ELECTRICAL ENGINEERING

Electromagnetic Theory

Comprehensive Theory
with Solved Examples *and* Practice Questions

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CHAPTER

1

Vector Analysis

1.1 INTRODUCTION

Vector analysis is a concise language or mathematical shorthand which greatly facilitates the analysis of electric and magnetic fields. The quantities of interest appearing in the study of EM theory can almost be classified as either a scalar or a vector.

Quantities that can be described by a magnitude alone are called **scalars**. Distance, temperature, mass etc. are examples of scalar quantities. Other quantities, called **vectors**, require both a magnitude and a direction to fully characterize them. Examples of vector quantities include velocity, force, acceleration etc.

In electromagnetics, we frequently use the concept of a **field**. A field is a function that assigns a particular physical quantity to every point in a region. In general, a field varies with both position and time. There are scalar fields and vector fields. Temperature distribution in a room and electric potential are examples of scalar fields. Electric field and magnetic flux density are examples of vector fields.

Note: Vectors are denoted by an arrow over a letter ($\vec{A}$) and scalars are denoted by simple letter (A).

1.1.1 Unit Vector

A unit vector $\hat{a}_A$ along $\vec{A}$ is defined as a vector whose magnitude is unity (*i.e.*, 1) and its direction is along $\vec{A}$, that is

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A} \qquad ...(1.1)$$

Thus we can write $\vec{A}$ as

$$\vec{A} = A\,\hat{a}_A = |\vec{A}|\hat{a}_A \qquad ...(1.2)$$

Remember: Any vector can be written as product of its magnitude and its unit vector.

1.1.2 Vector Addition and Subtraction

Two vectors $\vec{A}$ and $\vec{B}$ can be added together to give another vector $\vec{C}$; that is,

$$\vec{C} = \vec{A} + \vec{B} \qquad ...(1.3)$$

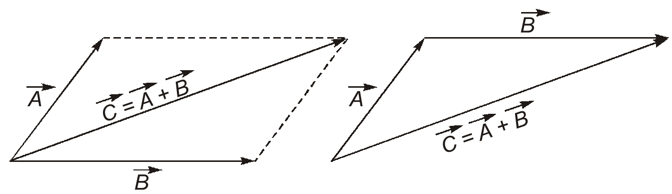


***Fig.:** Vector addition (a) parallelogram rule, (b) head-to-tail rule.*

NOTE

- $\vec{A}+\vec{B}=\vec{B}+\vec{A}$ (Commutative law).
- $(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$ (Associative law)

Vector subtraction is similarly carried out as

$$\vec{D} = \vec{A}-\vec{B}=\vec{A}+(-\vec{B}) \quad ...(1.4)$$

REMEMBER Graphically, vector addition and subtraction are obtained by either the parallelogram rule or the head-to-tail rule.

NOTE

- $k(\vec{A}+\vec{B})=k\vec{A}+k\vec{B}$ (Distributive law)
- $\dfrac{\vec{A}+\vec{B}}{k}=\dfrac{1}{k}\vec{A}+\dfrac{1}{k}\vec{B}$

1.1.3 Position and Distance Vectors:

A point P in Cartesian coordinates may be represented by (x, y, z).

The position vector $\vec{r}_p$ (or radius vector) of point P is defined as the directed distance from origin O to P.

$$\vec{r}_p = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z \quad ...(1.5)$$

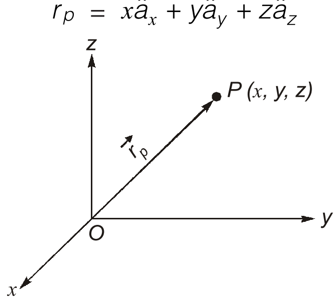


***Fig.:** Illustration of position vector* $\vec{r}_p = x\hat{a}_x + \hat{y}a_y + z\hat{a}_z$

The distance vector is the displacement from one point to another.

Consider point P with position vector $\vec{r}_p$ and point Q with position vector $\vec{r}_q$. The displacement from P to Q is written as

$$\vec{R}_{PQ} = \vec{r}_q - \vec{r}_p \quad ...(1.6)$$

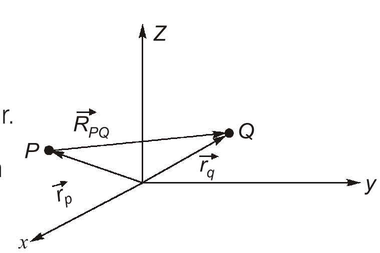


***Fig.:** Vector distance* $\vec{R}_{PQ}$

OBJECTIVE BRAIN TEASERS

Q.1 A point is represented in Cartesian coordinates as $P(3, 4, 5)$, the radial component ρ in cylindrical coordinates will be
(a) less than (b) greater than
(c) equal to (d) unrelated to
r in spherical coordinates.

Q.2 Consider a closed surface S surrounding a volume V. If $\vec{r}$ is the position vector of a point inside S, with $\hat{n}$ the unit normal on S, the value of the integral $\oiint_S 2\vec{r}\cdot\hat{n}\,dS$ is
(a) 3 V (b) 2 V
(c) 6 V (d) 4 V

Q.3 Consider a vector $\vec{E} = z\hat{a}_x + (x+y)\hat{a}_y$, the z component of the vector in cylindrical coordinates will be
(a) z
(b) $z\cos\phi + (x+y)\sin\phi$
(c) $-z\sin\phi + (x+y)\cos\phi$
(d) zero

Q.4 The direction of vector $\vec{A}$ is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and k is a constant. The value of n for which $\nabla\cdot\vec{A} = 0$ is
(a) – 2 (b) 2
(c) 1 (d) 0

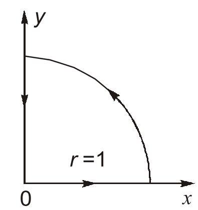
Q.5 Let a point in spherical and cylindrical coordinates are (r, θ, ϕ) and (ρ, ϕ, z). The radial component r in spherical coordinates is related to components in cylindrical coordinates as
(a) ρ (b) $\rho\cos\phi$
(c) $z\tan^{-1}\phi$ (d) $(\rho^2 + z^2)^{1/2}$

Q.6 Given the vector

$$\vec{A} = (\cos x)(\sin y)\hat{a}_x + (\sin x)(\cos y)\hat{a}_y,$$

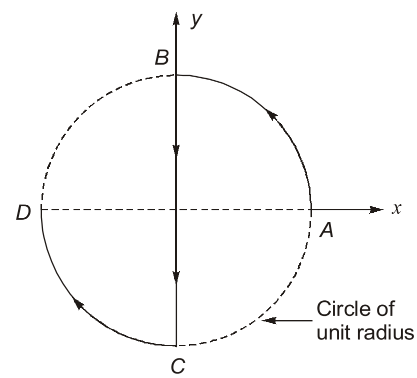
where $\hat{a}_x, \hat{a}_y$ denote unit vectors along x, y directions, respectively. The magnitude of curl of $\vec{A}$ is
(a) 0 (b) 1
(c) –1 (d) 2

Q.7 Given a vector field $\vec{A} = 2r\cos\phi\,\hat{a}_r$ in cylindrical coordinates. For the contour as shown below, $\oint \vec{A}\cdot d\vec{l}$ is



(a) 1
(b) $1 - (\pi/2)$
(c) $1 + (\pi/2)$
(d) – 1

Q.8



What is the value of the integral $\int_c d\vec{l}$ along the curve c (c is the curve ABCD in the direction of the arrow)?
(a) $2(\hat{a}_x + \hat{a}_y)/\sqrt{2}$
(b) $-2(\hat{a}_x + \hat{a}_y)/\sqrt{2}$
(c) $2\hat{a}_x$
(d) $-2\hat{a}_y$

ANSWERS KEY

1. (a)	2. (c)	3. (d)	4. (a)	5. (d)
6. (a)	7. (a)	8. (d)		

HINTS & EXPLANATIONS

1. (a)

$$P(3, 4, 5), \quad \rho = \sqrt{3^2 + 4^2} = 5$$

$$r = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

So, $r > \rho$

2. (c)

Position vector,

$$\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

$$\oiint_S 2\vec{r}\,\vec{n}\,dS = 2\iiint_V (\vec{\nabla}\cdot\vec{r})dV$$

$$\vec{\nabla}\cdot\vec{r} = \frac{\partial x}{\partial x}+\frac{\partial y}{\partial y}+\frac{\partial z}{\partial z} = 3$$

$$\oiint 2\vec{r}\cdot\vec{n}\,dS = 6\iiint_V dV = 6\text{ V}$$

3. (d)

$$\vec{E} = z\hat{a}_x + (x+y)\hat{a}_y$$

Z component of Cartesian and cylindrical component are equal.

Hence, z component is 0.

4. (a)

$$r^2 = x^2 + y^2 + z^2$$

$$|A| = kr^n$$

$$\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

$$\nabla\cdot\vec{A} = \nabla\cdot(kr^n\hat{a}_r) = k\nabla\cdot(r^n\,\hat{a}_r)$$

$$\nabla\cdot\vec{A} = k\nabla\cdot\left(r^n\frac{\hat{a}_r}{r_1}\right)$$

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 r^n) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^{n+2})$$

for $\nabla\cdot\vec{A} = 0,\ r^{n+2} = \text{constant}$

$\Rightarrow\quad n + 2 = 0$

5. (d)

$$r = \sqrt{x^2+y^2+z^2}$$

$$\rho = \sqrt{x^2+y^2} \quad\Rightarrow\quad r^2 = x^2 + y^2$$

$$\Rightarrow\quad r = \sqrt{\rho^2+z^2}$$

6. (a)

$$\vec{A} = \cos x\sin y\,\hat{a}_x + \sin x\cos y\,\hat{a}_y$$

$$\vec{\nabla}\times\vec{A} = \begin{bmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \cos x\sin y & \sin x\cos y & 0\end{bmatrix}$$

$$\vec{\nabla}\times\vec{A} = 0\cdot\hat{a}_x + (-0\cdot\hat{a}_y) + [\cos x\cos y - \cos x\cos y]\hat{a}_z$$

$$\Rightarrow\quad \vec{\nabla}\times\vec{A} = 0 \Rightarrow \left|\vec{\nabla}\times\vec{A}\right| = 0$$

7. (a)

$$\vec{A} = 2r\cos\phi\,\hat{a}_r$$

for Stokes's theorem

$$\oint\vec{A}\cdot dl = \iint_s(\vec{\nabla}\times\vec{A})\cdot dS$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{r}\begin{bmatrix}\hat{a}_r & r\hat{a}_\phi & \hat{a}_z\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ 2r\cos\phi & 0 & 0\end{bmatrix} = +2\sin\phi\,\hat{a}_z$$

$$\iint_s\left(\vec{\nabla}\times\vec{A}\right)\cdot ds = \int_0^{\pi/2}\int_0^1 +2r\sin\phi\,dr\,d\phi = \int_0^{\pi/2}\left[\frac{2r^2}{2}\right]_0^1\sin\phi\,d\phi$$

$$\Rightarrow\iint_s\left(\vec{\nabla}\times\vec{A}\right)\cdot ds = [-\cos\phi]_0^{\pi/2} = 1$$

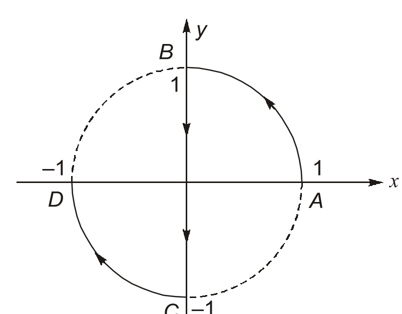
$$\Rightarrow\quad\oint\vec{A}\cdot\vec{dl} = 1$$

8. (d)

$$\int_C dl = \int_{AB}\vec{dl} + \int_{BC}\vec{dl} + \int_{CD}\vec{dl}$$

As $\int_{AB} dl$ and $\int_{CD}\vec{dl}$

are in opposite direction



i.e., $\int_{AB} dl = -\int_{CD} dl$

$$\Rightarrow\quad \int_C dl = \int_{BC} dl = -2\hat{a}_y$$

CONVENTIONAL BRAIN TEASERS

Q.1 Find the area of the curved surface of a right circular cylinder of radius *r* and height *h* using cylindrical coordinates.

1. (Sol.)

radius of cylinder, '*r*'; height of cylinder, '*h*'

Curved surface area, $$A = \int_0^{2\pi}\int_0^{h} r\,dz\,d\phi = r\int_0^{2\pi} h\,d\phi = 2\pi r h$$

Q.2 Calculate the volume in spherical coordinates defined by

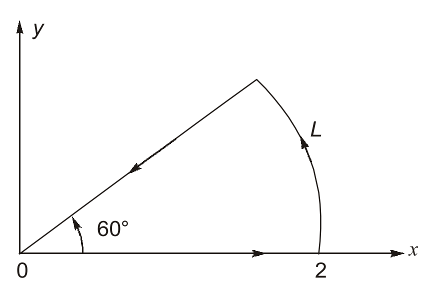
$$1 \le r \le 2\text{ m},\ 0 \le \theta \le \frac{\pi}{2},\ 0 \le \phi \le \frac{\pi}{2}$$

2. (Sol.)

$$\text{Volume, } V = \int_0^{\pi/2}\int_0^{\pi/2}\int_1^{2} r^2 \sin\theta\, dr\, d\theta\, d\phi = \int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{r^3}{3}\right]_1^2 \sin\theta\, d\theta\, d\phi$$

$$V = \frac{7}{3}\int_0^{\pi/2}[-\cos\theta]_0^{\pi/2}\, d\phi = \frac{7}{3}[\phi]_0^{\pi/2} = \frac{7}{6}\pi \text{m}^3$$

Q.3 Calculate the circulation of $A = \rho\cos\phi\,\hat{a}_\rho + z\sin\phi\,\hat{a}_z$ around the edge L of the wedge defined by $0 < \rho < 2,\ 0 \le \phi \le 60°,\ z = 0$ and shown in figure.



3. (Sol.)

$$A = \rho\cos\phi\,\hat{a}_\rho + z\sin\phi\,\hat{a}_z$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{\rho}\begin{bmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ \rho\cos\phi & 0 & z\sin\phi \end{bmatrix} = \frac{1}{\rho}[(z\cos\phi)\hat{a}_\rho - \rho\hat{a}_\phi[0] + (\rho\sin\phi)\hat{a}_z]$$

$$= \frac{z\cos\phi}{\rho}\hat{a}_\rho + \frac{\rho\sin\phi}{\rho}\hat{a}_z$$