



# POSTAL BOOK PACKAGE 2024

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### INSTRUMENTATION ENGINEERING

#### Objective Practice Sets

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## Vector Analysis

## MCQ and NAT Questions

**Q.1** If  $\vec{G} = 15r\hat{a}_\phi$ , then  $\oint \vec{G} \cdot d\vec{l}$  over the circular path

$r = 2 \text{ m}$ ,  $\theta = 30^\circ$ ,  $0 < \phi < 2\pi$  is

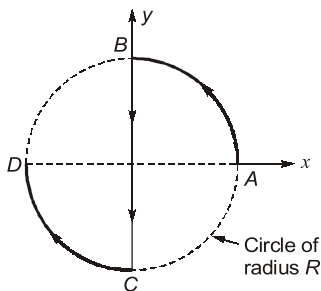
- (a)  $120\pi$  (b) 120  
(c)  $60\pi$  (d) 60

**Q.2** Which of the following is true?

- (a)  $\text{Curl}(\vec{A} \cdot \vec{B}) = \text{Curl } \vec{A} + \text{Curl } \vec{B}$   
(b)  $\text{Div}(\vec{A} \cdot \vec{B}) = \text{Div } \vec{A} \cdot \text{Div } \vec{B}$   
(c)  $\text{Div}(\text{Curl } \vec{A}) = 0$   
(d)  $\text{Div}(\text{Curl } \vec{A}) = \Delta \cdot \vec{A}$

**Q.3** What is the value of the integral  $\int_c d\vec{l}$  along the

curve  $c$  ( $c$  is the curve  $ABCD$  in the direction of the arrow)?



- (a)  $2R(\hat{a}_x + \hat{a}_y)/\sqrt{2}$  (b)  $-2R(\hat{a}_x + \hat{a}_y)/\sqrt{2}$   
(c)  $2R\hat{a}_x$  (d)  $-2R\hat{a}_y$

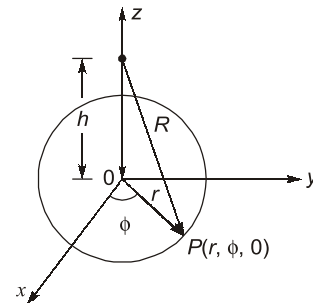
**Q.4** If  $uF = \nabla v$ , where  $u$  and  $v$  are scalar fields and  $F$  is a vector field, then  $F \cdot \text{curl } F$  is equal to

- (a) zero (b)  $\frac{\nabla^2 v}{u^2}$   
(c)  $\frac{(\nabla v \cdot \nabla) v}{u^2}$  (d) not defined

**Q.5** Laplacian of a scalar function  $V$  is

- (a) Gradient of  $V$   
(b) Divergence of  $V$   
(c) Gradient of the gradient of  $V$   
(d) Divergence of the gradient of  $V$

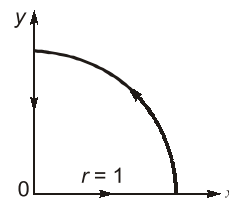
**Q.6** The unit vector  $\vec{a}_R$  which points from  $z = h$  on the  $z$ -axis towards  $(r, \phi, 0)$  in cylindrical co-ordinates as shown below is given by



- (a)  $\frac{h\vec{a}_r - r\vec{a}_z}{\sqrt{r^2 + h^2}}$  (b)  $\frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$   
(c)  $\frac{h\vec{a}_\phi - r\vec{a}_z}{\sqrt{r^2 + h^2}}$  (d)  $\frac{r\vec{a}_z - h\vec{a}_\phi}{\sqrt{r^2 + h^2}}$

**Q.7** Given a vector field  $\vec{A} = 2r\cos\phi\hat{a}_r$  in cylindrical coordinates. For the contour as shown below,

$\oint \vec{A} \cdot d\vec{l}$  is



- (a) 1 (b)  $1 - (\pi/2)$   
(c)  $1 + (\pi/2)$  (d) -1

**Q.8** If  $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$  and  $|\vec{r}| = r$  then find  $\text{div}(r^2 \nabla(\ln r))$

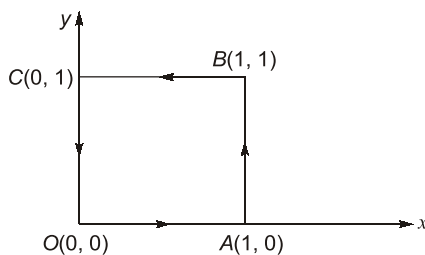
**Q.9** Which of the following statements is not true of a phasor?

- (a) It may be a scalar or a vector.
- (b) It is a time dependent quantity.
- (c) It is a complex quantity.
- (d) All are true.

**Q.10** The maximum space rate of change of the function which is in increasing direction of the function is known as

- (a) curl of the vector function
- (b) gradient of the scalar function
- (c) divergence of the vector function
- (d) Stokes theorem

**Q.11** Given vector  $\vec{A} = x^2y\hat{a}_x + 2xy^2\hat{a}_y$ , find circulation of  $\vec{A}$  along a closed path OABC as shown in figure below.



**Q.12** Given a vector field  $\vec{F}$ . The Stoke's theorem states that,

- (a)  $\oint \vec{F} \cdot d\vec{l} = \iint (\vec{\nabla} \times \vec{F}) \cdot d\vec{s}$
- (b)  $\oint \vec{F} \times d\vec{l} = \iiint (\vec{\nabla} \cdot \vec{F}) d\vec{s}$
- (c)  $\int \vec{F} \cdot d\vec{l} = \iint (\vec{\nabla} \times \vec{F}) \cdot d\vec{s}$
- (d)  $\int \vec{F} \times d\vec{l} = \iint (\vec{\nabla} \cdot \vec{F}) d\vec{s}$

**Q.13** The vector  $\vec{A}$  directed from  $(2, -4, 1)$  to  $(0, -2, 0)$  in Cartesian coordinates is given by

- (a)  $-2\vec{a}_x + 2\vec{a}_y + \vec{a}_z$
- (b)  $-2\vec{a}_x + 2\vec{a}_y - \vec{a}_z$
- (c)  $-\vec{a}_x - 2\vec{a}_y + 2\vec{a}_z$
- (d)  $\vec{a}_x - 2\vec{a}_y - \vec{a}_z$

**Q.14** The vector field given by

$$\vec{A} = yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z$$

- (a) rotational and solenoidal
- (b) rotational but not solenoidal
- (c) irrotational and solenoidal
- (d) irrotational but not solenoidal

**Q.15** If  $\vec{A} = \frac{\vec{a}_x}{\sqrt{x^2 + y^2}}$ , then the value of  $\nabla \cdot \vec{A}$  at  $(2, 2, 0)$

will be

- (a) -0.0884
- (b) 0.0264
- (c) -0.0356
- (d) 0.0542

**Q.16** If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ , then the value of

$$\vec{i} \times (\vec{r} \times \vec{i}) + \vec{j} \times (\vec{r} \times \vec{j}) + \vec{k} \times (\vec{r} \times \vec{k})$$

- (a)  $\vec{r}$
- (b)  $2\vec{r}$
- (c)  $3\vec{r}$
- (d)  $6\vec{r}$

**Q.17** What is the value of constant  $b$  so that the vector

$$\vec{V} = (x + 3y)\vec{i} + (y - 2x)\vec{j} + (x + bz)\vec{k}$$

is solenoidal?

- (a) 2
- (b) -1
- (c) 3
- (d) -2

**Q.18** For a vector field

$\vec{A} = xyz^3\hat{a}_x + xy^3z\hat{a}_y + x^3yz\hat{a}_z$ . Evaluate the surface integral for a surface of unit cube by  $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$ .

**Q.19** Which of the following option is not correct?

- (a) A vector field  $\vec{A}$  is solenoid, if  $\nabla \cdot \vec{A} = 0$
- (b) A vector field  $\vec{A}$  is irrotational, if  $\nabla \times \vec{A} = 0$
- (c) A vector field  $V$  is harmonics, if  $\nabla^2 V \neq 0$
- (d) All options are correct

**Q.20** Which of the following statements is not true regarding vector algebra?

- (a) Dot product of like unit vector is unity.
- (b) Dot product of unlike unit vector is zero.
- (c) Cross product of two like unit vectors is a third unit vector having positive sign for normal rotation and negative for reverse rotation.
- (d) All the above statements are true.

**Q.21** A vector  $\vec{P}$  is given by

$$\vec{P} = x^3 y \vec{a}_x - x^2 y^2 \vec{a}_y - x^2 y z \vec{a}_z$$

Which of the following statements is **TRUE**?

- (a)  $\vec{P}$  is solenoidal, but not irrotational
- (b)  $\vec{P}$  is irrotational, but not solenoidal
- (c)  $\vec{P}$  is neither solenoidal nor irrotational
- (d)  $\vec{P}$  is both solenoidal and irrotational

**Q.22** A rigid body is rotating with an angular velocity of  $\omega$  where,  $\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$  and  $v$  is the line velocity. If  $\vec{r}$  is the position vector given by  $\vec{r} = x \vec{i} + y \vec{j} + z \vec{k}$ , then the value of  $\text{curl } \vec{v}$  will be equal to

- (a)  $1/2 \omega$
- (b)  $\omega$
- (c)  $1/3 \omega$
- (d)  $2 \omega$

**Q.23** Which of the following identity is not true?

- (a)  $\vec{A}(\vec{B} \cdot \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$
- (b)  $\nabla \cdot (\nabla \times \vec{A}) = 0$
- (c)  $\nabla \times \nabla \phi \neq 0$
- (d) None of the above

**Q.24** The value of divergence of a vector quantity

$$\vec{A} = 4xy \hat{a}_x + xz \hat{a}_y + xyz \hat{a}_z \text{ at a point } P(1, -2, 3)$$

will be

- (a) 6
- (b) -16
- (c) -10
- (d) 12

**Q.25** Laplace equation in cylindrical coordinates is given by

$$(a) \nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \left( \frac{\partial^2 V}{\partial \phi^2} \right) + \frac{\partial^2 V}{\partial z^2} = 0$$

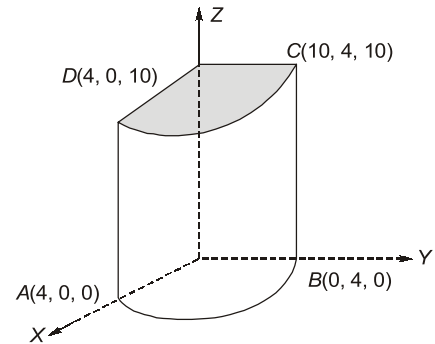
$$(b) \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$(c) \nabla^2 V = \frac{-\rho}{\epsilon}$$

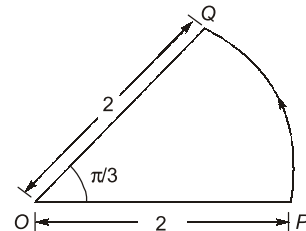
$$(d) \nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \left( -\frac{1}{r^2 \sin \theta} \right) \frac{\partial}{\partial \theta}$$

$$\left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} = 0$$

**Q.26** Consider the object shown in figure below calculate. The surface area  $ABCD$ ,



**Q.27** If  $\vec{A} = \hat{a}_\rho + \hat{a}_\phi + \hat{a}_z$ , the value of  $\oint \vec{A} \cdot d\vec{l}$  around the closed circular quadrant shown in the given figure is \_\_\_\_\_.



**Q.28** Given,  $W = x^2 y^2 + xy$ , compute  $\nabla W$  and the direction derivative  $dW/dl$  in the direction,

$$\vec{A} = 3\hat{a}_x + 4\hat{a}_y + 12\hat{a}_z \text{ at } (2, -1, 0) \text{ is } \underline{\hspace{2cm}}.$$

**Q.29** If  $\vec{E}$  is the electric field intensity then  $\vec{\nabla} \times (\vec{\nabla} \cdot \vec{A})$  is equal to

- (a)  $\vec{E}$
- (b)  $|\vec{E}|$
- (c) Null vector
- (d) zero

**Q.30** Divergence of the vector field,

$$V(x, y, z) = (x \sin xy) \hat{i} - (y \sin xy) \hat{j} + \sin z^2 \hat{k} \text{ is}$$

$$\text{Divergence} = 2z \cos z^2$$

- (a)  $2z \cos z^2$
- (b)  $\sin xy + 2z \cos z^2$
- (c)  $x \sin xy - \cos z$
- (d) none of these

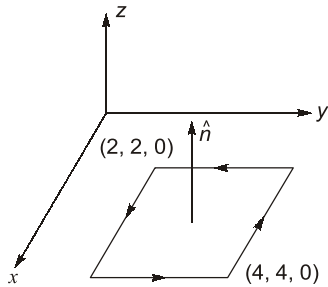
**Q.31** The line integral of the vector field

$$\vec{F} = 5x \vec{i} + 3y \vec{j} + x^2 z \vec{k} \text{ along a path from } (0, 0, 0)$$

to  $(1, 1, 1)$  parameterized by  $(t, t^2, t)$  is

**Q.32** If the vector  $V$  given below is irrotational, then the values of  $a$ ,  $b$  and  $c$  will be respectively

$$V = (x + 2y + az) \vec{i} + (bx - 3y - z) \vec{j} + (4x + cy + 2z) \vec{k}$$



Which of the following options is/are correct?

- (a)  $\iint_S \vec{F} \cdot \hat{n} ds = 80$   
 (b)  $\iint_S (\vec{F} \times \hat{n}) ds = 120\hat{a}_x - 112\hat{a}_y$   
 (c)  $\nabla \times \vec{F} = x^2\hat{a}_x - 2xy\hat{a}_y - 2xy\hat{a}_z$   
 (d)  $\iint_S (\nabla \times \vec{F}) \cdot \hat{n} ds = -120$

**Q.46** If  $[\vec{a}, \vec{b}, \vec{c}]$  represents the scalar triple product of

vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$ , then which of the below statements is/are true?

- (a)  $[\vec{a}, \vec{b}, \vec{c}] = [\vec{c}, \vec{b}, \vec{a}]$   
 (b)  $[\vec{a}, \vec{b} + \vec{a}, \vec{c}] = 0$   
 (c)  $[3\vec{b}, \vec{c}, \vec{a}] = 3[\vec{a}, \vec{b}, \vec{c}]$   
 (d) If  $[\vec{a}, \vec{b}, \vec{c}] = 0$ , the vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$  are coplanar.

**Q.47** The values of  $\alpha$  for which the vectors

$$\vec{A} = \alpha\hat{a}_x + 2\hat{a}_y + 10\hat{a}_z \text{ and } \vec{B} = 4\alpha\hat{a}_x + 8\hat{a}_y - 2\alpha\hat{a}_z$$

are perpendicular is/are

- (a) 1 (b) 2  
(c) 3 (d) 4

**Q.48** Which of the below vector identities are true?

- (a)  $A \times (B \times C) = (A \times B) \times C$   
 (b)  $A \times (B \times C) + C \times (A \times B) + B \times (C \times A) = 0$   
 (c)  $(B \times C) \times (C \times A) = C(A \cdot B \times C)$   
 (d)  $(A \times B) \cdot (C \times D) = (A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C)$

**Q.49** For the scalar function,  $\phi = x^2yz^3$ , which of the below statements is/are correct?

- (a) From the point  $(2, 1, -1)$  the directional derivative of  $\phi$  is maximum in the direction represented by vector  $-12\hat{i} - 4\hat{j} + 4\hat{k}$ .  
 (b) The magnitude of greatest rate of change of  $\phi$  from the point  $(2, 1, -1)$  is  $4\sqrt{11}$ .  
 (c)  $(x-2) + (y-1) - 3(z+1) = 0$  represents the tangent plane to the surface  $\phi = 0$  at point  $(2, 1, -1)$ .  
 (d)  $\phi$  satisfies the Laplacian equation.

■■■■

## Answers Vector Analysis

- |               |            |            |            |             |               |             |
|---------------|------------|------------|------------|-------------|---------------|-------------|
| 1. (c)        | 2. (c)     | 3. (d)     | 4. (a)     | 5. (d)      | 6. (b)        | 7. (a)      |
| 8. (3)        | 9. (a)     | 10. (b)    | 11. (0.33) | 12. (a)     | 13. (b)       | 14. (c)     |
| 15. (a)       | 16. (b)    | 17. (d)    | 18. (0.5)  | 19. (c)     | 20. (c)       | 21. (a)     |
| 22. (d)       | 23. (c)    | 24. (c)    | 25. (a)    | 26. (62.83) | 27. (2.09)    | 28. (-1.15) |
| 29. (d)       | 30. (a)    | 31. (3.75) | 32. (a)    | 33. (d)     | 34. (129.43)  | 35. (c)     |
| 36. (d)       | 37. (b)    | 38. (2)    | 39. (b)    | 40. (a)     | 41. (5)       | 42. (c)     |
| 43. (a, c, d) | 44. (b, d) | 45. (b, c) | 46. (c, d) | 47. (a, d)  | 48. (b, c, d) | 49. (b, c)  |

**Explanations Vector Analysis**

**1. (c)**

For spherical coordinate systems,

$$\vec{dl} = r \sin \theta d\phi \hat{a}_\phi$$

$$\begin{aligned}\oint \vec{G} \cdot \vec{dl} &= \int_0^{2\pi} 15 r \hat{a}_\phi \cdot r \sin \theta d\phi \hat{a}_\phi \\ &= 15 \cdot r^2 \cdot \sin \theta (2\pi) \\ &= 15 \cdot (2)^2 \times \sin 30^\circ (2\pi)\end{aligned}$$

$$\oint \vec{G} \cdot \vec{dl} = 60\pi$$

**2. (c)**

Divergence (Curl  $\vec{A}$ ) = 0

**3. (d)**

$$\int_{AB} \vec{dl} = \int_0^{\pi/2} R \cdot d\theta \hat{a}_\theta = \frac{\pi}{2} R \hat{a}_\theta$$

$$\int_{BC} \vec{dl} = \int_{+R}^{-R} dl (-\hat{a}_y) = -2R \hat{a}_y$$

$$\int_{CD} \vec{dl} = \int_{-\pi/2}^{-\pi} R \cdot d\theta \hat{a}_\theta = -\frac{\pi}{2} R \hat{a}_\theta$$

$$\therefore \int_C \vec{dl} = \int_{AB} \vec{dl} + \int_{BC} \vec{dl} + \int_{CD} \vec{dl} = -2R \hat{a}_y$$

**4. (a)**

$$\text{Given, } F = \frac{1}{u} \nabla v$$

$$\therefore \text{Curl } F = \nabla \times \left( \frac{1}{u} \nabla v \right)$$

$$\begin{aligned}\text{or, } \text{Curl } F &= \nabla \frac{1}{u} \times \nabla v + \frac{1}{u} \nabla \times (\nabla v) \\ &= \nabla \frac{1}{u} \times \nabla v\end{aligned}$$

$$\text{Hence, } F \cdot \text{Curl } F = \frac{1}{u} \nabla v \cdot \left( \nabla \frac{1}{u} \times \nabla v \right) = 0$$

**5. (d)**

$$\nabla^2 V = \nabla \cdot (\nabla V)$$

= divergence of gradient of V

**6. (b)**

Let the unit vector be given by  $\vec{a}_R$ .

$$\begin{aligned}\text{Now, } \vec{R} &= \text{Difference of two vectors} \\ &= r\vec{a}_r - h\vec{a}_z\end{aligned}$$

$$\therefore \text{Unit vector, } \vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$$

**7. (a)**

$$\begin{aligned}\oint \vec{A} \cdot \vec{dl} &= \int_0^1 2r \cos \phi \hat{a}_r \cdot dr \hat{a}_r \\ &\quad + \int_0^{\pi/2} 2r \cos \phi \cdot r d\phi (\hat{a}_r \cdot \hat{a}_\phi) \\ &\quad + \int_0^1 2r \cos \phi \cdot \hat{a}_r \cdot dr \hat{a}_r\end{aligned}$$

$$\text{As } \hat{a}_r \cdot \hat{a}_\phi \text{ and } \cos \frac{\pi}{2} = 0$$

$$\therefore \oint \vec{A} \cdot \vec{dl} = 1 + 0 + 0 = 1$$

**8. (3)**

$$\begin{aligned}\text{div}(r^2 \nabla(\ln r)) &= \text{div} \left( r^2 \left( \frac{\partial \ln r}{\partial r} \hat{r} \right) \right) \\ &= \text{div} \left( r^2 \cdot \frac{1}{r} \hat{r} \right) = \text{div}(|r| \hat{r})\end{aligned}$$

$$\begin{aligned}\text{Since, } \text{div}(\vec{A}) &= \frac{1}{r^2} + \frac{\partial r^2 A_r}{\partial r} + \frac{1}{r^2 \sin \theta} \\ &\quad + \frac{\partial \sin \theta A_\theta}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}\end{aligned}$$

$$\begin{aligned}\therefore \text{div}(r^2 \nabla(\ln r)) &= \text{div}(\vec{r}) \\ &= \frac{1}{r^2} \frac{\partial r^2 r}{\partial r} = \frac{1}{r^2} \frac{\partial r^3}{\partial r} = \frac{3r^2}{r^2} \\ \text{div} &= (r^2 \nabla \ln(r)) = 3\end{aligned}$$

**9. (a)**

A phasor is always a vector quantity.

**10. (b)**

Gradient of a scalar;

$\nabla A$  = maximum rate of change of scalar  $A$  with respect to given coordinates system.

**11. (0.33)**

In a closed path the circulation of vector  $\vec{A}$  is given as,

$$\begin{aligned}\oint \vec{A} \cdot \vec{dl} &= \int_{OA} + \int_{AB} + \int_{BC} + \int_{CO} (\vec{A} \cdot \vec{dl}) \\ \therefore \vec{A} &= x^2 y \hat{a}_x + 2xy^2 \hat{a}_y\end{aligned}$$

$$d\vec{l} = dx\hat{a}_x + d\hat{a}_y$$

$$\vec{A} \cdot d\vec{l} = A_x dx + A_y d\hat{a}_y$$

For path  $OA$ ,  $dy = 0$ ,  $y = 0$ ,  $A = 0$

For path  $AB$ ,  $dx = 0$ ,  $x = 1$ ,  $A = y\hat{a}_x + 2y^2\hat{a}_y$

$$\text{and } \int_{AB} \vec{A} \cdot d\vec{l} = \int_0^1 2y^2 dy = \left[ \frac{2y^3}{3} \right]_0^1 = \frac{2}{3}$$

For path  $BC$ ,  $dy = 0$ ,  $y = 1$

$$F = x^2\hat{a}_x + 2x\hat{a}_y$$

$$\text{and } \int_{BC} \vec{A} \cdot d\vec{l} = \int_1^0 x^2 dx = \left[ \frac{x^3}{3} \right]_1^0 = -\frac{1}{3}$$

For path  $CO$ ,

$$\int_{CO} \vec{A} \cdot d\vec{l} = 0$$

$$\text{Hence, } \oint \vec{A} \cdot d\vec{l} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

**13. (b)**

The vector  $\vec{A}$  is given as

$$\begin{aligned} \vec{A} &= (0-2)\vec{a}_x + [-2-(-4)]\vec{a}_y + (0-1)\vec{a}_z \\ &\quad \begin{array}{ccc} x & & y \\ \bullet & \xrightarrow{\quad} & \bullet \\ (2, -4, 1) & & (0, -2, 0) \end{array} \\ &= -2\vec{a}_x + 2\vec{a}_y - \vec{a}_z \end{aligned}$$

**14. (c)**

The vector field  $\vec{A}$  will be irrotational, if  $\nabla \times \vec{A} = 0$ .

$$\begin{aligned} \text{Now, } \nabla \times \vec{A} &= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} \\ &= \left[ \frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(xz) \right] \vec{a}_x \\ &\quad + \left[ \frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz) \right] \vec{a}_y \\ &\quad + \left[ \frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(yz) \right] \vec{a}_z \\ &= [x-x]\vec{a}_x + [y-y]\vec{a}_y + [z-z]\vec{a}_z \\ &= 0 \end{aligned}$$

Hence,  $\vec{A}$  is irrotational.

The vector field  $\vec{A}$  will be solenoidal, if  $\nabla \cdot \vec{A} = 0$

Here,

$$\begin{aligned} \nabla \cdot \vec{A} &= \left( \vec{a}_x \frac{\partial}{\partial x} + \vec{a}_y \frac{\partial}{\partial y} + \vec{a}_z \frac{\partial}{\partial z} \right) \cdot (yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z) \\ &= \vec{a}_x \cdot \vec{a}_x \frac{\partial}{\partial x}(yz) + \vec{a}_y \cdot \vec{a}_y \frac{\partial}{\partial y}(xz) + \vec{a}_z \cdot \vec{a}_z \frac{\partial}{\partial z}(xy) \\ &= 0 + 0 + 0 = 0 \end{aligned}$$

Hence,  $\vec{A}$  is solenoidal.

**15. (a)**

$$\text{Given, } \vec{A} = \frac{1}{\sqrt{x^2 + y^2}} \vec{a}_x$$

$$\begin{aligned} \therefore \nabla \cdot \vec{A} &= \frac{\partial}{\partial x}(A_x) + \frac{\partial}{\partial y}(A_y) + \frac{\partial}{\partial z}(A_z) \\ &= \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{x^2 + y^2}} \right) + 0 + 0 \\ &= \frac{\partial}{\partial x} (x^2 + y^2)^{-1/2} \\ &= -\frac{1}{2} (x^2 + y^2)^{-3/2} \cdot 2x \\ &= \nabla \cdot \vec{A} = -\frac{x}{\sqrt{(x^2 + y^2)}(x^2 + y^2)} \end{aligned}$$

$$\begin{aligned} \text{Now, } (\nabla \cdot \vec{A})_{2,2,0} &= -\frac{2}{\sqrt{(2^2 + 2^2)} \cdot (2^2 + 2^2)} \\ &= -\frac{2}{\sqrt{8} \cdot 8} = -0.0884 \end{aligned}$$

**16. (b)**

$$\text{Given, } \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\therefore \vec{r} \times \vec{i} =$$

$$(x\vec{i} + y\vec{j} + z\vec{k}) \times \vec{i} = -y\vec{k} + z\vec{j}$$

$$\text{Also, } \vec{i} \times (\vec{r} \times \vec{i}) =$$

$$\vec{i} \times (-y\vec{k} + z\vec{j}) = \vec{j}y + z\vec{k}$$

$$\text{Similarly, } \vec{j} \times (\vec{r} \times \vec{j}) = \vec{i}x + \vec{k}z$$

$$\text{and } \vec{k} \times (\vec{r} \times \vec{k}) = \vec{i}x + \vec{j}y$$

$$\text{Thus, } \vec{i} \times (\vec{r} \times \vec{i}) + \vec{j} \times (\vec{r} \times \vec{j}) + \vec{k} \times (\vec{r} \times \vec{k})$$

$$= 2(x\vec{i} + y\vec{j} + z\vec{k}) = 2\vec{r}$$

**17. (d)**

Since vector  $\vec{V}$  is solenoidal, therefore

$$\nabla \cdot \vec{V} = 0$$

$$\therefore \left[ \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right] \cdot \left[ \vec{i}(x+3y) + \vec{j}(y-2x) + \vec{k}(x+bz) \right] = 0$$

$$\text{or, } [1 + 1 + b] = 0 \text{ or } b = -2$$

**18. (0.5)**

Given unit cube, therefore the limits of integration are 0 to 1 for  $dx$ ,  $dy$  and  $dz$ .

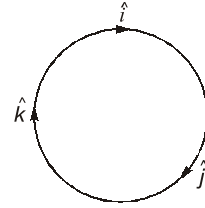
$$\begin{aligned}\oint \vec{A} \cdot d\vec{s} &= \int_0^1 \int_0^1 A_x dy dz + \int_0^1 \int_0^1 A_y dx dz + \int_0^1 \int_0^1 A_z dx dy \\ &= \int_0^1 \int_0^1 xyz^3 dy dz + \int_0^1 \int_0^1 xyz^3 dx dz + \int_0^1 \int_0^1 x^3 yz dx dy \\ &= I_1 + I_2 + I_3 \\ \therefore I_1 &= \int_0^1 \int_0^1 yz^3 dy dz \quad (\text{Since } x = 1 \text{ for } I_1) \\ &= \left( \frac{y^2}{2} \right)_0^1 \left( \frac{z^4}{4} \right)_0^1 \\ &= \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \\ I_2 &= \int_0^1 \int_0^1 xz dx dz \quad (\text{Since } y = 1 \text{ for } I_2) \\ &= \left[ \frac{x^2}{2} \right]_0^1 \left[ \frac{z^2}{2} \right]_0^1 \\ &= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \\ I_3 &= \int_0^1 \int_0^1 x^3 y dx dy \quad (\text{Since } z = 1 \text{ for } I_3) \\ &= \left( \frac{x^4}{4} \right)_0^1 \left( \frac{y^2}{2} \right)_0^1 \\ &= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8} \\ \therefore \oint \vec{A} \cdot d\vec{s} &= \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \frac{1}{2}\end{aligned}$$

**19. (c)**

A scalar field  $V$  is harmonics, if  $\nabla^2 V = 0$ . Hence, option (c) is not correct.

**20. (c)**

Option (c) is not correct because cross product of two unlike vectors is a third unit vector having positive sign for normal rotation and negative for reverse rotation while cross product of two like unit vectors is zero.



**21. (a)**

$$\vec{P} = x^3 y \vec{a}_x - x^2 y^2 \vec{a}_y - x^2 yz \vec{a}_z$$

For solenoidal,  $\nabla \cdot \vec{P} = 0$

$$\begin{aligned}\Rightarrow \nabla \cdot \vec{P} &= \frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z} \\ &= 3x^2 y - 2x^2 y - x^2 y \\ &= 0\end{aligned}$$

$\Rightarrow \vec{P}$  is solenoidal

For irrotational,  $\nabla \times \vec{P} = 0$

$$\begin{aligned}\Rightarrow \nabla \times \vec{P} &= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3 y & -x^2 y^2 & -x^2 yz \end{vmatrix} \\ &= \vec{a}_x(-x^2 z) + \vec{a}_y(2xyz) + \vec{a}_z(-2xy^2 - x^3) \\ &\neq 0\end{aligned}$$

$\Rightarrow \vec{P}$  is not irrotational.

**22. (d)**

Taking the curl, we have:

$$\nabla \times \vec{v} = \nabla \times \vec{\omega} \times \vec{r} \quad (\text{Since } \vec{v} = \vec{\omega} \times \vec{r})$$

$$\begin{aligned}\text{or, } \nabla \times \vec{v} &= \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \times \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix} \\ &= \left[ \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right] \times [\vec{i}(\omega_y z - \omega_z y) \\ &\quad + \vec{j}(\omega_z x - \omega_x z) + \vec{k}(\omega_x y - \omega_y x)] \\ &= (\vec{i} \times \vec{j}) \frac{\partial}{\partial x} (\omega_z x - \omega_x z) + (\vec{i} \times \vec{k}) \frac{\partial}{\partial x} (\omega_x y - \omega_y x) \\ &\quad + (\vec{j} \times \vec{i}) \frac{\partial}{\partial y} (\omega_y z - \omega_z y) \\ &\quad + (\vec{j} \times \vec{k}) \frac{\partial}{\partial y} (\omega_x y - \omega_y x) + (\vec{k} \times \vec{i}) \frac{\partial}{\partial z} (\omega_y z - \omega_z y) \\ &\quad + (\vec{k} \times \vec{j}) \frac{\partial}{\partial z} (\omega_z x - \omega_x z) \\ &= \vec{k}(\omega_z - 0) - \vec{j}(0 - \omega_y) - \vec{k}(0 - \omega_z) \\ &\quad + \vec{i}(\omega_x - 0) + \vec{j}(\omega_y - 0) - \vec{i}(0 - \omega_x)\end{aligned}$$



$$\begin{aligned}
 &= \vec{i}(\omega_x + \omega_x) + \vec{j}(\omega_y + \omega_y) + \vec{k}(\omega_z + \omega_z) \\
 &= 2\omega_x \vec{i} + 2\omega_y \vec{j} + 2\omega_z \vec{k} \\
 &= 2(\omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}) = 2\omega
 \end{aligned}$$

$$\therefore \nabla \times \vec{V} = \text{Curl } \vec{V} = 2\omega$$

**23. (c)**

$\vec{A} \times (\vec{B} \times \vec{C})$  is called "vector triple product" which is a correct expression.

$$\nabla = \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right)$$

and let  $\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$

Then, 
$$\nabla \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\begin{aligned}
 \therefore \nabla(\nabla \times \vec{A}) &= \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \times \left[ \vec{i} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \right. \\
 &\quad \left. + \vec{j} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \vec{k} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \right] \\
 &= \frac{\partial}{\partial x} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \frac{\partial}{\partial y} \left( \frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) + \frac{\partial}{\partial z} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\
 &= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} + \frac{\partial^2 A_z}{\partial y \partial x} - \frac{\partial^2 A_x}{\partial y \partial z} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \nabla \times \nabla \phi &= \left( \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right) \times \left( \frac{\partial \phi}{\partial x} \vec{i} + \frac{\partial \phi}{\partial y} \vec{j} + \frac{\partial \phi}{\partial z} \vec{k} \right) \\
 &= \left\{ (\vec{i} \times \vec{j}) \frac{\partial^2 \phi}{\partial x \partial y} + (\vec{i} \times \vec{k}) \frac{\partial^2 \phi}{\partial x \partial z} \right\} \\
 &\quad + \left\{ (\vec{j} \times \vec{i}) \frac{\partial^2 \phi}{\partial x \partial y} + (\vec{j} \times \vec{k}) \frac{\partial^2 \phi}{\partial y \partial z} \right\} \\
 &\quad + \left\{ (\vec{k} \times \vec{i}) \frac{\partial^2 \phi}{\partial z \partial x} + (\vec{k} \times \vec{j}) \frac{\partial^2 \phi}{\partial y \partial z} \right\} \\
 &= \vec{k} \frac{\partial^2 \phi}{\partial x \partial y} - \vec{j} \frac{\partial^2 \phi}{\partial x \partial z} - \vec{k} \frac{\partial^2 \phi}{\partial y \partial z} \\
 &\quad + \vec{i} \frac{\partial^2 \phi}{\partial y \partial z} + \vec{j} \frac{\partial^2 \phi}{\partial z \partial x} - \vec{i} \frac{\partial^2 \phi}{\partial z \partial y} = 0
 \end{aligned}$$

**24. (c)**

Divergence of a vector

$$\bar{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\begin{aligned}
 \bar{\nabla} \cdot \vec{A} &= \frac{\partial}{\partial x}(4xy) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(xyz) \\
 &= 4y + xy
 \end{aligned}$$

At point,  $P(1, -2, 3)$

$$\begin{aligned}
 \bar{\nabla} \cdot \vec{A} &= 4(-2) + (1)(-2) \\
 &= -8 - 2 = -10
 \end{aligned}$$

**25. (a)**

Laplace equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial^2 V}{\partial \phi^2} = 0$$

**26. (62.83)**

Although points  $A, B, C, D$  are given in Cartesian coordinate, it is obvious that the object has cylindrical coordinates. The points are transformed from Cartesian to cylindrical coordinates as follow:

$$A(4, 0, 0) \rightarrow A(4, 0, 0)$$

$$B(0, 4, 0) \rightarrow B\left(4, \frac{\pi}{2}, 0\right)$$

$$C(0, 4, 10) \rightarrow C\left(4, \frac{\pi}{2}, 10\right)$$

$$D(4, 0, 10) \rightarrow D(4, 0, 10)$$

For  $ABCD$ ,

$$ds = \rho d\phi dz, \rho = 4, \text{ hence}$$

$$\begin{aligned}
 \text{Area, } ABCD &= \int ds \\
 &= \int_{\phi=0}^{\pi/2} \int_{z=0}^{10} \rho d\phi dz \\
 &= 4 \int_{\phi=0}^{\pi/2} d\phi \int_{z=0}^{10} dz \\
 &= 4 \times \frac{\pi}{2} \times 10 = 20\pi = 62.83
 \end{aligned}$$

**27. (2.09)**

$$\oint \vec{A} \cdot d\vec{l} = \oint (\hat{a}_\rho + \hat{a}_\phi + \hat{a}_z) \cdot (\rho d\phi \hat{a}_\rho + \rho d\phi \hat{a}_\phi + dz \hat{a}_z)$$

For path  $OP \Rightarrow$  only  $\rho$  is varying

$$\oint \vec{A} \cdot d\vec{l} = \int_0^2 d\rho = 2$$