + \frac{1}{s}\times\frac{s}{s+1}} = \frac{(s+1)}{s(s+2)}$$

5. (c)

$$\text{T.F.} = \frac{1}{[s - (\sigma + j\omega)][s - (\sigma - j\omega)]}$$

$$= \frac{1}{[(s-\sigma) - j\omega)][(s-\sigma) + j\omega)]}$$

$$= \frac{1}{\left[(s-\sigma)^2 - (j\omega)^2\right]}$$

$$= \frac{1}{\left[(s-\sigma)^2 + \omega^2\right]}$$

For impulse response, taking its inverse Laplace transformation we get,

$$c(t) = e^{\sigma t}\sin\omega t \times c(t) = \frac{e^{\sigma t}\sin\omega t}{\omega}$$

Hence, option (c) is correct.

6. (a)

$$\frac{C(s)}{R(s)} = \frac{s+1}{s+2}$$

$\therefore$ $$C(s) = R(s)\cdot\frac{s+1}{s+2}$$

$$= \frac{1}{s^2}\cdot\frac{s+1}{s+2} = \frac{1}{s^2}\left(1 - \frac{1}{s+2}\right)$$

$$= \frac{1}{s^2} - \frac{1}{s^2(s+2)}$$

$$\frac{1}{s^2}\cdot\left(\frac{s+1}{s+2}\right) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2}$$

$$s+1 = As(s+2) + B(s+2) + Cs^2$$
$$= As^2 + 2As + Bs + 2B + Cs^2$$
$$\therefore \quad A + C = 0,\ 2A + B = 1 \text{ and } 2B = 1$$
$$\therefore \quad A = \frac{1}{2},\quad B = \frac{1}{4},\quad C = -\frac{1}{4}$$
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{1}{4s} + \frac{1}{2s^2} + \left(-\frac{1}{4}\right)\frac{1}{s+2}$$
$$= \frac{1}{4}u(t) + \frac{1}{2}tu(t) - \frac{1}{4}e^{-2t}u(t)$$

7. (d)

(i) Transfer function can be obtained from signal flow graph of the system.

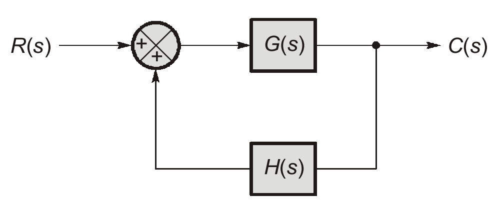
(ii) Transfer function typically characterizes to LTI systems.

(iii) Transfer function gives the ratio of output to input in s-domain of system.

$$\text{TF} = \left.\frac{L[\text{Output}]}{L[\text{Input}]}\right|_{\text{Initial conditions} = 0}$$

9. (d)

Block diagram of regenerating feedback system



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

11. (d)

Let $R(s)$ is the input Laplace transform of given differential equation is

$$s^2Y(s) + 6sY(s) + 9Y(s) = R(s)$$
$$(s^2 + 6s + 9)\ Y(s) = R(s)$$
$$\text{TF} = \frac{Y(s)}{R(s)} = \frac{1}{(s+3)^2}$$
$$IR = L^{-1}\left[\frac{1}{(s+3)^2}\right] = te^{-3t}u(t)$$

12. (c)

Given that :

$$r(t) = 1 - e^{-t},$$
$$c(t) = 1 - e^{-2t}$$
$$R(s) = \frac{1}{s} - \frac{1}{s+1} = \frac{1}{s(s+1)}$$
$$C(s) = \frac{1}{s} - \frac{1}{(s+2)} = \frac{2}{s(s+2)}$$
$$\text{TF} = \frac{C(s)}{R(s)} = \frac{2s(s+1)}{s(s+2)} = \frac{2(s+1)}{s+2}$$

13. (c)

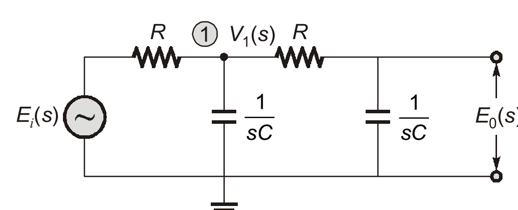
Given : $\quad c(t) = \frac{5}{2} + 5t - \frac{5}{2}e^{-2t}$

$$C(s) = \frac{5}{2s} + \frac{5}{s^2} - \frac{5/2}{s+2}$$
$$= \frac{5}{2}\left[\frac{1}{s} + \frac{2}{s^2} - \frac{1}{s+2}\right]$$
$$= \frac{5}{2}\left[\frac{s(s+2) + 2(s+2) - s^2}{s^2(s+2)}\right]$$
$$= \frac{5}{2}\left[\frac{s^2 + 2s + 2s + 4 - s^2}{s^2(s+2)}\right]$$
$$= \frac{5}{2}\frac{(4s+4)}{s^2(s+2)} = \frac{10(s+1)}{s^2(s+2)}$$

So transfer function,

$$\frac{C(s)}{R(s)} = \frac{\frac{10(s+1)}{s^2(s+2)}}{\frac{1}{s}} = \frac{10(s+1)}{s(s+2)}$$

14. (b)



KCL at node-1,

$$\frac{V_1(s)}{1/sC} + \frac{V_1(s)}{R + 1/sC} + \frac{V_1(s) - E_i(s)}{R} = 0$$

$$V_1(s) = \frac{E_i(s)/R}{sC + \frac{sC}{1+SCR} + \frac{1}{R}}$$

$$V_1(s) = \frac{E_i(s)(1+SCR)}{sC(1+SCR)R + SCR + 1 + SCR}$$

$$V_1(s) = \frac{E_i(s)(1+SCR)}{S^2R^2C^2 + 3SCR + 1}$$

$$E_0(s) = \frac{1/sC}{R + \frac{1}{sC}} \cdot V_1(s)$$

$$E_0(s) = \frac{1}{1+SCR} \times \frac{E_i(s)(1+SCR)}{S^2R^2C^2 + 3SCR + 1}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{S^2R^2C^2 + 3SCR + 1}$$

$$= \frac{1}{(s^2\tau^2 + 3\tau s + 1)}$$

15. (b)

Poles location, $s = 0,\ s = -2,\ s = -4$

Zero's location, $s = -3$

$$G(s) = \frac{k(s+3)}{s(s+2)(s+4)}$$

$$G(s)|_{s=1} = \frac{k(1+3)}{1 \times (1+2) \times (1+4)} = 4$$

$$\Rightarrow \quad k = \frac{4 \times 5 \times 3}{4} = 15$$

Hence, $$G(s) = \frac{15(s+3)}{s(s+2)(s+4)}$$

16. (b)

Poles location,

$$s_1 = -10 = P_1$$

$$s_{2,3} = (s^2 + 2s + 1) = (s + 1)^2$$

$$= -1, -1 = P_2, P_3$$

Since, $\frac{P_1}{P_2} > 5,\ \frac{P_1}{P_3} > 5$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{10}{\left(\frac{s}{10}+1\right)(s^2 + 2s + 1)}$$

Pole $\left(\frac{s}{10}+1\right)$ can be neglected, hence 2nd order approximation,

$$\frac{C(s)}{R(s)} = \frac{10}{(s^2 + 2s + 1)}$$

17. (b, d)

The relationship between the input and output of differentiator is given as

$$y(t) = \frac{dx}{dt}(t)$$

$$\Updownarrow \text{ L.T.}$$

$$Y(s) = sX(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = s$$

Put $s = j\omega$,

$$H(j\omega) = j\omega$$

$$|H(j\omega)| = \omega$$

$$\angle H(j\omega) = 90^\circ \text{ (always)}$$

So, it is constant with w.r.t. frequency.

18. (b)

Poles : $-1, -2$

Zeros : $-1 + j,\ -1 - j$

$\therefore$ Transfer function is given by,

$$G(s) = \frac{K((s+1)^2 + 1)}{(s+1)(s+2)},$$

where gain is assumed to be K

$$G(s) = \frac{K(s^2 + 2s + 2)}{(s+1)(s+2)}$$

at $s = 2$, $\quad G(s) = 15$

$$G(s) = 15 = \frac{K(2^2 + 2(2) + 2)}{(2+1)(2+2)}$$

$$\therefore \quad K = \frac{15 \times 3 \times 4}{10} = 18$$

Therefore, transfer function of the system is given as,

$$G(s) = \frac{18(s^2 + 2s + 2)}{(s+1)(s+2)}$$

19. (c)

Given : $y(t) = 2te^{-5t}$

$x(t) = u(t)$

Taking Laplace transform, we get,

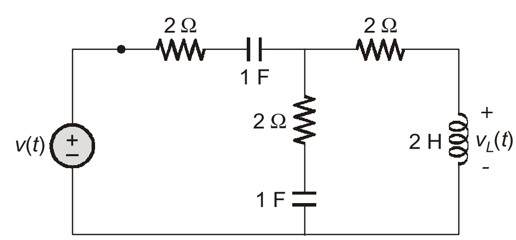
$$Y(s) = \frac{2}{(s+5)^2} \quad \text{and} \quad X(s) = \frac{1}{s}$$

$\therefore$ Overall transfer function,

$$\frac{Y(s)}{X(s)} = \frac{2s}{(s+5)^2}$$

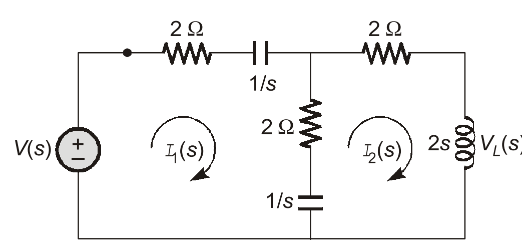
CONVENTIONAL BRAIN TEASERS

Q.1 Determine the transfer function, $G(s) = \dfrac{V_L(s)}{V(s)}$ for the network shown below using mesh analysis.



1. (Sol.)

For the given network assuming loop current as shown below.



Writing mesh equation using KVL,

$$-V(s) + 2I_1(s) + \frac{I_1(s)}{s} + (I_1(s) - I_2(s))\left(2 + \frac{1}{s}\right) = 0$$

$$V(s) = \left(4 + \frac{2}{s}\right)I_1(s) - \left(2 + \frac{1}{s}\right)I_2(s) \quad \text{...(1)}$$

Similarly, $(2+2s)I_2(s) + \left(2 + \dfrac{1}{s}\right)(I_2(s) - I_1(s)) = 0$

$$\left(4 + 2s + \frac{1}{s}\right)I_2(s) - \left(2 + \frac{1}{s}\right)I_1(s) = 0$$

$$-\left(2 + \frac{1}{s}\right)I_1(s) + \left(4 + 2s + \frac{1}{s}\right)I_2(s) = 0 \quad \text{...(2)}$$

Eliminating $I_1(s)$ using equation (2),

$$I_1(s) = \frac{\left(4+2s+\frac{1}{s}\right)I_2(s)}{\left(2+\frac{1}{s}\right)} = \left(\frac{4s+2s^2+1}{2s+1}\right)\cdot I_2(s)$$

Substituting equation (1),

$$V(s) = \left(4+\frac{2}{s}\right)\left(\frac{4s+2s^2+1}{2s+1}\right)I_2(s) - \left(2+\frac{1}{s}\right)I_2(s)$$

$$V(s) = \frac{4s+2}{s}\left(\frac{4s+2s^2+1}{2s+1}\right)I_2(s) - \left(\frac{2s+1}{s}\right)I_2(s)$$

$$V(s) = \frac{2}{s}(4s+2s^2+1)\,I_2(s) - \frac{(2s+1)}{s}\,I_2(s)$$

$$V(s) = \frac{(8s+4s^2+2-2s-1)}{s}\,I_2(s) = \frac{4s^2+6s+1}{s}\,I_2(s)$$

$$\frac{I_2(s)}{V(s)} = \frac{s}{4s^2+6s+1}$$

$$2s\frac{I_2(s)}{V(s)} = \frac{2s^2}{4s^2+6s+1} = \frac{V_L(s)}{V(s)}$$

■■■■

CHAPTER

1

Introduction

Section-B

1.1 PROCESS

- A process denotes an operation or a series of operation on fluid or solid materials by which the materials are converted into a more useful state, where the physical or chemical state of the materials is not necessarily altered. In terms of chemical engineering, process is a term which means change of chemical state and in terms of mechanical engineering the process means the change of physical states of the material.
- Bigger industrial process such as the chemical process occurs in the chemical plants. A chemical plant is an arrangement of processing units such as (heat exchanges, reactors, pumps, distillation column, absorbers, evaporators, tanks etc.). These plant auxiliaries are integrated with one another in a systematic and rational manner. The overall objective of the plant is to convert the available raw materials into the desired products in the most economical way.
- A process consists of various variable parameters which are to be controlled such as temperature, pressure, concentration, level etc. These variable programmes are called as the **process variables.**

  Example:

  Consider a water heating process as shown in the figure below:

- The purpose of the heater here is to maintain the supply of the heated water. This heater pipe acts as a heat exchange the heat of the hot water circulating in the heater pipe is transferred to the water present in the tank.

  So, the outlet water temperature of the tank marked '*c*' is the variable we want to control, so this variable '*c*' here is called as the **controlled variable.**
- The variable '*c*' is maintained by controlling the flow rate of the hot water through the heater pipe (i.e. variable '*m*'). So, here the flow rate of hot water '*m*' acts as the **manipulated variable**, and the incoming water temperature acts as the load variable, here the variable '*u*' is acts as **load variable**.

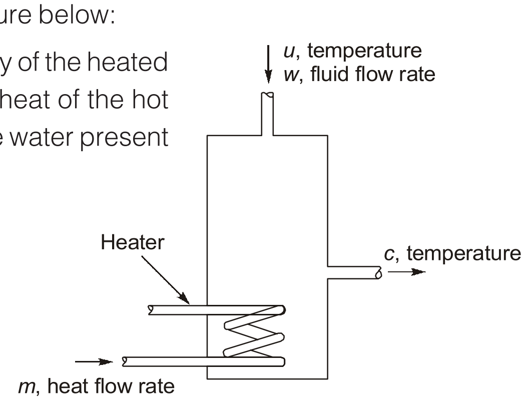


Figure : *A water-heating process*

1.2 AUTOMATIC CONTROL

- During the ongoing of the process it is strongly required that the controlled variable should be maintained at the desired set point, any deviation in the value of the controlled variable will lead to the change in the quality of the product obtained. So, we need a continuous monitoring of the operation of the chemical plant and has to avoid the effect of external disturbances, so as to achieve the error free output. This is accomplished by proper arrangement of equipment (i.e. measuring devices, valves, controllers, computers) and ensure a trained human intervention (plant designers or plant operators) which altogether forms a control system.
- Following are three basic needs that a control system is required to satisfy:
  1. Suppressing the influence of external disturbances.
  2. Ensuring the stability of a chemical process.
  3. Optimizing the performance of a chemical process.
- These all requirements can be achieved by installing an automatic control loop, which continuously monitors a process and keep the controlled variables at the desired point.

1.2.1 Controlling a Process

A process is controlled by using a feedback loop. The closed loop process containing the negative feedback keeps the controlled variable at the desired level. In a negative feedback closed loop process, wherever any deviation of the control variable from the desired point occurs the process automatically adjusts itself to the controlled variable set output.

1.2.2 Controlling the Process of a Stirred Tank Heater

- The figure below shows the process controlling the operation of a stirred tank heater.

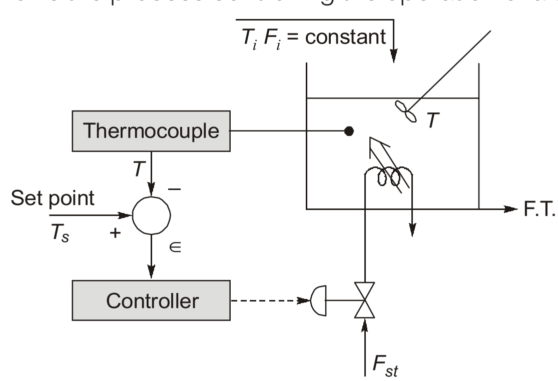


***Figure :** Feedback temperature control for a tank heater*

- The liquid enters the tank at a flow rate of F_i (ft^3/min) and at a temperature of T_i (°F). This liquid is heated by the flow of steam having the flow rate as F_{st} (lb/min) entering through the heater pipe.
- Let the flow rate and temperature of the steam leaving the tank be F and T respectively. The temperature T is uniformly distributed in the tank by the help of stirrer, which continuously mixes the water.
- The main objective of the process is
  1. To keep the effluent temperature (T) at a desired value T_s.
  2. To keep the volume of the liquid in the tank at desired value V_s.
- The operation of the heater is disturbed by the changes in the external factors such as change in the feed flow rate (F_i) and the temperature (T_i). If nothing changes then after attaining $T = T_s$ we can leave the system of steady state, but this is not possible as the F_i and T_i changes with time.

So a feedback loop is required to sense the change in the output and accordingly vary the manipulated variable to keep the control output stable at the set point.

- In the above figure we can see that we have to keep $T = T_s$ despite of the changes in the values of T_i and F_i. A thermocouple is installed which measures the temperature T of the liquid in the tank. The measured temperature. T is compared with the desired temperature. T_s which yield a deviation $\varepsilon = (T_s - T)$.
- The value of deviation ε is sent to the control mechanism which decides what should be done in order to keep the temperature T to return back to the desired value T_s (i.e. to make $\varepsilon = 0$).
- When $\varepsilon > 0$.
  i.e. when, $(T < T_s)$, the controller opens the steam valve so that more heat can be supplied.
- When $\varepsilon < 0$.
  i.e. when, $(T > T_s)$, the controller closes the steam value.
- When $\varepsilon = 0$.
  i.e. when $(T = T_s)$, controller does no work.
- The control system which measures the variable of direct importance (T in this case) after a disturbance had its effect on it, is called the **feedback control system**. The desired value T_s is called the set point and is supplied externally to the controller.
  Similarly, by the use of a feedback control loop we can control the liquid level in a tank by either manipulating the inlet flow rate or by manipulating the output flow rate.
- The figure below shows two alternative method of controlling the liquid level by the closed loop feedback configuration.

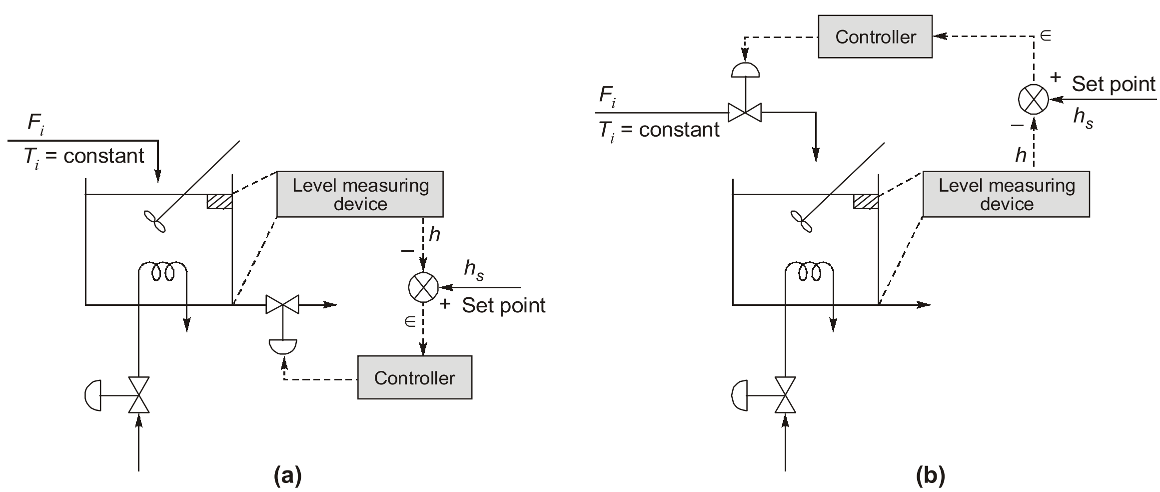


***Figure :** Alternative liquid-level control schemes*

- Here, the figure (a) controls the water level by manipulating the outlet flow rate (F_0) and figure (b) controls it by manipulating the inlet flow rate (F_i).

1.3 FEED FORWARD CONTROL TECHNIQUE

- The feed forward control does not wait until the effect of the disturbances has been felt in the output of the system but it act before the disturbances affects the output. So, we can say that the feedforward control anticipate the disturbance and do not let it affect the output of the system.

- In the above figure we can see a feedforward control loop used to control the temperature of the water in the tank. Our objective here is to keep the temperature $T = T_s$, when the T_i changes. Here we measure the temperature of inlet steam T_i and open or close the steam value to provide more or less steam. Such a control configuration is called the feed forward control loop.

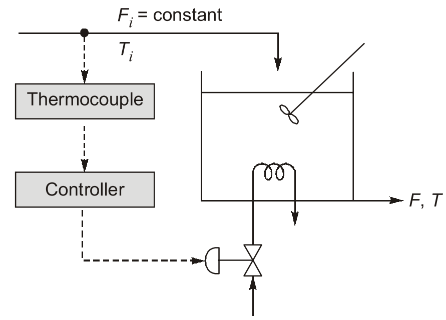


***Figure :** Feedforward temperature control for stirred tank heater*

REMEMBER

- Process dynamics and control is used to carry out the automatic control of the ongoing process.
- The controller compares the measured value to the set point value.
- For carrying out the automatic control of a process, the process variables are converted into the 4-20 mA current.
- The controller generates the error value which is used to vary the manipulated variable.
- When the error value is zero (i.e. $\varepsilon = 0$) the controller does not work.
- The feedforward control acts before the effect of disturbance has been felt in the system.
- The feedback control action acts after the effect of disturbance has taken place in the output.

OBJECTIVE BRAIN TEASERS

Q.1 What is the output of the controller?
(a) manipulated variable
(b) measured variable
(c) set point value
(d) manipulating variable

Q.2 Which of the following in the reference input given to the controller?
(a) manipulated variable
(b) measured variable
(c) set point value
(d) manipulating variable

Q.3 When the error, $\varepsilon = 0$, then the controller gives
(a) zero response (b) positive response
(c) negative response (d) none of the above

Q.4 The output of the controller is zero when
(a) $\bar{y}(s) = \bar{y}_{sp}(s)$ (b) $\bar{y}(s) > \bar{y}_{sp}(s)$
(c) $\bar{y}(s) < \bar{y}_{sp}(s)$ (d) $\bar{y}(s) >> \bar{y}_{sp}(s)$

Q.5 The feedforward control
(a) needs more in depth study of the process modelling.
(b) requires more number of measurements
(c) do not let the disturbance to affect the output
(d) all of the above

ANSWERS KEY

1. (a) 2. (c) 3. (a) 4. (a) 5. (d)