INSTRUMENTATION ENGINEERING

Communication

Comprehensive Theory
with Solved Examples *and* Practice Questions

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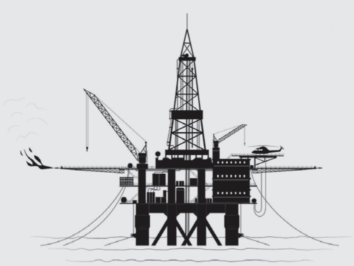
Communication



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CONTENTS

Communication

CHAPTER 4

CHAPTER 5

CHAPTER 6

CHAPTER 7

CHAPTER 8

Optimum Receivers for AWGN Channels....312-349

CHAPTER 9

Information Theory and Coding 350-402

CHAPTER 10

Multiple Access Technique and Communication Standards 403-418

CHAPTER

1

Introduction to Communication Systems

1.1 HISTORICAL SKETCH

The development of communication technology has proceeded in step with the development of electronic technology as a whole. For example, the demonstration of telegraphy by Joseph Henry in 1832 and by Samuel F.B. Morse in 1838 followed hard on the discovery of electromagnetism by Oersted and Ampere early in 1820's. Similarly, Hertz's verification late in the 1880's of Maxwell's postulation (1873) predicting the wireless propagation of electromagnetic energy led within 10 years of the radio-telegraph experiments of Marconi and Popov. The invention of diode by Flaming in 1904 and of triode by deforest in 1906 made possible the rapid development of long distance telephony, both by radio and wireless.

1.2 WHY STUDY COMMUNICATION

The rapidly changing face of technology necessitates learning of new technology. Today the question is no longer in the field of invention but of innovation. The question today in the twenty first century in not how to transmit data from point A to point B but how efficiently can we do it. To be able to answer this question, first we should be able to diagnose the problem. This can be done only by studying communication from the beginning to its modern form.

1.3 WHAT IS COMMUNICATION

In the most fundamental sense, communication involves implicitly the transmission of information from one point to another through a succession of processes, as described here:

1. The generation of a message signal: voice, music, picture, or computer data.
2. The description of that message signal with a certain measure of precisions, by a set of symbols: electrical, audio, or visual.
3. The encoding of these symbols in a form that is suitable for transmission over a physical medium of interest.
4. The transmission of the encoded symbols to the desired destination.
5. The decoding of the reproduction of the original symbols.
6. The re-creation of the original message signal, with a definable degradation in quality; the degradation is caused by imperfections in the system.

There are, of course, many other forms of communication that do not directly involve the human mind in real time. For example, in computer communications involving communication between two or more computers, human decisions may enter only in setting up the programs or commands for the computer, or in monitoring the results.

1.4 COMMUNICATION MODEL

The study of communication becomes easier, if we break the whole subject of communication in parts and then study it part by part. The whole idea of presenting the model of communication is to analyse the key concepts used in communication in isolated parts and then combining them to form the complete picture.

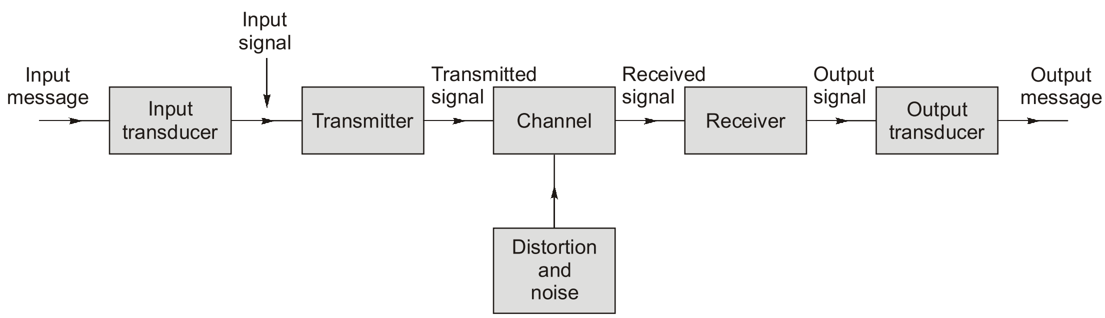


Figure : *Model of communication system*

Source : The source originates a message, such as a human voice, a television picture, an e-mail message, or data. If the data is non-electric (e.g., human voice, e-mail text, television video), it must be converted by an input transducer into an electric waveform referred to as the baseband signal or message signal through physical devices such as a microphone, a computer keyboard or a CCD camera.

Transmitter: The transmitter modifies the baseband signal for efficient transmission. The transmitter may consist of one or more subsystems: an A/D converter, an encoder and a modulator. Similarly, the receiver may consists of a demodulator, a decoder and a D/A converter.

Channel: The channel is a medium of choice that can carry the electric signals at the transmitter output over a distance. A typical channel can be a pair of twisted copper wires (telephone and DSL), coaxial cable (television and internet), an optical fibre or a radio link. Channel may be of two types.

1. **Physical channel:** When there is a physical connection between the transmitter and receiver through wires. eg. coaxial cable.
2. **Wireless channel:** When no physical channel is present and transmission is through air. eg. mobile communication.

It is inevitable that the signal will deteriorate during the process of transmission and reception as a result of some distortion in the system, or because of the introduction of noise, which is unwanted energy, usually of random character, present in a transmission system, due to a variety of causes. Since noise will be received together with the signal, it places a limitation on the transmission system as a whole. When noise is severe, it may mask a given signal so much that the signal becomes undetectable and therefore useless. Noise may interfere with signal at any point in a communications system, but it will have its greatest effect when the signal is weakest. This means that noise in the channel or at the input to the receiver is the most noticeable.

Receiver: The receiver reprocesses the signal received from the channel by reversing the signal modifications made at the transmitter and removing the distortions made by the channel. The receiver output is fed to the output transducer, which converts the electric signal to its original form i.e. the message signal.

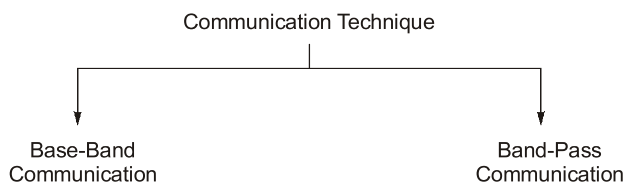
Destination: The destination is the unit to which the message is communicated.

1.5 MODES OF COMMUNICATION

There are two basic modes of communication:

1. **Broadcasting**, which involves the use of a single powerful transmitter and numerous receivers that are relatively inexpensive to build. Here information-bearing signals flow only in one direction.
2. **Point-to-point communication,** in which the communication process takes place over a link between a single transmitter and a receiver. In this case, there is usually a bidirectional flow of information-bearing signals, which requires the use of a transmitter and receiver at each end of the link.
3. **Multi casting:** It is similar to broadcasting but message transmission is intended for specific receiver not for all receivers.

1.5.1 Communication Technique



1. **Base Band Communication:** It is generally used for short distance communication. In this type of communication message is directly sent to the receiver without altering its frequency.
2. **Band Pass Communication:** It is used for long distance communication. In this type of communication, the message signal is mixed with another signal called as the carrier signal for the process of transmission. This process of adding a carrier to a signal is called as modulation.

1.5.2 Need of Modulation

1. To avoid the mixing of signals

   All messages lies within the range of 20 Hz - 20 kHz for speech and music, few MHz for video, so that all signals from the different sources would be inseparable and mixed up. In order to avoid mixing of various signals, it is necessary to translate them all to different portions of the electromagnetic spectrum.

2. To decrease the length of transmitting and receiving antenna

   For a message at 10 kHz, the antenna length 'l' for practical purposes is equal to $\lambda/4$ (from antenna theory) i.e.,

   $$\lambda = \frac{3\times10^8}{10\times10^3} = 3\times10^4 \text{ m} \quad \text{and } l = \frac{\lambda}{4} = \frac{3\times10^4}{4} = 7500 \text{ m}$$

   An antenna of this size is impractical and for a message signal at 1 MHz

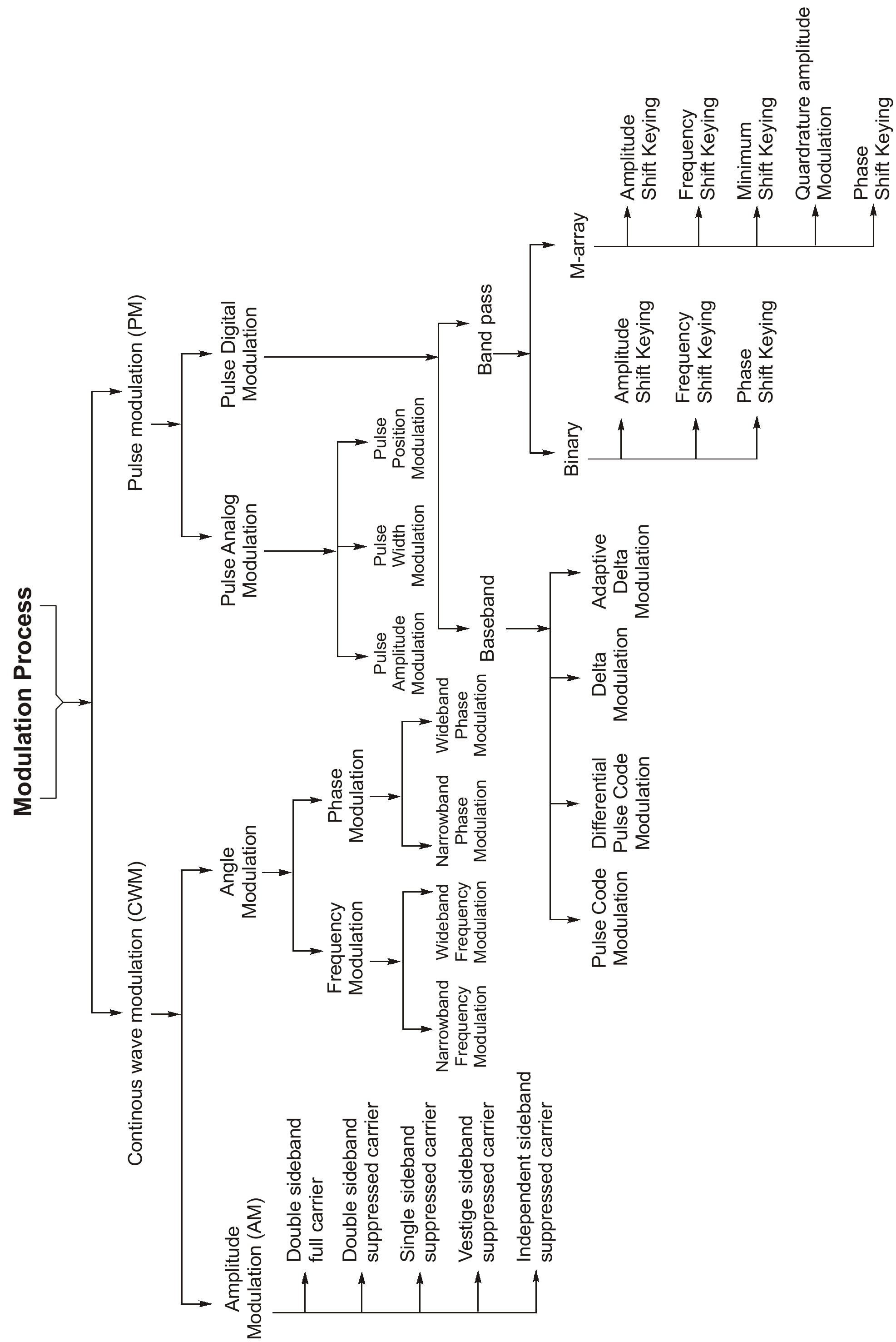
   $$\lambda = \frac{3\times10^8}{10^6} = 300 \text{ m} \quad \text{and} \quad l = \frac{\lambda}{4} = 75 \text{ m (practicable)}$$

3. To allow the multiplexing of signals

   By translating all signals from different sources to different carrier frequency, we can multiplex the signals and able to send all signals through a single channel.

4. To remove the interference
5. To improve the quality of reception i.e. increasing the value of S/N ratio
6. To increase the range of communication

1.6 TYPES OF MODULATION



1.7 AN EXAM ORIENTED APPROACH

Communication is a modern technology is undergoing many changes. The main focus of a student should be to single out on optimum path in which he develops a theoretically strong background of the subject while keeping in mind that he should be able to solve questions asked in various exams using the theory they have studied. Focusing on one aspect leads to failure in written exam or in the interview. Thus this book and communication both have the same approach and that is "optimization" and being a communication engineer one should have this approach too.

Frequency (f) range	Wavelength (λ) range	EM Spectrum Nomenclature	Typical Application
30 – 300 Hz	$10^7 - 10^6$ m	Extremely low frequency (ELF)	Power line communication
0.3 – 3 kHz	$10^6 - 10^5$ m	Voice frequency (VF)	Face to face speech, communication intercom
3 – 30 kHz	$10^5 - 10^4$ m	Very low frequency (VLF)	Submarine communication
30 – 300 kHz	$10^4 - 10^3$ m	Low frequency (LF)	Marine communication
0.3 – 3 MHz	$10^3 - 10^2$ m	Medium frequency (MF)	AM broadcasting
3 – 30 MHz	$10^2 - 10^1$ m	High frequency (HF)	Landline telephony
30 – 300 MHz	$10^1 - 10^0$ m	Very high frequency (VHF)	FM broadcasting, TV
0.3 – 3 GHz	$10^0 - 10^{-1}$ m	Ultra high frequency (UHF)	TV, Cellular telephony
3 – 30 GHz	$10^{-1} - 10^{-2}$ m	Super high frequency (SHF)	Microwave oven, radar
30 – 300 GHz	$10^{-2} - 10^{-3}$ m	Extremely high frequency (EHF)	Satellite communication, radar
0.3 – 3 THz	0.1 – 1 mm	Experimental	For all new explorations
3 – 430 THz	100 – 0.7 μm	Infrared	LED, Laser, TV remote
430 – 750 THz	0.7 – 0.4 μm	Visible light	Optical communication
750 – 3000 THz	0.4 – 0.1 μm	Ultraviolet	Medical application
> 3000 THz	< 0.1 μm	X-rays, gamma rays, cosmic rays	Medical application

Table: *EM Spectrum*

1.8 IMPORTANT FOURIER TRANSFORM

S. No.	$x(t)$	$X(\omega)$	$X(f)$	Comment
1.	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$ $\quad a > 0$	$\frac{1}{a+j(2\pi f)}$	Asymmetric, complex
2.	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$ $\quad a > 0$	$\frac{1}{a-j(2\pi f)}$	Asymmetric, complex
3.	$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$, $\quad a > 0$	$\frac{2a}{a^2+(2\pi f)^2}$	Real and even symmetric
4.	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$, $\quad a > 0$	$\frac{1}{(a+j2\pi f)^2}$	Multiplication of t
5.	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$, $\quad a > 0$	$\frac{n!}{(a+j2\pi f)^{n+1}}$	Multiplication of t^n
6.	$\delta(t)$	1	1	Real and even symmetric

S. No.	$x(t)$	$X(\omega)$	$X(f)$	Comment
7.	A	$2\pi A\delta(\omega)$	$A\delta(f)$	Real and even symmetric
8.	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$\delta(f-f_0)$	Frequency shifting
9.	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	$\frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]$	Used in modulation property
10.	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	$\frac{j}{2}[\delta(f+f_0)+\delta(f-f_0)]$	Used in modulation property
11.	$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$	$\frac{1}{j2\pi f}+\frac{\delta(f)}{2}$	Unit step function
12.	$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$	Imaginary and odd symmetric
13.	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	$\frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]+\frac{j2\pi f}{(2\pi f_0)^2-(2\pi f)^2}$	
14.	$\sin\omega_0 t\, u(t)$	$j\frac{\pi}{2}[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	$\frac{j}{2}[\delta(f+f_0)-\delta(f-f_0)]+\frac{2\pi f_0}{(2\pi f_0)^2-(2\pi f)^2}$	
15.	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2},\ a>0$	$\frac{2\pi f_0}{(a+j2\pi f)^2+2\pi f_0^2},\ a>0$	Decaying sin function
16.	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2},\ a>0$	$\frac{a+j2\pi f}{(a+j2\pi f)^2+2\pi f_0^2},\ a>0$	Decaying cosine function
17.	$\mathrm{rect}\,(t/\tau)$	$\tau S_a\left(\frac{\omega\tau}{2}\right)$	$\tau\mathrm{sinc}(f\tau)$	Rectangular function
18.	$W\mathrm{sinc}(Wt)$	$\mathrm{rect}\left(\frac{\omega}{2\pi W}\right)$	$\mathrm{rect}\left(\frac{f}{W}\right)$	Sinc function
19.	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)$	$f_0\sum_{k=-\infty}^{\infty}\delta(f-kf_0)$	Sampling function
20.	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2(2\pi f)^2/2}$	Gaussian signal

1.9 TRANSMISSION OF SIGNALS THROUGH LINEAR TIME-INVARIANT SYSTEMS

A system refers to any physical entity that produces an output signal in response to an input signal. It is customary to define the input signal as the excitation and the output signal as the response. In a linear system, the principle of superposition holds: that is, the response of a linear system to a number of excitation applied simultaneously is equal to the sum of the responses of the system when each excitation is applied individually.

1.9.1 The Time Domain Response

In the time domain, a linear system is usually described in terms of its impulse response. The impulse response of a linear system is the response of the system (with zero initial conditions) to a unit impulse or delta function $\delta(t)$ applied to the input of the system at time $t = 0$.

Suppose that a system described by the impulse response $h(t)$ is subjected to an arbitrary excitation $x(t)$. The resulting response of the system $y(t)$, is defined in terms of the impulse response $h(t)$ by

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Which is called the convolution integral. Equivalently, we may write

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

Input = $x(t) \longrightarrow$ $\boxed{\text{LTI system } h(t)}$ $\longrightarrow$ Output = $y(t)$

$$y(t) = x(t) * h(t)$$

1.9.2 Examining the Convolution Integral

We see that three different time scales are involved: excitation time τ, response time t, and system memory time $t-\tau$. This relation is the basis of time-domain analysis of linear time-invariant systems. The present value of the response of a linear time-invariant system is an integral over the past history of the input signal, weighted according to the impulse response of the system. Thus, the impulse response acts as a memory function of the system.

1.9.3 Frequency Response

Let $X(f)$, $H(f)$, and $Y(f)$ denote the Fourier transforms of the excitation $x(t)$, the impulse response of the system $h(t)$ and the output $y(t)$.

Equivalently, we may write

input, $\quad x(t) \xrightleftharpoons{FT} X(f)$

and output, $\quad y(t) \xrightleftharpoons{FT} Y(f)$

and with a transfer function,

$$h(t) \xrightleftharpoons{FT} H(f)$$

then its input/output relationship is given by:

Input = $x(t) \longrightarrow$ $\boxed{\text{LTI system } h(t)}$ $\longrightarrow$ Output = $y(t)$

$$y(t) = x(t) * h(t)$$

apply convolution-time theorem of Fourier transform,

we get,

$$Y(\omega) = X(\omega)\,H(\omega)$$

$\therefore \quad H(\omega) = \dfrac{Y(\omega)}{X(\omega)}$ = frequency response of continuous time LTI system.

or $\quad H(f) = \dfrac{Y(f)}{X(f)}$

The new frequency function $H(f)$ is called the transfer function or frequency response of the system. The frequency response of a linear time-invariant system is defined as the ratio of the Fourier transform of the response of the system to the Fourier transform of the excitation applied to the system.

In general, the frequency response $H(f)$ is a complex quantity, so we may express it in the form.

$$H(f) = |H(f)|\exp[j\beta(f)]$$

Where $|H(f)|$ is called the magnitude response, and $\beta(f)$ is the phase response,.

$$|H(f)| = |H(-f)|$$

and
$$\beta(f) = -\beta(-f)$$

That is, the magnitude response $|H(f)|$ of a linear system with real-valued impulse response is an even function of frequency, whereas the phase $\beta(f)$ is an odd function of frequency.

An alternate way of representing the signal in the log scale is

$$\ln H(f) = \alpha(f) + j\beta(f)$$

where
$$\alpha(f) = \ln|H(f)|$$

The function $\alpha(f)$ is called the gain of the system, it is measured in nepers. The phase $\beta(f)$ is measured in radians. Equation indicates that the gain $\alpha(f)$ and phase $\beta(f)$ are, respectively, the real and imaginary parts of the (natural) logarithm of the transfer function $H(f)$.

$$\text{where the gain} = \alpha(f) = 20\log_{10}|H(f)| \text{ in (db)}$$

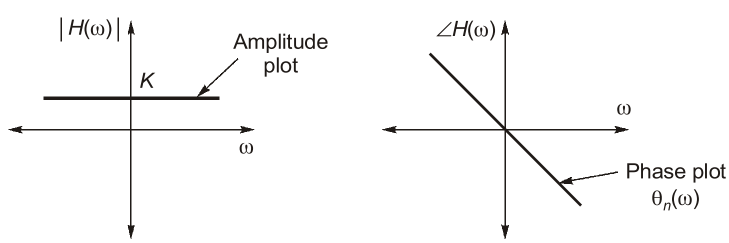
1.9.4 Frequency Response of LTI System

Distortionless Transmission

For a distortionless transmission, the output waveform is the exact replica of the input waveform. A delayed output that retains the input waveform is also considered distortionless.

For a delayed transfer function by "t_d", the system is as below.

$$x(t) \longrightarrow \boxed{Kh(t-t_d)} \longrightarrow y(t)$$



So, if
$$h(t) = \delta(t)$$

then,
$$y(t) = Kx(t-t_d)$$

$\Rightarrow$
$$Y(\omega) = KX(\omega)\,e^{-j\omega t_d} \longrightarrow \text{(using time-shifting property)}$$

But we know,

$$Y(\omega) = H(\omega)\,X(\omega)$$

$\therefore$
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = Ke^{-j\omega t_d}$$

$\Rightarrow$ It is transfer function (T.F.) required for distortionless transmission.

NOTE
$\Rightarrow$ Phase plot of such functions is odd symmetric.
$\Rightarrow$ $|H(\omega)| = K$ and $\theta_n(\omega) = -\omega t_d$

1.10 IDEAL FILTERS

These filters allow distortionless transmission of a certain band of frequencies and suppresses all the remaining frequencies.

Ideal low-pass filter frequency response and its impulse response

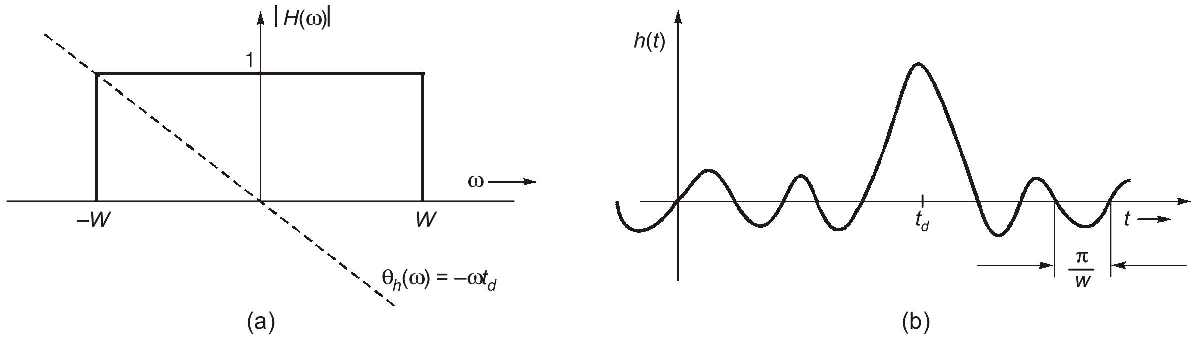


Figure: *(a) Frequency response of ideal LPF and (b) impulse response of ideal LPF*

Ideal high-pass and band-pass filter frequency response

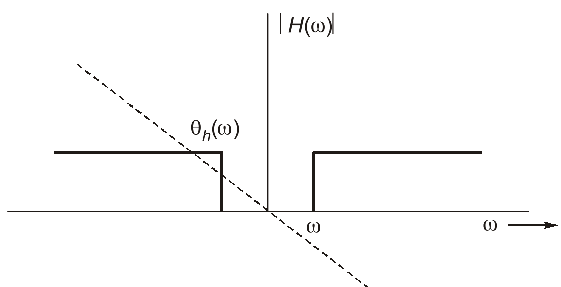


Figure: *High-pass filter frequency response*

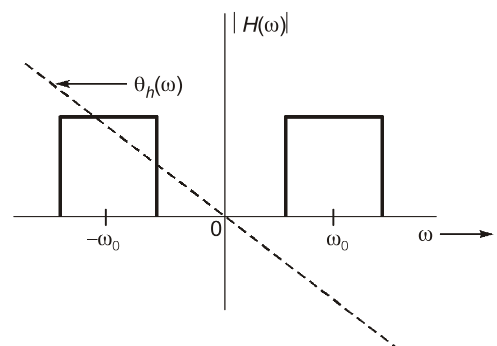
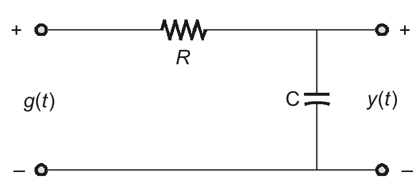


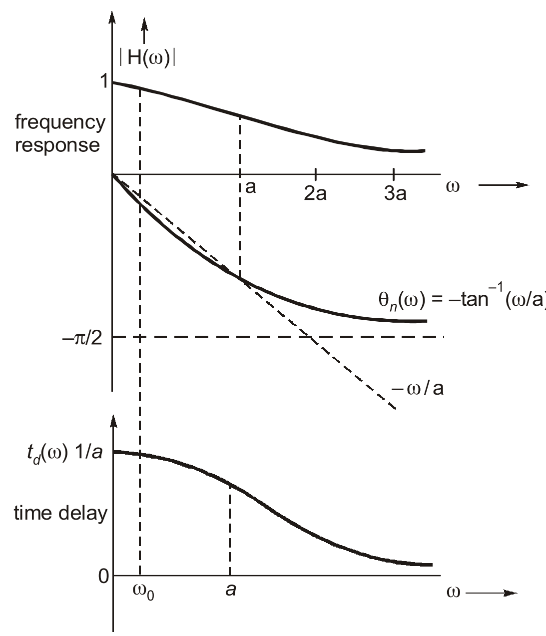
Figure: *Band-pass filter frequency response*

For a physically realizable system $h(t)$ must be causal, that is $h(t) = 0$ for $t < 0$.

1.10.1 Practical RC Filter And its Response



Transfer Function,
$$H(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{a}{a + j\omega} \qquad \left(a = \frac{1}{RC}\right)$$



1.11 HILBERT TRANSFORM

In an LTI system the output $y(t)$ can be written as

$$y(t) = x(t) * h(t)$$

where $x(t)$ is the input signal and $h(t)$ is the impulse response of the LTI system.

Now, in frequency domain the equation can be written as

$$Y(f) = X(f)\,H(f)$$

or $$|Y(f)| = |X(f)|\,|H(f)|$$

and $$\theta_y(f) = \theta_x(f) + \theta_h(f)$$

If we design a filter with $|H(j\omega)| = 1$, then the above expression will reduce to.

$$|y(t)| = |X(t)|$$

and $$\theta_y(f) = \theta_x(f) + \theta_h(f)$$

From our study of Fourier transform, we know that a phase addition is caused by a time delay

∴ $$y(t) = x(t - T_d)$$

Now, in this we consider a special case where,

$$\theta_y(f) = \begin{cases} -\dfrac{\pi}{2} + \theta_x(f) & ; \quad f > 0 \\ \dfrac{\pi}{2} + \theta_x(f) & ; \quad f < 0 \end{cases}$$

and $$|Y(f)| = |X(f)| \;\forall\, f \neq 0$$

Alternately we can write, $$H(f) = -j\,\mathrm{sgn}(f)$$

and $$-j\,\mathrm{sgn}(f) \xrightleftharpoons{\text{FT}} \frac{1}{\pi t}$$

$$h(t) = \frac{1}{\pi t}$$

When $x(t)$ is the input to a Hilbert transformer, we denote its output as $\hat{x}(t)$

where, $$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{(t-\tau)} d\tau \quad \text{and} \quad \hat{X}(f) = -j\text{sgn}(f)\, X(f)$$

$\hat{x}(t)$ is called the Hilbert transform of $x(t)$.

Note: Unlike other transforms, both $x(t)$ and $\hat{x}(t)$ are functions of the same variable (t in our case).

Hilbert Transform (HT) will prove quite useful later on in the study of bandpass signals and single sideband signals. For the present, let us look at some examples of HT.

EXAMPLE : 1.1 Hilbert transform of $\delta(t)$. For $x(t) = \delta(t)$, find Hilbert transform $\hat{x}(t)$.

Solution:

As $\delta(t) \longleftrightarrow 1$, we have

$$F[\hat{\delta}(t)] = -j\text{sgn}(f)$$

$$\Rightarrow \quad \hat{x}(t) = \hat{\delta}(t) = \frac{1}{\pi t}$$

This also establishes the relation,

$$\left[\delta(t) * \frac{1}{\pi t}\right] = \frac{1}{\pi t}$$

EXAMPLE : 1.2 Hilbert transform of a cosine signal. For $x(t) = \cos(2\pi f_0 t)$, find Hilbert transform $\hat{x}(t)$.

Solution:

$$X(f) = \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$$

$$\hat{X}(f) = -j\,\text{sgn}(f) X(f)$$

i.e. $$\hat{X}(f) = \frac{1}{2j}[\delta(f - f_0) - \delta(f + f_0)]$$

That is, $\hat{x}(t) = \sin(2\pi f_0 t)$.

Alternatively,

If $x_1(t) = e^{j2\pi f_0 t}$, then $\hat{x}_1(t) = e^{j(2\pi f_0 t - \pi/2)}$ and if $x_2(t) = e^{-j2\pi f_0 t}$, then $\hat{x}_2(t) = e^{-j(2\pi f_0 t - \pi/2)}$

Hence, $$x(t) = \cos(2\pi f_0 t) = \frac{1}{2}[x_1(t) + x_2(t)] \text{ has } \hat{x}(t)$$

$$= \cos\left(\omega_0 t - \frac{\pi}{2}\right) = \sin(\omega_0 t)$$

Similarly, we can show that if $x(t) = \sin(2\pi f_0 t)$, then $\hat{x}(t) = \cos\omega_0 t$.

OBJECTIVE BRAIN TEASERS

Q.1 A modulated signal is given by,
$s(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$
where the baseband signal $m_1(t)$ and $m_2(t)$ have bandwidths of 10 kHz and 15 kHz, respectively. The bandwidth of the modulated signal, in kHz, is
(a) 10 (b) 15
(c) 25 (d) 30

Q.2 Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt$ is
(a) 1 (b) –1
(c) 0 (d) $\frac{\pi}{2}$

Q.3 If a signal $f(t)$ has energy E, the energy of the signal $f(5t)$ is equal to
(a) E (b) $\frac{E}{5}$
(c) $5E$ (d) $10E$

Q.4 The trigonometric Fourier series of an even function of time does not have
(a) the dc term (b) cosine terms
(c) sine terms (d) odd harmonic terms

Q.5 The trigonometric Fourier series of a periodic time function can have only
(a) cosine terms
(b) sine terms
(c) cosine and sine terms
(d) dc and cosine terms

Q.6 The expression of trigonometrical Fourier series coefficient b_n in terms of exponential Fourier series coefficient C_n is
(a) $j(C_n + C_{-n})$ (b) $j\left(\frac{C_n + C_{-n}}{2}\right)$
(c) $j(C_n - C_{-n})$ (d) $j\left(\frac{C_n - C_{-n}}{2}\right)$

Q.7 Consider a real time domain signal $x(t)$ whose Fourier transform is $X(j\omega)$. Which of the following properties are true:
(i) $\text{Even}\{x(t)\} \longleftrightarrow \text{Re}\{X(j\omega)\}$
(ii) $\text{Odd}\{x(t)\} \longleftrightarrow jIm\{X(j\omega)\}$
(iii) $x^*(t) \longleftrightarrow X^*(j\omega)$
(iv) $\int_{-\infty}^{t} x(\tau)\,d\tau \longleftrightarrow \frac{X(j\omega)}{j\omega}$
(a) (i) and (ii) (b) (i), (ii) and (iii)
(c) (i) and (iii) (d) All the above are true

Q.8 Consider two periodic signal $x_1(t)$ and $x_2(t)$, these signal can be represented in terms of linear combination of complex exponential as:

$$\text{If } x_1(t) = \sum_{k=-100}^{100} \cos(k\pi)e^{jk\left(\frac{2\pi}{50}\right)t}$$

$$\text{and } \quad x_2(t) = \sum_{k=-100}^{100} j\sin(k\pi)e^{jk\left(\frac{2\pi}{50}\right)t}$$

then which of the following option is true
(a) $x_1(t)$ is real and even
(b) $x_2(t)$ is real and even
(c) $x_1(t)$ and $x_2(t)$ are real and even
(d) $x_2(t)$ is imaginary and odd

Q.9 If $f(t)$ is an even function, then what is its Fourier transform $F(j\omega)$?
(a) $\int_0^{\infty} f(t)\cos(2\omega t)dt$ (b) $2\int_0^{\infty} f(t)\cos(\omega t)dt$
(c) $2\int_0^{\infty} f(t)\sin(\omega t)dt$ (d) $\int_0^{\infty} f(t)\sin(2\omega t)dt$

Q.10 If the Fourier transform of $f(t)$ is $f(j\omega)$, then what is the Fourier transform of $f(-t)$?
(a) $f(j\omega)$
(b) $f(-j\omega)$
(c) $-F(j\omega)$
(d) complete conjugate of $f(j\omega)$

Q.11 The trigonometric Fourier series expansion of an odd function shall have
(a) only sine terms
(b) only cosine terms
(c) odd harmonics of both sine and cosine terms
(d) none of the these

ANSWERS KEY

1. (d)	2. (a)	3. (b)	4. (c)	5. (c)
6. (c)	7. (a)	8. (a)	9. (b)	10. (b)
11. (a)				

HINTS & EXPLANATIONS

1. (d)

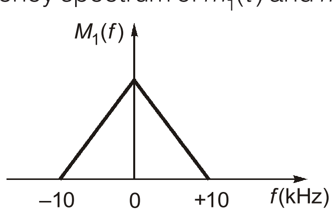
$s(t) = m_1(t) \cdot \cos 2\pi f_c t + m_2(t) \cdot \sin 2\pi f_c t$

Taking Fourier transform,

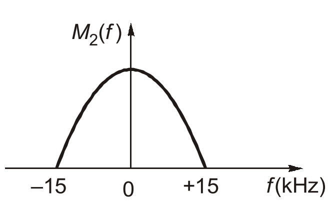
$$S(f) = \frac{1}{2}[M_1(f - f_C) + M_1(f + f_c)]$$
$$+\frac{1}{2j}[M_2(f - f_c) - M_2(f + f_c)]$$

$m_1(t)$ and $m_2(t)$ have bandwidths of 10 kHz and 15 kHz respectively, we can assume the frequency spectrum of $m_1(t)$ and $m_2(t)$ as below:

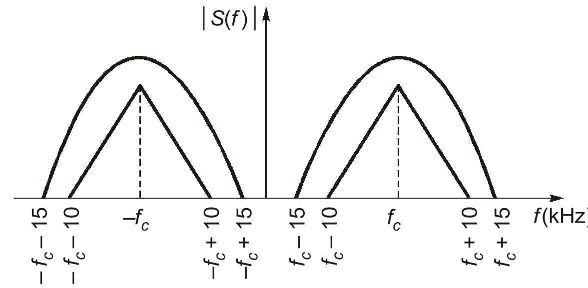
***$M_1(f)$*:**



***$M_2(f)$*:**



Using above, the frequency spectrum of the modulated signal can be drawn as below:



Bandwidth of modulated signal

$$= (f_c + 15) - (f_c - 15) = 30 \text{ kHz}$$

2. (a)

Let, $$I = \int_{-\infty}^{\infty} \delta(t) \cdot \cos\left(\frac{3t}{2}\right) \cdot dt$$

Using the property of impulse function,

$$\int_{-\infty}^{\infty} \delta(t) \cdot x(t) \cdot dt = x(0)$$

Hence, $I = \cos 0° = 1$

3. (b)

Given, the energy of signal $f(t)$ is E. Hence,

$$E = \int_{-\infty}^{\infty} |f(t)|^2 \cdot dt$$

The energy of the signal $f(5t)$ can be calculated as

$$E' = \int_{-\infty}^{\infty} |f(5t)|^2 \cdot dt$$

Let, $5t = u \Rightarrow 5dt = du$

$$E' = \frac{1}{5}\int_{-\infty}^{\infty} \left|f(u)^2\right| du = \frac{E}{5}$$

4. (c)

For the trigonometric Fourier series,

$$b_n = \frac{2}{T_o}\int_{-T/2}^{T_o/2} x(t) \cdot \sin(n\omega_o t) \cdot dt$$

$$b_n = \frac{2}{T_o}\int_{-T_o/2}^{0} x(t) \cdot \sin(n\omega_o t) + \frac{2}{T_o}\int_{0}^{T_o/2} x(t) \cdot \sin(n\omega_o t)\, dt$$

In the first integral, substitute $t = -t$.

For an even signal, $x(t) = x(-t)$.

$$b_n = \frac{2}{T_o}\int_{T_o}^{T_o/2} x(t) \cdot [-\sin(n\omega_o t)]\, dt$$
$$+\frac{2}{T_o}\int_{T_o}^{T_o/2} x(t) \cdot \sin(n\omega_o t) \cdot dt$$

$\Rightarrow \qquad b_n = 0$

Hence, the trigonometric Fourier series of an even function does not have sine terms.

5. (c)

The trigonometric Fourier series representation of a periodic signal is given as

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_o t + b_n \sin n\omega_o t)$$

It can be expressed as

$$x(t) = \sum_{n=0}^{\infty} (a_n \cos n\omega_o t + b_n \sin n\omega_o t)$$

Hence, the trigonometric Fourier series of a periodic time function has cosine and sine terms.

6. (c)

We have,

$$C_n = \frac{1}{T}\int_{-T_o/2}^{T_o/2} x(t)\cdot e^{-jn\omega_o t}\cdot dt$$

$$C_n = \frac{1}{T}\int_{-T_o/2}^{T_o/2} x(t)\left[\cos(n\omega_o t) - j\sin(n\omega_o t)\right]dt$$

$$C_n = \frac{1}{T}\left[\int_{-T_o/2}^{T_o/2} x(t)\cos(n\omega_o t)dt - j\int_{-T_o/2}^{T_o/2} x(t)\sin(n\omega_o t)dt\right]$$

$$\therefore \qquad C_n = \frac{a_n}{2} - j\frac{b_n}{2} \qquad ..(i)$$

Similarly, $C_{-n} = \frac{a_n}{2} + j\frac{b_n}{2}$ ...(ii)

Subtracting equation (i) and (ii),

$$C_n - C_{-n} = -jb_n$$

$$\therefore \qquad b_n = -\frac{1}{j}(C_n - C_{-n})$$

$$b_n = j(C_n - C_{-n})$$

7. (a)

(i) Even $\{x(t)\} = \frac{x(t)+x(-t)}{2}$

$$\text{Even } \{x(t)\} \xleftrightarrow{\text{F.T.}} \frac{X(j\omega)+X(-j\omega)}{2}$$

For real signal

$$X(-j\omega) = X^*(j\omega)$$

Hence,

$$\text{Even}\{x(t)\} \xleftrightarrow{\text{F.T.}} \frac{X(j\omega)+X^*(j\omega)}{2} = \text{Re}\{X(j\omega)\}$$

(ii) Odd $\{x(t)\} = \frac{x(t)-x(-t)}{2}$

$$\text{Odd } \{x(t)\} \xleftrightarrow{\text{F.T.}} \frac{X(j\omega)-X(-j\omega)}{2}$$

$$\text{Odd}\{x(t)\} \xleftrightarrow{\text{F.T.}} \frac{X(j\omega)+X^*(j\omega)}{2}$$

$$= j\,\text{Im}\{X(j\omega)\}$$

(iii) For a real signal, $x(t) = x^*(t)$

Hence, $x^*(t) \xleftrightarrow{\text{F.T.}} X(j\omega)$

(iv) $\int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{\text{F.T.}} \frac{X(j\omega)}{j\omega} + \pi X(0)\,\delta(\omega)$

Hence, only properties (i) and (ii) are correct.

8. (a)

Signal	Trigonometric Fourier series	Complex Fourier series
Real	$a_o, a_n, b_n \neq 0$	$c_{-n} = c_n^*$
Real and even	$a_o, a_n \neq 0$ $b_n = 0$ a_n is real and even	c_n is real and even
Real and odd	$a_o, a_n = 0$ $b_n \neq 0$ b_n is imaginary and odd	c_n is imaginary and odd

For $x_1(t)$, $\quad b_k = 0$ and

$\qquad a_k = \cos(k\pi) \rightarrow$ Real and even

Hence, $x_1(t)$ is real and even.

For $x_2(t)$, $a_0 = 0$, $a_k = 0$ and

$b_k = j\sin(k\pi) \rightarrow$ Imaginary and odd.

Hence, $x_2(t)$ is real and odd.

9. (b)

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\cdot\left[\cos\omega t - j\sin\omega t\right]dt$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\cos\omega t dt - j\int_{-\infty}^{\infty} f(t)\cdot\sin\omega t\cdot dt$$

$$F(j\omega) = \int_{-\infty}^{0} f(t)\cos\omega t\cdot dt + \int_{0}^{\infty} f(t)\cos\omega t\cdot dt$$

$$-j\int_{-\infty}^{0} f(t)\cdot\sin\omega t\cdot dt - j\int_{0}^{\infty} f(t)\sin\omega t\cdot dt$$

Substituting $t = -u$ in first and third integral.
Hence, $dt = -$ du.
Since, $f(-t) = f(t)$, we get,

$$F(j\omega) = \int_{\infty}^{0} f(u)(\cos\omega u)(-du) + \int_{0}^{\infty} f(t)(\cos\omega t)dt$$

$$-j\int_{\infty}^{0} f(u)(-\sin\omega u)(-du) - j\int_{0}^{\infty} f(t)(\sin\omega t)dt$$

$$F(j\omega) = \int_{0}^{\infty} f(u)(\cos\omega u)\cdot du + \int_{0}^{\infty} f(t)(\cos\omega t)dt$$

$$+j\int_{0}^{\infty} f(u)(\sin\omega u)du - j\int_{0}^{\infty} f(t)\cdot(\sin\omega t)\cdot dt$$

$$F(j\omega) = 2\int_{0}^{\infty} f(t)\cos\omega t\cdot dt$$

10. (b)

$$F[f(-t)] = \int_{-\infty}^{\infty} f(-t)\cdot e^{-j\omega t}\cdot dt$$

Let $-t = u \quad \Rightarrow -dt = du$

$$F[f(-t)] = \int_{\infty}^{-\infty} f(u)\cdot e^{j\omega u}(-du)$$

$$F[f(-t)] = \int_{-\infty}^{\infty} f(u)\cdot e^{-j(-\omega)u}du$$

$$F[f(-t)] = f(-j\omega)$$

11. (a)

For an odd function,

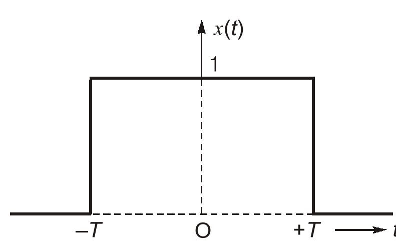
$$a_o = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\cdot dt = 0$$

$$a_o = \frac{2}{T}\int_{-T/2}^{T/2} \underbrace{f(t)\cdot\cos n\omega_o t\cdot dt}_{\text{Odd function}} = 0$$

Since, $a_o = 0$ and $a_n = 0$, therefore the trigonometric Fourier series expansion of an odd signal have only sine terms ($\because\ b_n \neq 0$).

CONVENTIONAL BRAIN TEASERS

Q.1 For the rectangular pulse shown in the figure below, determine the Fourier Transform of $x(t)$ and sketch the magnitude spectrum with respect to frequency.



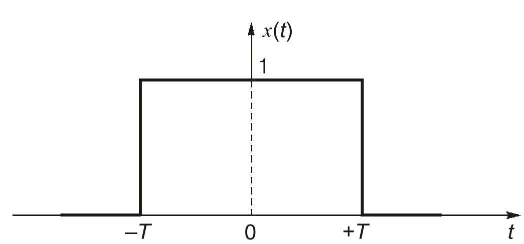
1. (Sol.)

The Fourier transform of signal $x(t)$ is given as

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\cdot dt$$

We have,

$$x(t) = \begin{cases} 1 & ; \quad -T \le t \le T \\ 0 & ; \quad \text{otherwise} \end{cases}$$



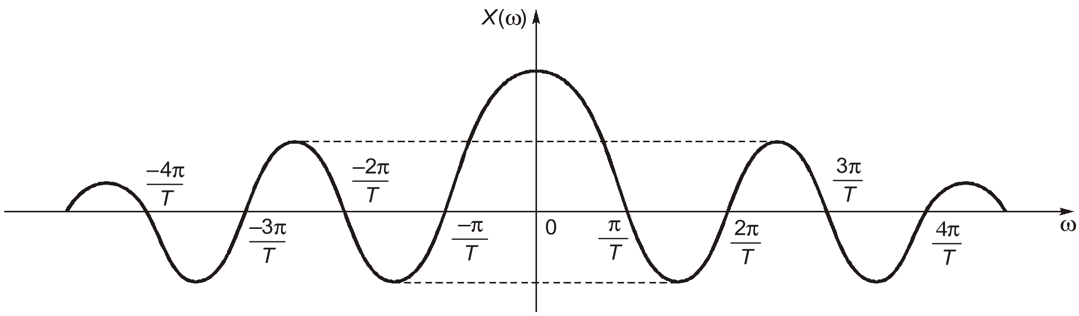
Hence, $$X(\omega) = \int_{-T}^{T} e^{-j\omega t} \cdot dt = -\frac{1}{j\omega} X\left[e^{-j\omega t}\right]_{-T}^{T} = \frac{e^{j\omega T} - e^{-j\omega T}}{j\omega} = \frac{2\sin(\omega T)}{\omega} = 2T\left[\frac{\sin(\omega T)}{\omega T}\right]$$

$$X(\omega) = 2TS_a(\omega T)$$

Magnitude spectrum: The zero crossings occur at

$$\omega T = 0, \pm\pi, \pm 2\pi \ldots$$

$$\omega = 0, \pm \pi/T, \pm 2\pi/T \ldots$$



Q.2 State and prove convolution theorem in Fourier transform.

2. (Sol.)

According to the convolution theorem, the Fourier transform of the convolution of two signals in time domain is equal to the multiplication of their Fourier transform in frequency domain.

Mathematically, $$x_1(t) * x_2(t) \xleftrightarrow{\text{F.T.}} X_1(\omega) \cdot X_2(\omega)$$

Proof: $$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\lambda) \cdot x_2(t-\lambda) \cdot d\lambda$$

The Fourier transform of $y(t) = x_1(t) * x_2(t)$ is given as

$$Y(\omega) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x_1(t) \cdot x_2(t-\lambda)\, d\lambda\right] e^{-j\omega t} \cdot dt$$

$$Y(\omega) = \int_{-\infty}^{\infty} x_1(\lambda) \int_{-\infty}^{\infty} x_2(t-\lambda)\, e^{-j\omega t} \cdot dt \cdot d\lambda$$

Let $t - \lambda = u \Rightarrow dt = du$ $$Y(\omega) = \int_{-\infty}^{\infty} x_1(\lambda) \cdot \int_{-\infty}^{\infty} x_2(u) \cdot e^{-j\omega(u+\lambda)}\, dt \cdot d\lambda$$

$$Y(\omega) = \left[\int_{-\infty}^{\infty} x_1(\lambda) \cdot e^{-j\omega\lambda} \cdot d\lambda\right] \times \left[\int_{-\infty}^{\infty} (x) \cdot e^{-j\omega u} \cdot du\right]$$

$$Y(\omega) = X_1(\omega) \cdot X_2(\omega)$$

Q.3 Using time shifting and time differentiation properties, find the Fourier transform of the trapezoidal signal shown.