Thoroughly Revised and Updated

Reasoning & Aptitude *for* GATE 2024 and ESE Pre 2024

Comprehensive Theory *with* **Examples**
and **Solved Questions of**
GATE *and* **ESE Prelims**

Also useful for
UPSC (CSAT), MBA Entrance, Wipro, SSC, Bank (PO), TCS , Railways, Infosys,
various Public Sector Units and other Competitive Exams conducted by UPSC

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Reasoning & Aptitude for GATE 2024 & ESE Prelims 2024

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Director's Message

Engineering is one of the most chosen graduation fields, choosing to become an engineer after high school is usually a matter of interest but this eventually develops into "the purpose of being an engineer" and then a student thinks of cracking various competitive exams like ESE, GATE, PSUs exams, and other state engineering services exams. With the objective nature of these competitive exams and with increasing competition, it becomes necessary for the student to study and practice every topic and also get acclimatize with the style of questions asked in the exam.

Studying engineering in university is one aspect but studying to crack different prestigious competitive exams requires altogether different strategies, crystal clear concepts and rigorous practice of previous years' questions. Every student can achieve great results through proper guidance and exam-oriented study material, and hence we have come up with this book covering all the previous years' questions. This book will help aspirants to develop an understanding of important and frequently asked areas in the exam and will also help in strengthening concepts. MADE EASY Team has put sincere efforts in framing accurate and detailed explanations for all the previous years' questions. The explanation provided for each question is not only question specific but it will also give insight on the concept as a whole which will beneficial for the student from the exam point of view to handle similar questions.

All the previous years' questions are segregated subject wise and further, they have been categorized topic-wise for easy learning and this certainly assists aspirants to solve all previous years' questions of a particular area in one place. I would like to acknowledge the efforts of the entire MADE EASY team who worked hard to solve previous years ' questions with accuracy. I hope this book will stand up to the expectations of aspirants and my desire to serve the student community by providing the best study material will get accomplished.

B. Singh (Ex. IES)
CMD, MADE EASY Group

Contents

A

Section

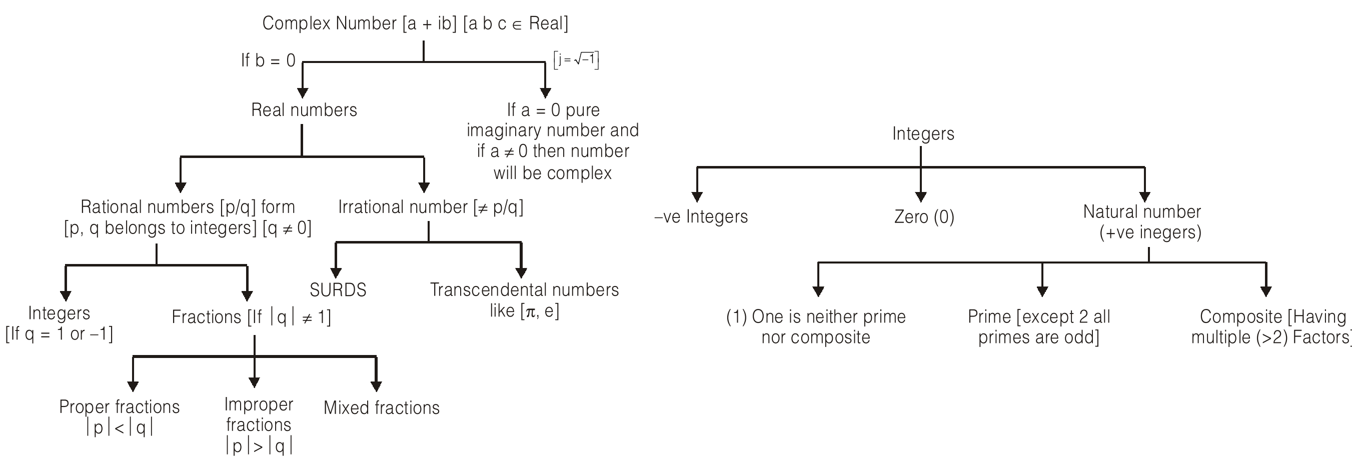
Arithmetic

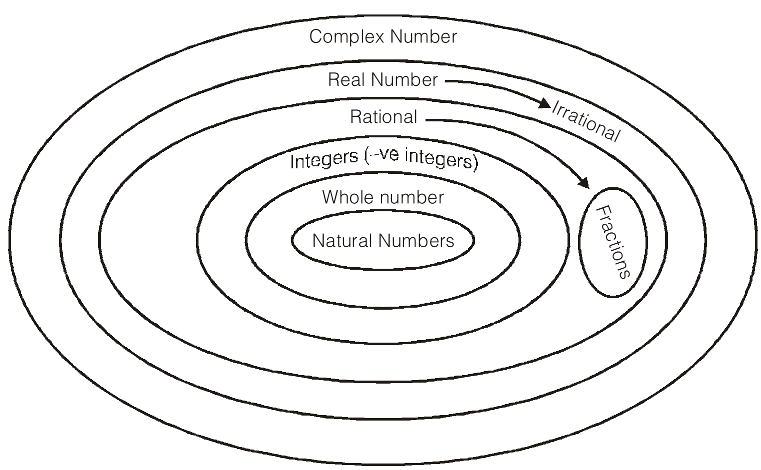
1.1 CHAPTER

Number System

In Quantitative Aptitude (QA), Number System is one of the modules which is of critical importance. We can consider this module as the back bone as well as basic foundation and building block for QA as well as for reasoning. Applications of concepts of numbers can be easily found in puzzles, reasoning based questions, number series and many more reasoning areas. This is why it is our suggestion to students to understand the concepts discussed in the module thoroughly alongwith understanding of applications.

Classifications of Numbers





Our main focus in this module of numbers in on **real number system**. However in context of imaginary numbers only following property is important.

Imaginary Numbers

$i = \sqrt{-1} \quad \Rightarrow \quad i^{4K+1} \equiv \sqrt{-1} \equiv i$

$i^2 = -1 \quad \Rightarrow \quad i^{4K+2} \equiv -1 \equiv i^2$

$i^3 = -i \quad \Rightarrow \quad i^{4K+3} \equiv -i \equiv i^3$

$i^4 = 1 \quad \Rightarrow \quad i^{4K} \equiv 1 \equiv i^4$

Ex.1

What is the value of expression

$$\frac{i^{12}+i^{13}+i^{14}+i^{15}}{i^{18}+i^{19}+i^{20}+i^{21}}?$$

(a) i^2 (b) -1

(c) $1/i^2$ (d) None of these

Ans. (d)

$$\frac{i^{12}\left(1+i+i^2+i^3\right)}{i^{18}\left(1+i+i^2+i^3\right)}$$

If we commit a mistake of cancelling out common terms in numerator and denominator options a, b, c all one correct hence my answer should be (d) but

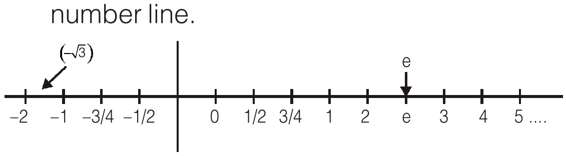
Expression $1 + i + i^2 + i^3$

$= 1 + i + (-1) + (-i) = 0$

Hence expression in question leading to undetermined form $\left[\frac{0}{0}\right]$ hence correct answer is option (d).

Real Number System

Entire real numbers group of rational and irrational numbers combined forms the set of real number, which is represented by symbol → R. All real numbers can be represented as points on a real number line.



Rational Number

All the numbers in p/q ($q \neq 0$) form are rational numbers [p, q are integers]. Set of rational number is represented by → Q.

Rational Numbers have following forms of representations.

(a) Terminating decimal forms

for example 0.125

$$\Rightarrow \quad 0.125 = \frac{125}{1000} \Rightarrow \text{Rational}$$

(b) Nonterminating but recurring decimal forms.

(i) For example

Q = 0.37373737 ...

100 Q = 37.373737...

99Q = 37 ⇒ Q = 37/99 ⇒ rational

(ii) For example

Q = 0.37292929 ...

100Q = 37.292929 ...

10000Q = 3729.292929 ...

9900Q = (3729 – 37)

$$Q = \left(\frac{3729-37}{9900}\right)$$

$$= \frac{p}{q} \text{ form} \Rightarrow \text{rational}$$

Fraction

All rational numbers in which $|q| \neq 1$ comprise the set of fractions.

Proper Fraction

If $|p| < |q|$

then fraction is proper fraction. Value of proper fraction is always in between (–1 to +1) i.e., [$-1 < p/q < 1$]

Improper Fraction

If $|p| > |q|$

than fraction is improper fraction. Value of improper fraction is < –1 or > 1.

Mixed Fraction

Just a modified form of improper fraction.

$$\text{Eg.} \quad \underbrace{\frac{13}{4}}_{\text{Improper fraction}} \Rightarrow \underbrace{3\frac{1}{4}}_{\text{equivalent mixed fraction}}$$

Integers

The set of all rational numbers in p/q form [$|q| = 1$] is called as integers. It is denoted by

I = { .... –3, –2, –1, 0, 1, 2, 3, ...}

It includes.

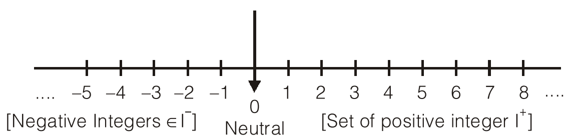
Negative Integers

$I^- = \{... -7, -6, -5, -4, -3, -2, -1\}$

Positive Integers

$I^+ = \{1, 2, 3 .... \}$

Note: Status of 0 (zero) is neutral neither positive nor negative.



Natural Numbers

All counting numbers or set of positive integers is considered as set of natural numbers. It denoted by set [N or I^+]

N = {1, 2, 3, 4, ...}

Whole Number

Set of all nonnegative integers are considered as whole number; it is denoted by set W = {0, 1, 2, 3, 4 ......}

Note: If terms "numbers" is used without any qualifier than it means natural number henceforth.

Even Numbers & Odd Numbers

1. **Even Numbers**

   All numbers divisible by 2 are considered as even numbers.

   Note: Property evenness is applicable in entire integral number line. Hence [–2, –4, –6, ...] are even integers but they are not even numbers.

2. **Odd Number**

   All numbers not divisible by 2 are odd.

   [1, 3, 5, 7, ...] are odd numbers.

   [.......... –5, –3, –1 ....] are odd integers.

Properties of numbers based on even & odd

Even + Even = Even
Odd + Odd = Even
Odd + Odd + Odd = Odd
Odd × Odd = Odd
Odd × Even = Even
Even × Even = Even
$(\text{Even})^{\text{Odd}} \Rightarrow$ Even
$(\text{Odd})^{\text{Even}} \Rightarrow$ Odd
$(\text{Even})^{\text{Odd}} \Rightarrow$ Even
$(\text{Odd})^{\text{Odd}} \Rightarrow$ Odd

These properties can be used extensively to find out alternative method to get answers quickly with the help of options. Here are few examples.

Ex. 1

There are two, 2-digit numbers ab and cd, ba is the another two digit number prepared by reversing the digits of ab, if ab × cd = 493, ba × cd = 2059, what is value 'g' sum of (ab + cd) = ?

(a) 43 (b) 45
(c) 47 (d) 46

Ans: (d)

Value 'g' = ab × cd is odd.
It means ab and cd both are odd.
Hence there sum must be even, only one option is there which is even. Hence answer is option d.

Ex. 2

I have multiple gift vouchers of value, Rs. 101, 107, 111, 121, 131, 141, 151, 171. I have to pick exactly 10 vouchers to make payment of Rs. 1121. In how many ways I can do that?

(a) one (b) two
(c) more than two (d) none of these

Ans. (d)

Reasoning is very simple, if I'll add 10 odd numbers their sum will be always even. Hence there is no way to accomplish this.

Prime Number & Composite Numbers

Prime Numbers

Number which are perfectly divisible either by 1 or by itself only are called prime numbers. 25 prime number are there which are less than 100. 2 is the only even prime number. All prime numbers greater than 5 can be expressed as $(6K \pm 1)$ $(K \in N)$ form but all the numbers in form of $(6K \pm 1)$ form are not necessarily prime.

Composite Numbers

All the numbers which can be factorized into multiple prime numbers are called composite number.

Number (1) one is neither prime nor composite.

How to check whether given number is prime or not?

1. Take the square root of number
2. Consider the prime numbers, starting from 2 till the number. Take all prime numbers upto this square root value or nearest higher integer.

3. If number is divisible by any of these prime numbers, then number is composite.

Learn it by example:

Suppose we want to check, is 629 prime or not?
Square root of 627 is just more than 25. Then prime no. till 25 are 2, 3, 7, 5, 11, 13, 17, 19, 23, 29.
629 is not divisible by 2, 3, 5, 7, 11, 13 but is divisible by 17.
Hence it is not prime number

One more example: 179

Square root of 179 is more than 13. Hence we need to check divisibility of 179 against 2, 3, 5, 7, 11, 13, 17
179 is not divisible by either of these hence it is a prime number.

Test of Divisibility

1. Divisibility by 2
A number is divisible by 2 if the unit digit is zero or divisible by 2.
Eg.: 22, 42, 84, 3872 etc.

2. Divisibility by 3
A number is divisible by 3 if the sum of digit in the number is divisible by 3.
Eg.: 2553
Here 2 + 5 + 5 + 3 = 15, which is divisible by 3 hence 2553 is divisible by 3.

3. Divisibility by 4
A number is divisible by 4 if its last two digit are divisible by 4.
Eg.: 2652, here 52 is divisible by 4 so 2652 is divisible by 4.
Eg.: 3772, 584, 904 etc.

4. Divisibility by 5
A number is divisible by 5 if the units digit in number is 0 or 5.
Eg.: 50, 505, 405 etc.

5. Divisibility by 6
A number is divisible by 6 if the number is even and sum of digits is divisible by 3.
Eg.: 4536 is an even number also sum of digit 4 + 5 + 3 + 6 = 18 is divisible by 3.
Eg: 72, 8448, 3972 etc.

6. Divisibility by 8
A number is divisible by 8 if last three digit of it is divisible by 8.
Eg.: 47472 here 472 is divisible by 8 hence this number 47472 is divisible by 8.

7. Divisibility by 9
A number is divisible by 9 if the sum of its digit is divisible by 9.
Eg.: 108936 here 1+0+8+9+3+6 is 27 which is divisible by 9 and hence 108936 is divisible by 9.

8. Divisibility by 10
A number is divisible by 10 if its unit digit is 0.
Eg.: 90, 900, 740, 34920 etc.

9. Divisibility by 11
A number is divisible by 11 if the difference of sum of digit at odd places and sum of digit at even places is either 0 or divisible by 11.
Eg.: 1331, the sum of digits at odd place is 1+3 and sum of digit at even places is 3+1 and their difference is 4 – 4 = 0. so 1331 is divisible by 11.

HCF and LCM of Numbers

H.C.F.

(Highest Common Factor) of two or more number is the greatest number that divides each one of them exactly. For example 8 is the highest common factor of 16 and 40.

HCF is also called greatest common divisior (G.C.D.)

L.C.M.

(Least Common Multiple) of two or more number is the least or a lowest number which is exactly divisible by each of them.

For example LCM of 8 and 12 is 24, because it is the first number which is multiple of both 8 and 12.

LCM and HCF of Fractions

Fractions are written in form of $\frac{\text{Numerator}}{\text{Denominator}}$. Where denominator is not equal to zero.

$$\text{H.C.F of Fraction} = \frac{\text{(H.C.F. of Numerators)}}{\text{(LCM of Denominators)}}$$

$$\text{L.C.M of Fraction} = \frac{\text{(LCM of Numerators)}}{\text{(HCF of Denominators)}}$$

All Fractions have to be in their simplest form:

Example: Find HCF & LCM of $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{7}$

$$\text{H.C.F.} = \frac{\text{H.C.F. of } (1,2,3)}{\text{L.C.M } (2, 3, 7)} = \frac{1}{42}$$

$$\text{L.C.M} = \frac{\text{L.C.M. of } (1,2,3)}{\text{H.C.F. of } (2, 3, 7)} = \frac{6}{1} = 6$$

Important Algebraic Formulae

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a-b)(a+b) = a^2 - b^2$
4. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
5. $(a+b)^2 - (a-b)^2 = 4ab$
6. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
   $= a^3 + b^3 + 3ab(a+b)$
7. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
   $= a^3 - b^3 - 3ab(a-b)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
10. $\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca} = (a+b+c)$
11. $a^4 - b^4 = (a^2)^2 - (b^2)^2 = (a^2 + b^2)(a^2 - b^2)$
    $= (a^2 + b^2)(a+b)(a-b)$

[Condition of Divisibility for Algebric Function

1. $a^n + b^n$ is exactly divisible by a+b only when n is odd
   Ex.: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ is divisible by a+b, also $a^5 + b^5$ is divisible by a+b
2. $a^n + b^n$ is never divisible by a-b (whether n is odd or even)
   Ex.: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ is not divisible by (a-b)
   $a^7 + b^7$ is also not divisible by (a – b)
3. $a^n - b^n$ is always divisible by (a – b) (whether n is odd or even)
   Ex.: $a^9 - b^9$ is exactly divisible by (a – b) also $a^{12} - b^{12}$ is also exactly divisible by (a – b).
4. $a^n - b^n$ is divisible by a + b only when 'n' is even natural number.
   Ex. : $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a - b)(a + b)(a^2 + b^2)$. Hence $a^4 - b^4$ is always divisible by (a + b) but $a^3 - b^3$ will not be.]

Factors of Composite Number

Composite numbers are the numbers which can be factorised into prime factors, or simply we can say that composite number are those numbers which are not prime.

For eg.: 8 is a composite number since it can be factorised into

$8 = 2 \times 2 \times 2$

Similarly 9 is also a composite number, i e

$9 = 3 \times 3$

Composite number $= P_1^{\lambda_1} \times P_2^{\lambda_2} \times P_3^{\lambda_3} \ldots\ldots P_n^{\lambda_n}$ here, P_1, P_2, P_3 ..... P_n are distinct prime numbers and λ_1, λ_2, ... λ_n are their respective powers.

Factors of composite number =

$(\lambda_1 + 1).(\lambda_2 + 1)\ldots(\lambda_n + 1)$

For eg.: $18 = 2 \times 3 \times 3 = 2^1 \times 3^2$

Factors of 18 = $(1 + 1) \times (2 + 1) = 2 \times 3 = 6$

Clearly it contains six factors 1, 2, 3, 6, 9 and 18

Factors of other Composite numbers $6 = 2^1 \times 3^1$

Factors = $(1 + 1) \times (1 + 1) = 4$ = 1, 2, 3 and 6

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

Factors = $(3 + 1) \times (2 + 1) = 12$

Ex.1 Find the factors of composite number 360

Sol.: $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$= 2^3 \times 3^2 \times 5^1$

Factors = (3 + 1) (2 + 1) (1 + 1) = 24.

□□□□

Counting Number of Trailing Zeros

Sometimes we come across problems in which we have to count number of zeros at the end of factorial of any number. For example

Number of zero at the end of 10!

$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Here basically we have to count number of fives, because multiplication of five by any even number will result in 0 at the end of final product. In 10! we have 2 fives thus total number of zeros are 2.

Short Cut:

Counting number of zeros at the end of n!

Value will be $\frac{n}{5}+\frac{n}{5^2}+\frac{n}{5^3}+\frac{n}{5^4}\ldots$

The integral value of this sum will be the total number of zeros.

Ex. 1 Number of zeros at the end of 100!

Sol.: $\frac{100}{5}+\frac{100}{5^2}+\frac{100}{5^3}+$

integral value will be

20 + 4 = 24 zeros

Ex.2 Number of zeros at the end of 126!

Sol.: $\frac{126}{5}+\frac{126}{5^2}+\frac{126}{5^3}+\frac{126}{5^4}$

integral value will be

25 + 5 + 1 = 31 zeros.

Cyclicity

Cyclicity of a number is used mainly for the calculation of unit digits.

1. **Cyclicity of 1.**
   In 1^n, unit digit will always be 1.

2. **Cyclicity of 2.**
   $2^1 = 2$
   $2^2 = 4$
   $2^3 = 8$
   $2^4 = 16$
   $2^5 = 32$
   $2^6 = 64$
   $2^7 = 128$
   $2^8 = 256$

After every four intervals it repeats so cycle of 2 is 2, 4, 8, 6.

Ex.1 Unit digit of 2^{323}

Sol.: Here 2, 4, 8, 6 will repeat after every four interval till 320 next digit will be 2, 4, $\boxed{8}$. So unit digit of 2^{323} will be 8.

Ex.2 Find unit digit of $12^{12} \times 22^{22}$

Sol.: Unit digit of 12^{12} will be 6 and 22^{22} will be 4. So unit digit of $12^{12} \times 22^{22}$ will be

$6 \times 4 = 2\boxed{4}$; 4 Ans.

3. **Cyclicity of 3.**
   $3^1 = 3$
   $3^2 = 9$
   $3^3 = 27$
   $3^4 = 81$
   $3^5 = 243$
   $3^6 = 729$
   $3^7 = 2187$
   $3^8 = 6561$

After every four intervals 3,9, 7 and 1 are repeated. So cycle of 3 is 3, 9, 7, 1.

Ex.1 Find unit digit of 133^{133}.

Sol.: Cycle of 3 is 3, 9, 7, 1 which repeats after every four intervals till 133^{132}. So next unit digit will be 3.

4. **Cyclicity of 4.**
   $4^1 = 4$
   $4^2 = 16$
   $4^3 = 64$
   $4^4 = 256$
   Cycle is 4, 6, i.e.
   Unit digit of 4^n depends on value of n.
   If n is odd unit digit is 4 and if n is even digit is 6.

Ex.1 Find unit digit of 4^{1024}.

Sol.: Since 1024 is even number unit digit will be 6.

Ex.2 Find unit digit of $133^{63} \times 4^{49}$.

Sol.: Unit digit of 133^{63} is 7 and unit digit of 4^{49} is 4 so unit digit of $133^{63} \times 4^{49}$ will be $7 \times 4 = 28$ i.e. 8.

5. **Cyclicity of 5.**
   $5^1 = 5$
   $5^2 = 25$
   $5^3 = 125$
   $5^4 = 625$
   Unit digit will always be 5.

6. **Cyclicity of 6.**
   $6^1 = 6$
   $6^2 = 36$
   $6^3 = 216$
   $6^4 = 1296$
   Unit digit will always be 6.

Ex.1 Find unit digit of $4^{69} \times 6^5$

Sol.: Unit digit of 4^{69} is 4 and unit digit of 6^5 is 6 so unit digit of $4^{69} \times 6^5$ will be $4 \times 6 = 24$ i.e. 4.

7. Cyclicity of 7.

$7^1 = 7$

$7^2 = 49$

$7^3 = 343$

$7^4 = 2401$

$7^5 = 16807$

$7^6 = 117649$

$7^7 = 823543$

$7^8 = 5764801$

Cycle of 7 is 7, 9, 3, 1

Ex.1 Find unit digit of $17^{17} \times 27^{27}$

Sol.: Unit digit of 17^{17} is 7 and unit digit of 27^{27} is 3. So unit digit of $17^{17} \times 27^{27}$ will be $7 \times 3 = 21$ i.e. 1.

8. Cyclicity of 8.

$8^1 = 8$

$8^2 = 64$

$8^3 = 512$

$8^4 = 4096$

$8^5 = 32768$

So cycle of 8 is 8, 4, 2, 6.

Ex. 1 Find unit digit of $18^{18} \times 28^{28} \times 288^{288}$.

Sol.: Unit digit of 18^{18} is 4, unit digit of 28^{28} is 6, unit digit of 288^{288} is 6. So unit digit of $18^{18} \times 28^{28} \times 288^{288}$ will be $4 \times 6 \times 6 = 144$ i.e. 4.

9. Cyclicity of 9.

$9^1 = 9$

$9^2 = 81$

$9^3 = 729$

$9^4 = 6561$

Cycle of 9 is 9, 1.

In 9^n unit digit will be 9 if n is odd and unit digit will be 1 if n is even.

Ex. 1 Find unit digit of

$11^{11} + 12^{12} + 13^{13} + 14^{14} + 15^{15}$

Sol.: Unit digit of 11^{11} is 1

Unit digit of 12^{12} is 6

Unit digit of 13^{13} is 3

Unit digit of 14^{14} is 6

Unit digit of 15^{15} is 5

So unit digit of given sum will be

$1 + 6 + 3 + 6 + 5 = 21$ i.e. 1.

Remember

Cyclicity table

1 : 1

2 : 2, 4, 6, 8

3 : 3, 9, 7, 1

4 : 4, 6

5 : 5

6 : 6

7 : 7, 9, 3, 1

8 : 8, 4, 2, 6

9 : 9, 1

0 : 0

□□□□

Remainder Theorem

Remainder of expression $\frac{a \times b \times c}{n}$ [i.e. $a \times b \times c$ when divided by n] is equal to the remainder of expression $\frac{a_n \times b_n \times c_n}{n}$ [i.e. $a_n \times b_n \times c_n$ when divided by n], where

a_n is remainder when a is divided by n,

b_n is remainder when b is divided by n, and

c_n is remainder when c is divided by n.

Ex.1 Find the remainder of $15 \times 17 \times 19$ when divided by 7.

Sol.: Remainder of expression $\frac{15 \times 17 \times 19}{7}$ will be equal to $\frac{1 \times 3 \times 5}{7} = \frac{15}{7} = \frac{1}{7}$.

i.e. 1.

On dividing 15 by 7 we get 1 as remainder

On dividing 17 by 7 we get 3 as remainder

On dividing 19 by 7 we get 5 as remainder and combined remainder will be equal to remainder of $\frac{15}{7}$ i.e. 1.

□□□□

Polynomial Theorem

This is very powerful theorem to find the remainder.

According to polynomial theorem.

$$(x + a)^n = x^n + {}^nC_1\, x^{n-1} + {}^nC_2\, x^{n-2}\, a^2 + {}^nC_3\, x^{n-3}\, a^3 \ldots {}^nC_{n-1}\, x^1 a^{n-1} + a^n \quad \ldots(1)$$

$$\therefore \frac{(x+a)^n}{x} = \frac{\left(\begin{array}{l} {}^nC_0\, x^n + {}^nC_1\, x^{n-1}\, a^1 + {}^nC_2\, x^{n-2}\, a^2 + \\ {}^nC_3\, x^{n-3} + \ldots {}^nC_{n-1}\, x^1\, a^{n-1} + {}^nC_n a^n \end{array} \right)}{x} \quad \ldots(2)$$

remainder of expression (2) will be equal to remainder of $\frac{a^n}{x}$ because rest of the term contains x are completely divisible by x.

Ex.1 Find the remainder of $\frac{9^{99}}{8}$.

Sol.: $\frac{9^{99}}{8} = \frac{(8+1)^{99}}{8}$

According to polynomial theorem remainder will be equal to remainder of the expression $\frac{1^{99}}{8}$ which is equal to 1.

Ex.2 Find remainder of $\frac{5^{100}}{7}$

Sol.: $\frac{5^{100}}{7} = \left[\frac{3\times7+4}{7}\right]^{50} \Rightarrow \frac{(4)^{50}}{7}$

$\Rightarrow \frac{2^{100}}{7} \Rightarrow \frac{(2^3)^{33}\times 2}{7} \Rightarrow \frac{(7+1)^{33}}{7}\times 2 \Rightarrow \frac{1\times 2}{7}$

$\Rightarrow$ Remainder is 2.

More on Remainders

Case-I

On dividing a number by a, b & c if we get a-k, b-k and c-k as remainder respectively then that number will be **n × LCM of [a, b, c]-k.**

For ex (I): On dividing a number by 4, 5 & 6 we get 3, 4, & 5 as remainder. Find the number.

Sol.:

	4,	5,	6
Remainder	3,	4,	5,
which is equal to	(4 – 1),	(5 – 1),	(6 – 1),

so that number will be:

n × LCM of (4, 5, 6) – 1, = 60n – 1

If n = 1, 60 – 1 = 59 is smallest such natural number.

Note: n such numbers are possible. Here we have taken n as 1. Other numbers are 119, 179, 239, etc. Where value of n is 2, 3, & 4 respectively.

Ex.1 On dividing a number by 5, 6 and 7 we get 3, 4 and 5 as remainder. Find the number.

Sol.:

	5,	6,	7
Remainder	3,	4,	5
which is equal to	(5 –2),	(6–2),	(7–2)

that number will be:

n × LCM of (5, 6, 7) – 2 = 210 –2 = 208.

Note: Here we have taken value of n as 1.

Ex.2 On dividing a number by 4, 5 and 6 we get 2, 3 and 4 as remainder find highest possible three digit such number.

Sol.:

	4,	5,	6
Remainder	2,	3,	4

which is equal to (4 –2), (5 – 2),(6 – 2), that number will be:

n × LCM of [4, 5, 6]–2 = n × 60 – 2

When n = 1 we get 58. Highest possible three digit such number will be 958.

Ex.3 On dividing a number by 5, 6 and 7 we get 3, 4 and 5 as remainder. Find highest possible three digit such number.

Sol.:

	5,	6,	7
Remainder	3,	4,	5

which is equal to (5 –2), (6 – 2),(7 – 2) that number will be:

n ∗ LCM (5, 6, 7)–2= n × 210 – 2

Highest possible three digit number will be 838.

Case-II

On dividing a number a, b and c if we get k as remainder always, then that number will be (n – 1) LCM of (a, b, c)+ k.

Ex.1 On dividing a number by 5, 6 and 7 if we get 2 as remainder always, find that number

Sol.: That number will be

(n – 1) × LCM of [5, 6, 7]+ 2

$\Rightarrow$ 2 is such smallest number

next number will be = 210 + 2 = 212

Case-III

If a number after adding k is exactly divisible by a , b and c then that number will be.

$n \times \text{LCM}(a, b, c) - k$

Ex.1 Find a number which after adding 7 is divisible by 10, 11 and 12.

Sol.: That number will be

$n \times \text{LCM of } [10, 11, 12] - 7$

if n = 1 then

660 – 7 = 653 Ans.

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Squares of Numbers

Squares of numbers are frequently used for calculations on various types of problems. It is advisable to remember square of at least first thirty numbers.

$1^2 = 1$	$11^2 = 121$
$2^2 = 4$	$12^2 = 144$
$3^2 = 9$	$13^2 = 169$
$4^2 = 16$	$14^2 = 196$
$5^2 = 25$	$15^2 = 225$
$6^2 = 36$	$16^2 = 256$
$7^2 = 49$	$17^2 = 289$
$8^2 = 64$	$18^2 = 324$
$9^2 = 81$	$19^2 = 361$
$10^2 = 100$	$20^2 = 400$

From following table we come to know that square of a number always ends with 0, 1, 4, 5, 6 & 9 as unit digit. Square of a number can never have 2, 3, 7 & 8 in its unit place.

On observing squares of numbers between 21 to 29 we get following pattern.

$21^2 = 4\boxed{41}$	$29^2 = 8\boxed{41}$
$22^2 = 4\boxed{84}$	$28^2 = 7\boxed{84}$
$23^2 = 5\boxed{29}$	$27^2 = 7\boxed{29}$
$24^2 = 5\boxed{76}$	$26^2 = 6\boxed{76}$
$25^2 = 6\boxed{25}$	

Last two digits are common.

Observation

Square of two digit number having 5 in unit places can be calculated very easily

n5 here n may 1 to 9.

$(n5)^2 = [n * (n + 1)]25$

Ex.1 $65^2 = ?$

Sol.: $[6 \times (6 + 1)]25 = 4225$

Ex.2 $85^2 = ?$

Sol.: $[8 \times (8 + 1)]25 \Rightarrow 7225$

Ex.3 $95^2 = ?$

Sol.: $[9 \times (9 + 1)]25 \Rightarrow 9025$

Base System

The Number system is used to represent any number using a set of symbols (digits /letters). The base defines the number of symbols in particular base system. We generally work in Decimal system as there are 10 digits (0, 1, 2, ...9). Some others systems are;

Binary base system: 2 symbols: 0, 1

Octal base system: 8 symbols: 0,1,2,3,4,5,6,7

Hexadecimal system: 16 symbols:

0,1,2,3,4,5,6,7,8,9, A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

Converting any number from any Base system to Decimal number system:

$$abcd.efg_B = a \times B^3 + b \times B^2 + c \times B^1 + d \times B^0 + e \times B^{-1} + f \times B^{-2} + g \times B^{-3}$$

Example:

$$1234.56_8 = 1 \times 8^3 + 2 \times 8^2 + 3 \times 8^1 + 4 \times 8^0 + 5 \times 8^{-1} + 6 \times 8^{-2}$$

= 512 + 128 + 24 + 4 + 0.625 + 0.093750

= 668.718750

Converting any number from Decimal to other Base system:

Divide the number by base and get the first remainder r_1 and Quotient q_1.

Now divided q_1 by base and get remainder r_2 and Quotient q_2.

Repeat the following process till we get the quotient $q_n = 0$.

Now the decimal number in base b is $r_n r_{n-1} \ldots r_3 r_2 r_1$.

Example 1:

1. $(149)_{10} = (\ )_7$

7	149	Remainder
7	21	2
7	3	0
	0	3

$(149)_{10} = (302)_7$

2. Add $(432)_7 + (355)_7$

$$\begin{array}{l} \ \ 11 \leftarrow \text{carry} \\ (432)_7 \\ \underline{(355)_7} \\ 1120 \end{array}$$

as $2 + 5 = (7)_{10} = (10)_7$

$3 + 5 + 1 = (9)_{10} = (12)_7$

$1 + 4 + 3 = (8)_{10} = (11)_7$

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Solved Examples

1. The sum of the digits of a two-digit number is 10, while when the digits are reversed, the number decreases by 54. Find the the changed number.

(a) 28 (b) 19
(c) 37 (d) 46

Ans: (a)

Going through options we get 82 – 28 = 54

2. The sum of two numbers is 15 and their geometric mean is 20% lower than their arithmetic mean. Find the numbers.

(a) 11, 4 (b) 12, 3
(c) 13, 2 (d) 10, 5

Ans: (b)

Going through options only 12 and 3 satisfices the conditon

$$AM = \frac{12+3}{2} = 7.5$$

$GM = \sqrt{12 \times 3} = 6\sqrt{3}$ which is 20% less than 7.5.

3. If A381 is divisible by 11, find the value of the smallest natural number A?

(a) 5 (b) 6
(c) 7 (d) 9

Ans. (c)

A 381 is divisible by 11 if and only if (A + 8) – (3 + 1) is divisible by 11.

So, A=7 Satisfies the condition

4. Find the LCM of 5/2, 8/9, 11/14.

(a) 280 (b) 360
(c) 420 (d) None of these

Ans: (d)

$$\text{LCM of fraction} = \frac{\text{LCM of numerators}}{\text{H. C. F of Denominators}}$$

Here, 5/2, 8/9, 11/14, so

$$\text{LCM} = \frac{\text{LCM of }(5,\ 8,\ 11)}{\text{HCF of }(2,\ 9,\ 14)} = \frac{440}{1} = 440$$

5. Find the number of divisors of 1420.

(a) 14 (b) 15
(c) 13 (d) 12

Ans: (d)

$1420 = 142 \times 10 = 71 \times 2 \times 2 \times 5 = 2^2 \times 5^1 \times 71^1$

No. of divisor = (2+1) (1+1) (1+1) = 12

6. A milkman has three different qualities of milk. 403 gallons of 1st quality, 465 gallons of 2nd quality and 496 gallons of 3rd quality. Find the least possible number of bottles of equal size in which different milk of different qualities can be filled without mixing?

(a) 34 (b) 46
(c) 26 (d) 44

Ans: (d)

It is given that gallons of

1st quality : 403

2nd quality : 465

3rd quality : 496

least number of bottles will be in size of HCF (403, 465 and 496)

$403 = 13 \times 31$

$465 = 15 \times 31$

$496 = 16 \times 31$

HCF = 31. So we required 13+15+16 = 44 bottles.

7. What is the greatest number of 4 digits that when divided by any of the numbers 6, 9, 12, 17 leaves a remainder of 1?

(a) 9997 (b) 9793
(c) 9895 (d) 9487

Ans: (b)

LCM of 6, 9, 12, 17 = 612

greatest number of 4 digit divisible by 612 is 9792, to get remainder 1 number should be 9792+1

8. Which of the following is not a perfect square?

(a) 100858 (b) 3, 25, 137
(c) 945723 (d) All of these

Ans: (d)

Square of number never ends up with 2, 3, 7, 8

9. The LCM of $(16 - x^2)$ and $(x^2 + x - 6)$ is
   (a) $(x - 3)(x + 3)(4 - x^2)$
   (b) $4(4 - x^2)(x + 3)$
   (c) $(4 - x^2)(x - 3)$
   (d) None of these
   Ans: (d)

$$16 - x^2 = (4 - x)(4 + x)$$

$$(x^2 + x - 6) = (x + 3)(x - 2)$$

$$\text{LCM will } (16 - x^2)(x^2 + x - 6)$$

10. GCD of $x^2 - 4$ and $x^2 + x - 6$ is
   (a) $x + 2$ (b) $x - 2$
   (c) $x^2 - 2$ (d) $x^2 + 2$
   Ans: (b)

$$x^2 - 4 = (x - 2)(x + 2)$$

$$(x^2 + x - 6) = (x + 3)(x - 2)$$

$$\text{GCD} = (x - 2)$$

11. Decompose the number 20 into two terms such that their product is the greatest.
   (a) $x_1 = x_2 = 10$ (b) $x_1 = 5, x_2 = 15$
   (c) $x_1 = 8, x_2 = 12$ (d) None of these
   Ans: (a)
   If x + y = constant then xy will be maximum when x = y
   here, $x_1 + x_2 = 20$
   $x_1 = x_2 = 10$

12. Find the GCD of the polynomials $(x + 3)^2 (x - 2)(x + 1)^2$ and $(x + 1)^3 (x + 3)(x + 4)$.
   (a) $(x + 3)^3 (x + 1)^2 (x - 2)(x + 4)$
   (b) $(x + 3)(x - 2)(x + 1)(x + 4)$
   (c) $(x + 3)(x + 1)^2$
   (d) None of these
   Ans: (c)

$$\text{GCD of } (x+3)(x-2)(x+1)^2 \text{ and}$$

$$(x+1)^3(x+3)(x+4) \text{ will be } (x+3)(x+1)^2$$

13. Find the LCM of $(x + 3)(6x^2 + 5x - 4)$ and $(2x^2 + 7x + 3)(x + 3)$
   (a) $(2x + 1)(x + 3)(3x + 4)$
   (b) $(4x^2 - 1)(x + 3)^2 (3x + 4)$
   (c) $(4x^2 - 1)(x + 3)(3x + 4)$
   (d) $(2x - 1)(x + 3)(3x + 4)$
   Ans: (b)

$$(x+3)(6x^2+5x-4) = (x+3)(2x-1)(3x+4)$$

$$(2x^2+7x+3)(x+3) = (2x+1)(x+3)(x+3)$$

$$\text{LCM} = (2x+1)(2x-1)(x+3)^2(3x+4)$$

$$= (4x^2-1)(x+3)^2(3x+4)$$

14. The product of three consecutive natural numbers, the first of which is an even number, is always divisible by
   (a) 12 (b) 24
   (c) 8 (d) All of these
   Ans: (d)
   Three consecutive number will be n(n + 1)(n + 2) if n is even number then (n + 2) will also be an even number and one of them will be divisible by 3. Hence number is always divisible by 12.

15. Find the pairs of natural numbers whose least common multiple is 78 and the greatest common divisor is 13.
   (a) 58 and 13 or 16 and 29
   (b) 38 and 23 or 36 and 49
   (c) 18 and 73 or 56 and 93
   (d) 78 and 13 or 26 and 39
   Ans: (d)
   LCM = 78 and GCD = 13
   Clearly 13, 78 and 26, 39 are the two numbers

16. Fill in the blank indicated by a star in the number 4* 56 so as to make it divisible by 33
   (a) 3 (b) 4
   (c) 5 (d) None of these
   Ans. (a)
   4*56 is divisible by 33 if and only if it is divisible by 3 and 11.
   4* 56 will be divisible by 3 if * will be equal to 0, 3, 6, 9
   4*56 is divisible by 11 if (4 + 5) – (* + 6) will be divisible by 11 so * should be 3.

17. Find the least number which being divided by 9, 12, 16 and 30 leaves in each case a remainder 3?
   (a) 623 (b) 723
   (c) 728 (d) None of these

Ans. (b)

LCM of 9, 12, 16 and 30 is 720 so required number is LCM + 3 = 723

18. Find the greatest number less than 10000 which is divisible by 48, 60 and 64

(a) 9600 (b) 8500

(c) 7600 (d) None of these

Ans. (a)

The required number will be the largest four digit number in form of n*(LCM) of 48, 60 and 64 LCM of 48, 60 and 64 is 960

So the largest four digit number will be 9600

19. Find the least multiple of 11 which when divided by 8, 9, 12, 14 leaves 4 as remainder in each case.

(a) 1012 (b) 1037

(c) 1090 (d) None of these

Ans. (a)

The number is divisible by 11 and can be written in form n(LCM) +4, LCM of 8, 9, 12, 14 is 504

So the number may be 508 & 1012 but 508 is not divisible by 11 so it is 1012

20. The LCM of two number is 12 times their HCF. The sum of HCF and LCM is 403. If one number is 93 find the other.

(a) 134 (b) 124

(c) 128 (d) None of these

Ans. (b)

It is given that LCM=12 times HCF

also LCM + HCF = 403

So, 13 HCF = 403, $\Rightarrow$ HCF = 31

LCM = 372 also we know that HCF

$$\text{HCF} \times \text{LCM} = \text{Number}(1) \times \text{Number}(2)$$

$$31 \times 372 = 93 \times N2 \quad \therefore \quad N2 = 124$$

21. I have to spend 1/10 of my income on house rent, 1/10 of remainder on conveyance 1/3 of further remainder on children's education after which I have Rs. 648 left over. What is my income?

(a) Rs. 1200 (b) Rs. 1400

(c) Rs. 1700 (d) None of these

Ans. (a)

One alternate method

Let I have x rupees

After spending $\frac{1}{10}$ of it on house rent I have $\frac{9x}{10}$.

Now out of $\frac{9x}{10}$ I spent $\frac{1}{10}$ of it i.e., $\frac{9}{100}x$ on conveyance so remainder will be

$$\frac{9}{10}x - \frac{9}{100}x = \frac{81x}{100}.$$

Further I spent $\frac{1}{3}$ of $\frac{81x}{100}$ i.e. $\frac{27x}{100}$ into childrens education now I have $\frac{54x}{100}$,

So, $\frac{54x}{100} = 648, \quad x = 1200$

22. A man had two sons. To the elder he gave $\frac{5}{11}$ of his property, to the younger $\frac{5}{11}$ of remainder, the rest to the widow. Find the Share of the sons if the widow gets Rs. 3600.

(a) Rs. 1200, 1000 (b) Rs. 6000, 2000

(c) Rs. 7500, 1000 (d) None of these

Ans. (d)

Younger son gets

$$3600 \times \left(\frac{1}{1-\frac{5}{11}}\right) \times \frac{5}{11} = \text{Rs. } 3000$$

Elder son gets

$$3000 \times \left(\frac{1}{1-\frac{5}{11}}\right) = \text{Rs. } 5500$$

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I Practice Exercise

1. $\sqrt{3\sqrt{80}+\dfrac{3}{9+4\sqrt{5}}}=?$
(a) $\sqrt{3\sqrt{5}}$ (b) 3
(c) $3\sqrt{3}$ (d) $3+2\sqrt{5}$

2. x and y are integers and If $\dfrac{x^2}{y^3}$ is even integer then which of the following must be an even integer?
(a) $x - y$ (b) y + 1
(c) $\dfrac{x^2}{y^4}$ (d) xy

3. What is the tens' digit of the sum of the first 50 terms of 1, 11, 111, 1111, 11111, 111111,.......................?
(a) 2 (b) 4
(c) 5 (d) 8

4. If $81^y = \dfrac{1}{27^x}$, in terms of y, x=?
(a) $\dfrac{3y}{4}$ (b) $-\dfrac{3y}{4}$
(c) $\dfrac{4y}{3}$ (d) $-\dfrac{4y}{3}$

5. If $\dfrac{1}{n+1}<\dfrac{1}{31}+\dfrac{1}{32}+\dfrac{1}{33}<\dfrac{1}{n}$; then n ?
(a) 9 (b) 10
(c) 11 (d) 12

6. If one integer is greater than another integer by 3, and the difference of their cubes is 117, what could be their sum?
(a) 11 (b) 7
(c) 8 (d) 9

7. Which of these has total 24 positive factors?
(a) $21^5 \times 2^3$ (b) $2^7 \times 12^3$
(c) $2^6 \times 3^4$ (d) 63×55

8. Two numbers, x and y are such that when divided by 6, they leave remainder 4 and 5 respectively. Find the remainder when $x^3 + y^3$ is divided by 6?
(a) 2 (b) 3
(c) 4 (d) 5

9. What is the remainder when N = $(1! + 2! + 3! + ...1000!)^{40}$ is divided by 10?
(a) 1 (b) 3
(c) 7 (d) 8

10. Set A is formed by selecting some of the numbers from the first 100 natural numbers such that the HCF of any two numbers in the set A is 5, what is the maximum number elements that set A can have?
(a) 7 (b) 8
(c) 9 (d) 10

11. Let x and y be positive integers such that x is prime and y is composite. Then,
(a) $y - x$ cannot be an even integer
(b) $\dfrac{x+y}{x}$ cannot be an even integer
(c) $(x + y)$ cannot be even.
(d) None of the above statements are true

12. Let N = 1421 × 1423 × 1425. What is the remainder when N is divided by 12?
(a) 0 (b) 9
(c) 3 (d) 6

13. When a four digit number is divided by 85 it leaves a remainder of 39. If the same number is divided by 17 the remainder would be ?
(a) 2 (b) 5
(c) 7 (d) 9

14. Integers 34041 and 32506 when divided by a three-digit integer n leave the same remainder. What is n?
(a) 289 (b) 367
(c) 453 (d) 307

15. A box contains 100 tickets, numbered from 1 to 100. A person picks out three tickets from the box, such that the product of the numbers on two of the tickets yields the number on the third ticket. Which of the following tickets can never be picked as third ticket?
(a) 10 (b) 12
(c) 25 (d) 26

16. N is a natural number , then how many values of N are possible such that $\dfrac{6N^3+3N^2+N+24}{N}$ is also a Natural Number?
(a) 6 (b) 7
(c) 8 (d) 9

17. What is the unit digit of $39^{53} \times 27^{23} \times 36^{12}$?

(a) 2 (b) 4

(c) 6 (d) 8

18. How many number of zeros are there if we multiply all the prime numbers between 0 and 200.

(a) 1 (b) 2

(c) 3 (d) 4

19. A man wrote all the natural numbers starting from 1 in a series. What will be the 50th digit of the number?

(a) 1 (b) 2

(c) 3 (d) 4

20. N = n(n + 1) (n + 2) (n + 3) (n + 4); where n is a natural number. Which of the following statement/s is/are true?

1. Unit digit of N is 0.
2. N is perfectly divisible by 24.
3. N is perfect square.
4. N is odd.

(a) 3 only (b) 3 and 4 only

(c) 1 only (d) 1 and 2 only

21. How many factors of

$N = 12^{12} \times 14^{14} \times 15^{15}$ are multiple of

$K = 12^{10} \times 14^{10} \times 15^{10}$

(a) $2 \times 4 \times 5$ (b) $3 \times 5 \times 6$

(c) $8 \times 7 \times 4 \times 5$ (d) $9 \times 8 \times 6 \times 5$

22. In a certain base

137 + 254 = 402 then

What is the sum of 342 + 562 in that base

(a) 904 (b) 1014

(c) 1104 (d) 1024

Answers

1. (c)	2. (d)	3. (b)	4. (d)	5. (b)
6. (b)	7. (d)	8. (b)	9. (a)	10. (c)
11. (d)	12. (c)	13. (b)	14. (d)	15. (c)
16. (c)	17. (a)	18. (a)	19. (c)	20. (d)
21. (d)	22. (b)			

Solutions

1. (c)

Method (i) $\sqrt{3\sqrt{80}+\frac{3}{9+4\sqrt{5}}}$ using rationalization

$$= \sqrt{3\sqrt{80}+\frac{3}{9+4\sqrt{5}}\times\left(\frac{9-4\sqrt{5}}{9-4\sqrt{5}}\right)}$$

$$= \sqrt{3\sqrt{80}+\frac{(3\times9-3\times4\sqrt{5})}{9^2-(4\sqrt{5})^2}}$$

$$= \sqrt{3\sqrt{80}+\frac{27-12\sqrt{5}}{81-80}}$$

$$= \sqrt{3\sqrt{16\times5}+27-12\sqrt{5}}$$

$$= \sqrt{3\times4\times\sqrt{5}+27-12\sqrt{5}}$$

$$= \sqrt{12\sqrt{5}+27-12\sqrt{5}}$$

$$= \sqrt{27} = 3\sqrt{3}$$

Alternative Method

$$\sqrt{\left(3\sqrt{80}+\frac{3}{9+4\sqrt{5}}\right)}$$

$$3\sqrt{80} \cong 3\sqrt{81} \cong 27$$

and $\frac{3}{9+4\sqrt{5}} < 1$

Thus, $\sqrt{3\sqrt{80}+\frac{3}{9+4\sqrt{5}}} \cong \sqrt{3\sqrt{81}}$

$$\cong \sqrt{3\times9} = 3\sqrt{3}$$

2. (d)

if $\frac{x^2}{y^3}$ = even

$x^2 = y^3$ even

$\Rightarrow x^2 \Rightarrow$ even

and x is integer

$\Rightarrow x$ = even

so only xy must be even.

3. (b)

$$\begin{array}{r} 1 \\ 1\,1 \\ 1\,1\,1 \\ \vdots\;\vdots\;\vdots \\ \text{50 terms..............}1\,1\,1 \\ \hline 40 \end{array}$$

unit digit (1 + 1...... 50 times)= 0

and carry = 5

tens digit (1 + 1 + ... 49 times)+ carry 5 = 4