ELECTRICAL ENGINEERING

SIGNALS AND SYSTEMS

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Signals and Systems

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Signals and Systems

INTRODUCTION TO SIGNALS AND SYSTEMS

This book starts with basic and extensive chapter on signals in which continuous and discrete-time case are discussed in parallel. A variety of basic signals, functions with their mathematical description, representation and properties are incorporated. A substantial amount of examples are given for quick sketching of functions. A chapter on systems is discussed separately which deals with classification of systems, both in continuous and discrete domain and more emphasize is given to LTI systems and analytical as well as graphical approach is used to understand convolution operation. These two chapters makes backbone of the subject.

Further we shall proceed to transform calculus which is important tool of signal processing. A logical and comprehensive approach is used in sequence of chapters. The continuous time Fourier series which is base to the Fourier transform, deals with periodic signal representation in terms of linear complex exponential, is discussed.

The Fourier transform is discussed before Laplace transform. The sampling, a bridge between continuous-time and discrete-time, is discussed to understand discrete-time domain.

A major emphasis is given on proof of the properties so that students can understand and analyzes fundamental easily.

A point wise recapitulation of all the important points and results in every chapter proves helpful to students in summing up essential developments in the chapter which is an integral part of any competitive examination.

CHAPTER

1

Introduction to Signals

1.1 INTRODUCTION

A signal is any quantity having information associated with it. It may also be defined as a function of one or more independent variables which contain some information.

The function defines mapping from one set to another and similarly a signal may also be defined as mapping from one set (domain) to another (range). e.g.

- A speech signal would be represented by acoustic pressure as a function of time.
- A monochromatic picture would be represented by brightness as a function of two spatial variables.
- A voltage signal is defined by a voltage across two points varying as function of time.
- A video signal, in which color and intensity as a function of 2-dimensional space (2D) and 1-dimensional time (i.e. hybrid variables).

NOTE: In this course of "signals and systems", we shall focus on signals having only one variable and will consider 'time' as independent variable.

1.2 ELEMENTARY SIGNALS

These signals serve as basic building blocks for construction of somewhat more complex signals. The list of elementary signals mainly contains singularity functions and exponential functions.

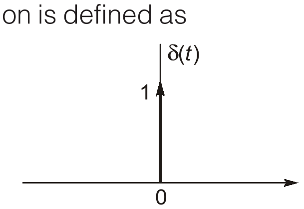
These elementary signals are also known as basic signals/standard signals.

Let us discuss these basic signals one-by-one.

1.2.1 Unit Impulse Function

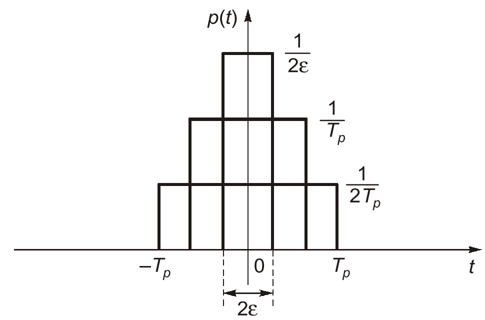
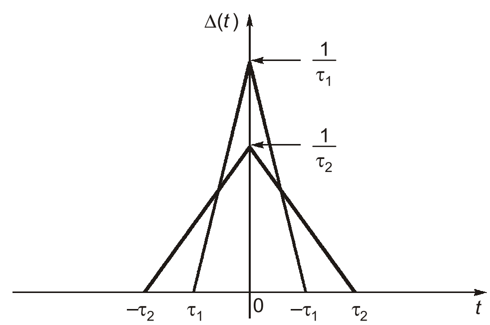
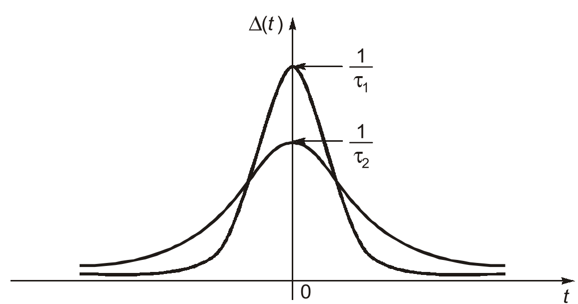
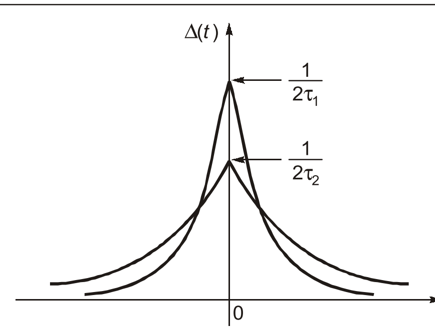
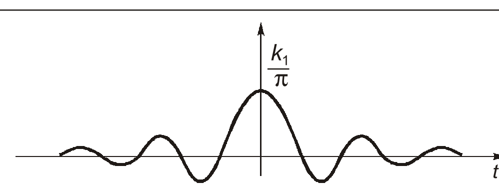
A continuous-time unit impulse function δ(t), also called as Dirac delta function is defined as

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$



The unit-impulse function is represented by an arrow with strength of '1' which represents its 'area' or 'weight'.

The above definition of an impulse function is more generalised and can be represented as limiting process without any regard to shape of a pulse. For example, one may define impulse function as a limiting case of rectangular pulse, triangular pulse Gaussian pulse, exponential pulse and sampling pulse as shown below:

Sl. No.	Type of Impulse	Graph
1.	**Rectangular Pulse** $\delta = (t) \lim_{\varepsilon \to 0} p(t)$	
2.	**Triangular Pulse** $\delta(t) = \begin{cases} \lim_{\tau \to 0} \frac{1}{\tau}\left[1 - \frac{\lvert t \rvert}{\tau}\right] & ; \lvert t \rvert < \tau \\ 0 & ; \lvert t \rvert > \tau \end{cases}$	
3.	**Gaussian Pulse** $\delta(t) = \lim_{\tau \to 0} \frac{1}{\tau}\left[e^{-\pi t^2/\tau^2}\right]$	
4.	**Exponential Pulse** $\delta(t) = \lim_{\tau \to 0} \frac{1}{2\tau}\left[e^{-\lvert t \rvert/\tau}\right]$	
5.	**Sampling Function** $\int_{-\infty}^{\infty} \frac{k}{\pi} Sa(kt)dt = 1$	

Properties of Continuous Time Unit Impulse Function

(i) Scaling property:

$$\delta(at) = \frac{1}{|a|}\delta(t) \quad ; \quad \text{'}a\text{' is a constant, postive or negative}$$

Proof: $\delta(at) = \frac{1}{|a|}\delta(t)$

Integrating above equation on both the sides with respect to 't'.

$$\int_{-\infty}^{+\infty}\delta(at)\,dt = \int_{-\infty}^{+\infty}\frac{1}{|a|}\delta(t)\,dt$$

Let $at = \tau$

$a \cdot dt = d\tau$; 'a' is a constant, positive or negative or $|a| \cdot dt = d\tau$

Now, $\int_{-\infty}^{+\infty}\delta(at)\,dt = \int_{-\infty}^{+\infty}\delta(\tau)\cdot\frac{d\tau}{|a|} = \int_{-\infty}^{+\infty}\frac{1}{|a|}\delta(t)\cdot dt$ By definition, $\int_{-\infty}^{+\infty}\delta(t)dt = \int_{-\infty}^{+\infty}\delta(\tau)d\tau = 1$

(ii) Product property/multiplication property:

$$x(t)\delta(t-t_o) = x(t_o)\,\delta(t-t_o)$$

Proof:

The function $\delta(t-t_o)$ exists only at $t = t_o$. Let the signal $x(t)$ be continuous at $t = t_o$.

Therefore, $x(t)\,\delta(t-t_o) = x(t)|_{t=t_o}\cdot\delta(t-t_o) = x(t_o)\,\delta(t-t_o)$

Important Expressions

- $\delta(at \pm b) = \frac{1}{|a|}\delta\left(t \pm \frac{b}{a}\right)$
- $\delta(-t) = \delta(t)$ $\because$ $\delta(t)$ is an even function of time.
- $x(t)\,\delta(t) = x(0)\,\delta(t)$
- $\int_{-\infty}^{+\infty}x(t)\,\delta(t)\,dt = x(0)$

(iii) Sampling property:

$$\int_{-\infty}^{+\infty}x(t)\,\delta(t-t_o)\,dt = x(t_o)$$

Proof :

Using product property of impulse function

$$x(t)\,\delta(t-t_o) = x(t_o)\,\delta(t-t_o)$$

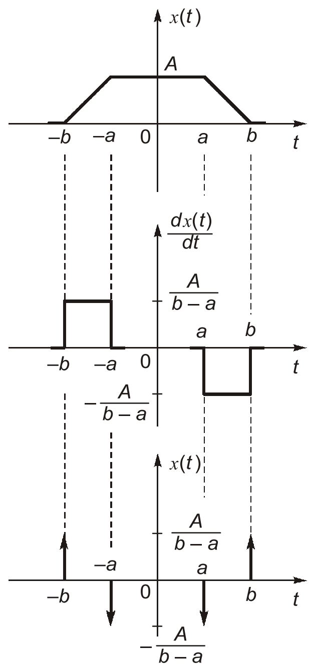
Integrating above equation on both the sides with respect to 't'.

$$\int_{-\infty}^{+\infty}x(t)\,\delta(t-t_o)dt = \int_{-\infty}^{+\infty}x(t)\,\delta(t-t_o)dt = x(t_o)\int_{-\infty}^{+\infty}\delta(t-t_o)dt = x(t_o)$$

$$\int_{-\infty}^{+\infty}x(t)\delta(t)\,dt = x(0)$$

Solution :

(i)

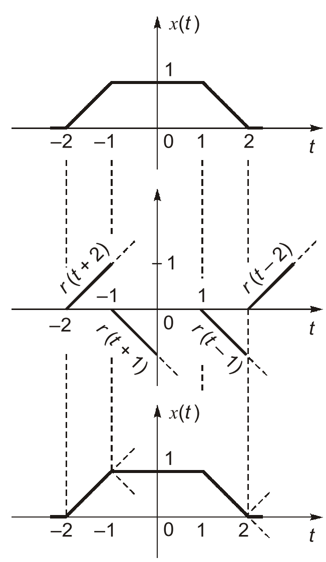


$$x(t) = \begin{cases} \dfrac{A}{b-a}(t+b), & -b \le t \le -a \\ A \quad , & -a \le t + a \\ -\dfrac{A}{b-a}(t-b), & a \le t \le b \end{cases}$$

$$\frac{dx(t)}{dt} = \frac{A}{b-a}\{u(t+b) - u(t+a)\} - \frac{A}{b-a}\{u(t-a) - u(t-b)\}$$

$$\frac{d^2x(t)}{dt^2} = \frac{A}{b-a}\{\delta(t+b) + \delta(t-b)\} - \frac{A}{b-a}\{\delta(t+a) + \delta(t-a)\}$$

(ii)



$$x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$

OBJECTIVE BRAIN TEASERS

Q.1 The odd component of the signal $x(t) = e^{-2t} \cos t$ is

(a) $\cosh(2t) \cos t$ (b) $-\sinh(2t) \cos t$

(c) $-\cosh(2t) \cos t$ (d) $\sinh(2t) \cos t$

Q.2 A discrete time system is given as:

$$x[n] = \cos\left(\frac{n}{4}\right) \cdot \sin\left(\frac{\pi n}{4}\right)$$

The signal is

(a) periodic with 8 (b) periodic with $8(\pi + 1)$

(c) periodic with 4 (d) non-periodic

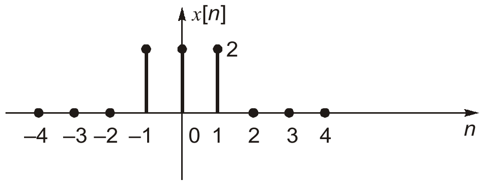
Q.3 The power of signal $x[n] = (-1)^n u[n]$ is _____ W.

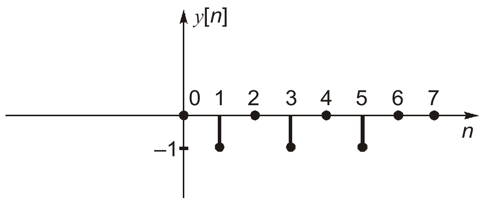
Q.4 A discrete time signal is given as

$$x[n] = \cos\left(\frac{\pi n}{3}\right) \cdot (u[n] - u[n-6])$$

The energy of the signal is __________ J.

Q.5 Two functions $x[n]$ and $y[n]$ are shown in following figures.





If $y[n] = \alpha x\left[\frac{n - n_0}{k}\right]$ then value of $n_0 + \alpha + k$ is _____.

Q.6 Consider a discrete time signal as follows:

$$x[n] = \begin{cases} 1 & ; \; n = 1 \\ -1 & ; \; n = -1 \\ 0 & ; \; \text{otherwise} \end{cases}$$

If $y[n] = x[n] + x[-n]$, then the energy of the signal $y[n]$ will be

(a) 0 (b) 1
(c) 2 (d) 4

Q.7 A continuous time signal is defined as,

$$x(t) = 4\cos\left(\frac{2\pi}{3}t + 40^\circ\right) + 3\sin\left(\frac{4\pi}{5}t + 20^\circ\right).$$

The fundamental time period of $x(t)$ is

(a) 30π sec (b) 15π sec
(c) 15 sec (d) 30 sec

Q.8 The conjugate antisymmetric part of the sequence, $x(n) = [-5, \underset{\uparrow}{-3+2j}, 4j, 8+9j]$ is

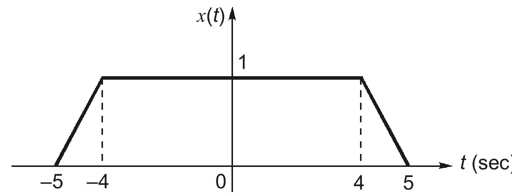
(a) $[-4+4.5j, -2.5+2j, \underset{\uparrow}{-2j}, -2.5+2j, 4+4.5j]$

(b) $[-4+4.5j, -2.5+2j, \underset{\uparrow}{2j}, 2.5+2j, 4+4.5j]$

(c) $[-4-4.5j, -2.5+2j, \underset{\uparrow}{-2j}, 2.5+2j, 4+4.5j]$

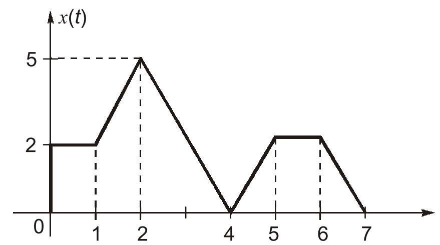
(d) $[-4-4.5j, -2.5+2j, \underset{\uparrow}{-2j}, 2.5-2j, -4+4.5j]$

Q.9 Consider the trapezoidal pulse $x(t)$ shown in the figure below :



The energy of $x(t)$ is __________ .

Q.10 A signal $x(t)$ is given by,

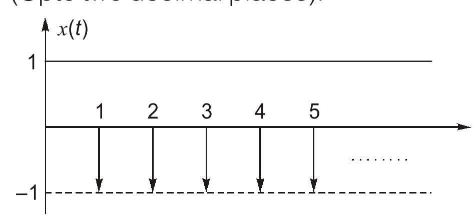


The value of the integral

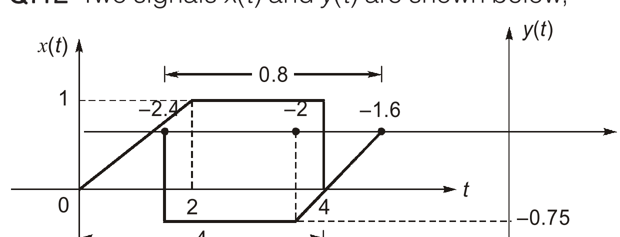
$$I = \int_{-\infty}^{\infty} x(-t+1)\delta'(t+2.5)dt \text{ is } \_\_\_\_\_\_\_\_\_\_.$$

Q.11 For the signal $x(t)$ shown below, the value of $\left.\int_{-\infty}^{t} x(\tau)\, d\tau\right|_{t=0.5}$ is ________ .

(Upto two decimal places).



Q.12 Two signals $x(t)$ and $y(t)$ are shown below,



If $y(t) = ax(bt + c)$ then the value of $4a + b - c$ is _______.

ANSWERS KEY

1.	(b)	**2.**	(d)	**3.**	(0.5)	**4.**	(3)	**5.**	(4.5)
6.	(a)	**7.**	(c)	**8.**	(b)	**9.**	(8.67)		
10.	(2.5)	**11.**	(0.5)	**12.**	(0)				

HINTS & EXPLANATIONS

1. (b)

$$x(t) = e^{-2t}\cos t$$
$$x(-t) = e^{2t}\cos(-t) = e^{2t}\cos t$$

Odd component,

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$
$$x_o(t) = \cos t\left[\frac{e^{-2t} - e^{2t}}{2}\right] = -\cos t\left[\frac{e^{2t} - e^{-2t}}{2}\right]$$
$$\Rightarrow \quad x_o(t) = -\sinh(2t)\cos t$$

2. (d)

$$x[n] = \cos\left(\frac{n}{4}\right)\sin\left(\frac{nx}{4}\right) \; ; \quad \frac{\frac{1}{4}}{2\pi} = \frac{m}{N}$$
$$\Rightarrow \quad \frac{m}{N} = \frac{1}{8\pi} \quad \text{which is irrational}$$

Hence, the signal is aperiodic.

3. (0.5)

$$x[n] = (-1)^n u[n]$$

Power, $P = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} x^2[n]$

$$P = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=0}^{N}(-1)^{2n} = \lim_{N\to\infty}\frac{N+1}{2N+1}$$

By using L-Hospital's rule, $P = \frac{1}{2}\text{ W} = 0.5\text{ W}$

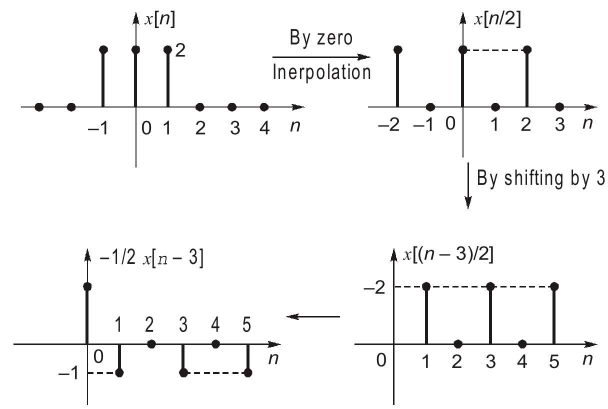
4. (3)

$$x[n] = \cos\frac{n\pi}{3}[u[n] - u[n-6]]$$

Energy, $E = \sum_{n=-\infty}^{\infty} x^2[n]$

$$E = \sum_{n=0}^{5}\left[\cos\frac{n\pi}{3}\right]^2$$
$$\Rightarrow E = [\cos 0]^2 + \left[\cos\frac{\pi}{3}\right]^2 + \left[\cos\frac{2\pi}{3}\right]^2 + [\cos\pi]^2 + \left[\cos\left[\frac{4\pi}{3}\right]\right]^3 + \cos\left[\frac{5\pi}{3}\right]^2$$
$$E = 1 + \frac{1}{4} + \frac{1}{4} + 1 + \frac{1}{4} + \frac{1}{4}$$
$$E = 3\text{ J}$$

5. (4.5)

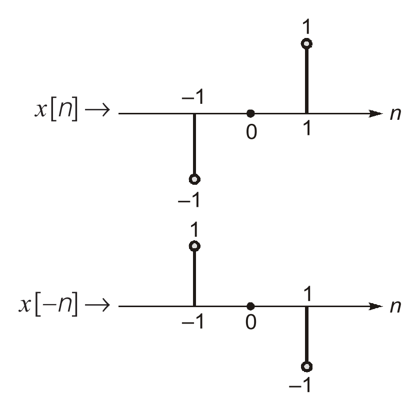


So, $y[n] = -\frac{1}{2}x\left[\frac{n-3}{2}\right] = \alpha x\left[\frac{n-n_o}{K}\right]$

$$\alpha = -\frac{1}{2} \; ; \; n_o = 3 \; ; \; K = 2$$

So, $n_o + \alpha + K = 4.5$

6. (a)



$$\therefore \quad y[n] = x[n] + x[-n] = 0$$
$$\Rightarrow \text{Energy of } y[n] = \sum_{n=-\infty}^{\infty}|y(n)|^2 = 0$$

7. (c)

$$x(t) = \underbrace{4\cos\left(\frac{2\pi}{3}t + 40^\circ\right)}_{x_1(t)} + \underbrace{3\sin\left(\frac{4\pi}{5}t + 20^\circ\right)}_{x_2(t)}$$
$$\omega_1 = \frac{2\pi}{3} \quad \Rightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2\pi/3} = 3$$

CONVENTIONAL BRAIN TEASERS

Q.1 Consider a signal $x[n]$,

$$x[n] = \sum_{k=-\infty}^{\infty} g[n-k] + \sum_{k=-\infty}^{\infty} g\left[\frac{n}{N} - k\right] \qquad \text{where, } g[n] = \delta[n]$$

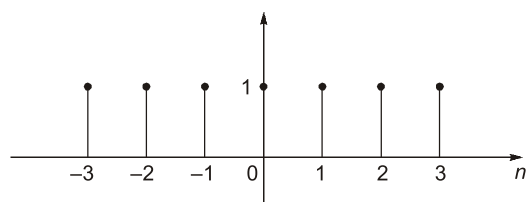
N is an even integer with $0 < N < 4$.

(i) Draw the waveform of signal $x[n]$. **(ii)** Find the power of the signal $x[n]$.

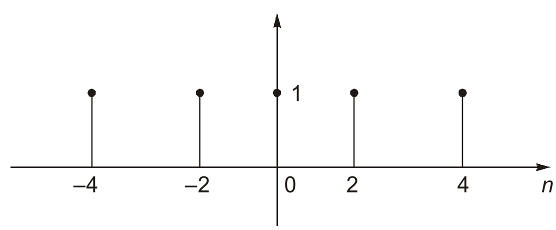
(iii) Find the value of $\sum_{n=-4}^{5} x[n]$.

1. (Sol.)

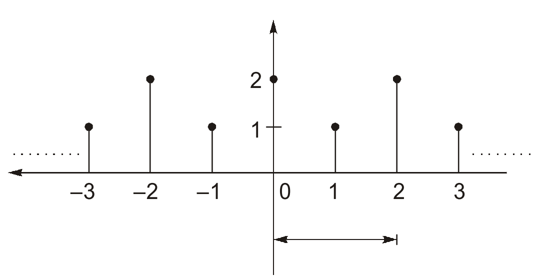
(i) $\sum_{k=-\infty}^{\infty} \delta[n-k]$



$$\sum_{k=-\infty}^{\infty} \delta\left[\frac{n}{2} - k\right]$$



$x[n] \Rightarrow$



$x[n]$ is periodic with period. $M = 2$.

(ii) Power of $x[n]$ $\quad P_x = \frac{1}{M}\sum_{n=0}^{1} |x[n]|^2 = \frac{1}{2}[4+1] = \frac{5}{2} = 2.5 \text{ W}$

(iii) $\sum_{n=-4}^{5} x[n] = 2 + 1 + 2 + 1 + 2 + 1 + 2 + 1 + 2 + 1 = 15$

■■■■

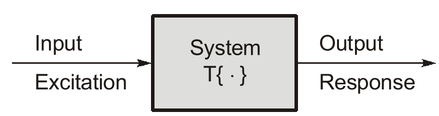
CHAPTER

2

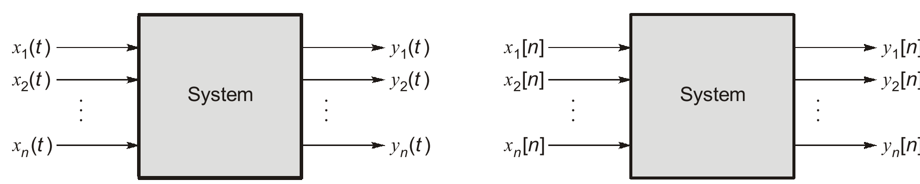
Introduction to Systems

2.1 INTRODUCTION

A system is an operator which maps the relation between input signal and output signal by the process of transformation. A system may also be defined as set of elements which produces expected output with available input. The examples of systems are electrical system, mechanical system, electromechanical system etc.



In brief, a system is mathematical identity which maps a set of input ($x(t)$ or $x[n]$) to set of output ($y(t)$ or $y[n]$).



System may be multi-input multi-output systems. In this subject, we consider Single Input Single Output (SISO) systems.

Methods of Representing Systems:

(i) Input-output relation $y(t) = x^2(t)$ or $y[n] = x^2[n]$

(ii) Differential equation $\dfrac{dy(t)}{dt} + y(t) = x(t)$

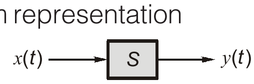
(a) Difference equation $y[n] - y[n-1] = x[n]$

(iii) Transfer function equation $\{H(s), H(\omega), H(z)\}$, $H(s) = \dfrac{Y(s)}{X(s)}$

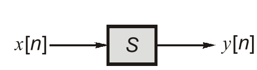
(iv) Impulse response equation ($h(t)$, $h[n]$) $y(t) = h(t) * x(t)$ or $y[n] = h[n] * x[n]$

(v) By its physical definition.

(vi) Block diagram representation



or

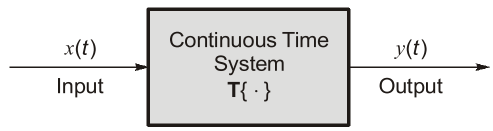


$x(t) \xrightarrow{s} y(t)$ $x[n] \xrightarrow{s} y[n]$

(vii) State variable approach

2.2 CONTINUOUS-TIME AND DISCRETE-TIME SYSTEMS

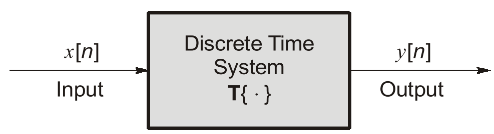
A continuous time system (CTS) is one in which continuous time input signals are transformed into continuous time output signals.



$y(t) = T\{x(t)\}$

e.g. integrator, differentiator, filters etc.

A discrete time system (DTS) is one which transform discrete time input signal into discrete time output signal.



$y[n] = T\{x(n)\}$

Moreover, a continuous time signal can be processed by a discrete time system. This is done, because discrete time systems have several significant advantages over continuous time systems.

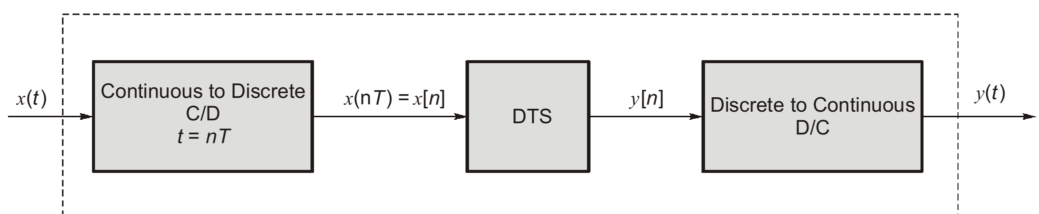


Figure : *Transformation of continuous time signals using discrete time systems*

2.3 CLASSIFICATION OF SYSTEMS

Systems are broadly classified into continuous time and discrete time systems both CTS and DTS are further classified according to the way they interact with input signals. These are:

(a) Linear and non-linear systems
(b) Time variant and time invariant systems
(c) Causal and non causal systems
(d) Static and dynamic systems
(e) Stable and unstable systems
(f) Invertible and non-invertible systems

2.3.1 Linear and non-linear systems

A system is said to be linear if it satisfies to properties: (i) Additivity (ii) Homogeneity (scaling)

(i) Additivity

It states that, if an input $x_1(t)$ produces output $y_1(t)$, and another input $x_2(t)$ also acting along produces output $y_2(t)$, then, when both inputs acting on the system simultaneously, produces output $y_1(t) + y_2(t)$.

If $x_1(t) \xrightarrow{S} y_1(t)$ and $x_2(t) \xrightarrow{S} y_2(t)$

then $x_1(t) + x_2(t) \xrightarrow{S} y_1(t) + y_2(t)$

Similarly, in discrete-time system

If $x_1[n] \xrightarrow{S} y_1[n]$ and $x_2[n] \xrightarrow{S} y_2[n]$

then $x_1[n] + x_2[n] \xrightarrow{S} y_1[n] + y_2[n]$