ELECTRICAL ENGINEERING

ELECTRIC CIRCUITS

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Electric Circuits

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CONTENTS

Electric Circuits

CHAPTER 6

First Order RL and RC Circuits 113-147

CHAPTER 7

Second Order RLC Circuits 148-179

CHAPTER 8

Sinusoidal Steady-State Analysis 180-223

CHAPTER 9

AC Power Analysis 224-248

CHAPTER 10

Magnetically Coupled Circuits 249-272

CHAPTER 11

Frequency Response & Resonance 273-299

CHAPTER 12

Two Port Network 300-334

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CHAPTER

Basics Components and Electric Circuits

1

1.1 INTRODUCTION

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are build. Many branches of electrical engineering such as electric machines, control system, electronics, communication and instrumentation are based on electric circuit theory.

A circuit is an energy or signal/information processor. Each circuit consists of interconnections of "simple" circuit elements, or device. In other word we can say that an electric circuit is an interconnection of electric elements. For example, a device (or circuit element) called a 'source' produces a voltage or a current signal. This signal may serve as a source of energy for the rest of the circuit, or it may represent information. Information in the form of such voltage or current signals is processed by the circuit to produce new signals or new/different information.

1.2 CHARGE

One of the most fundamental concepts in electric circuit analysis is that of charge conservation. We know from basic physics that there are two types of charges : Positive and Negative. For the most part, this text is concerned with circuits in which only electron flow is relevant. We continuously transfer charges between different parts of a circuit, we do nothing to change the total amount of charge. In other words, we neither create nor destroy electrons (or protons) when they are running in the electric circuits. Charge in motion represents a current. The fundamental unit of charge is the Coulomb (C).

The following points should be noted about electric charge :

1. A single electron has a charge of -1.602×10^{-19} C and a single proton has a charge of $+1.602 \times 10^{-19}$ C.
2. Charge is quantized, i.e., the charge on a body will always be an integral multiple of the elementary charge.
3. A constant charge may be represented by either Q or q, but an amount of charge that changes over time must be represented by the lower case letter q.

EXAMPLE : 1.1 How much charge is represented by 4600 electrons?

Solution :

Each electron has -1.602×10^{-19} C.

Hence, 4600 electrons will have, -1.602×10^{-19} C/electron $\times$ 4600 electrons $= -7.369 \times 10^{-16}$ C

1.3 CURRENT

Electric current is the time rate of change of charge, measured in amperes (A). Mathematically, the relationship between current i, charge q, and time t is

$$i(t) = \frac{dq(t)}{dt} \qquad ...(1.1)$$

The net movement of 1 Coulomb (1C) of charge through a cross section of a conductor in 1 second (1s) produces an electric current of 1 ampere (1A).

The charge transferred between time t_0 and t_1 is obtained by integrating both sides of Equation (1.1). We gets

$$q(t) = \int_{t_0}^{t} i(t)\,dt \qquad ...(1.2)$$

(as t is variable of integration and the limit of integration and variable should not be same)

EXAMPLE : 1.2

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution :

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t\sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\,\pi t \cos 4\pi t) \text{ mA}$$

at $t = 0.5$, $\quad i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42$ mA

EXAMPLE : 1.3

Determine the total charge entering a terminal between $t = 1s$ and $t = 2s$ if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution :

$$Q = \int_{t=1}^{2} i\,dt = \int_{1}^{2}(3t^2 - t)\,dt = \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (8-2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

1.3.1 Reference Direction for Current

The direction of an electric current is by convention, i.e., the direction in which a positive charge would move. Thus, the current in the electric circuit is directed away from the positive terminal and towards the negative terminal of the battery.

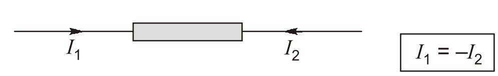
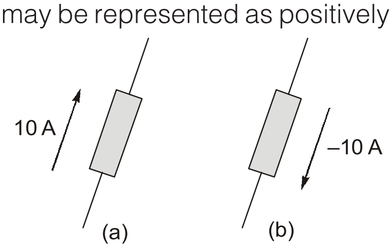


Fig. : Current in a circuit element

Electrons actually move through the wires in the opposite direction of current.

A current can be completely described by a value (which can be positive or negative) and a direction (indicated by arrow).

For example, a current of 10 A may be represented as positively or negatively as shown in below figure.



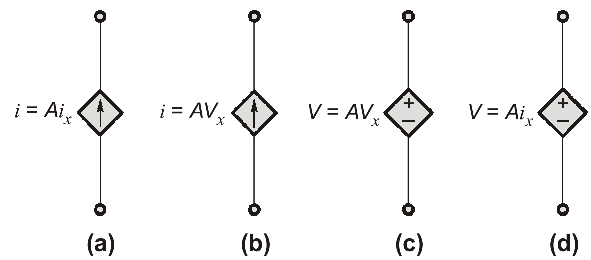


Fig.: *The four different types of dependent sources*
(a) A current-controlled current source (b) A voltage-controlled current source
(c) A voltage-controlled voltage source (d) A current-controlled voltage source

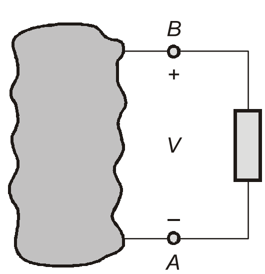
NOTE

- A different symbol, in the shape of a diamond, is used to represent dependent sources.
- Dependent sources are very useful in describing certain types of electronic circuits.
- A dependent source may absorb or supply power.

■■■■

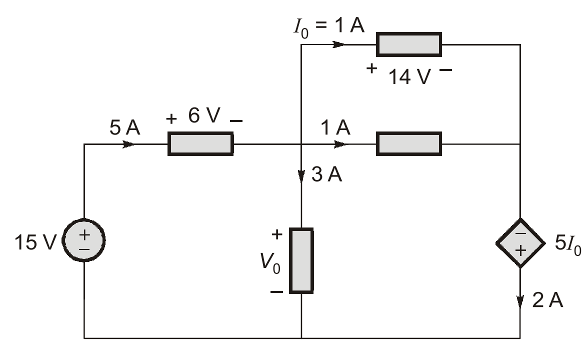
OBJECTIVE BRAIN TEASERS

Q.1 In the figure shown below, when 12 coulombs of charge passes through the element from A to B, the energy absorbed by the element is 60 J. The voltage V across the element is _____.



(a) 5 V
(b) –5 V
(c) 10 V
(d) –10 V

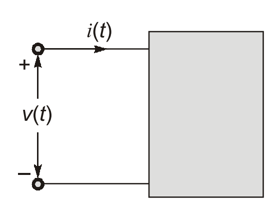
Q.2 In the circuit shown in figure, voltage V_0 is ______ Volts.



Q.3 For the circuit element shown in figure voltage and current are given as

$v(t) = 200e^{-50t}\sin 150t$ V and

$i(t) = 10e^{-50t}\sin 150t$ A



The power absorbed by the element at $t = 20$ ms (in watts) is _____.

Q.4 If $q = (10 - 10e^{-2t})$ mC, the current at $t = 0.5$s is _____ A.

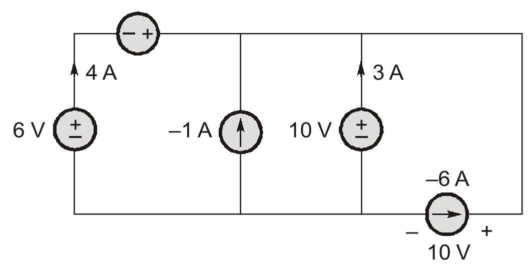
For maximum power,

$$\frac{dP}{dI_o} = 0$$

$$\Rightarrow \quad I_o = \sqrt{\frac{25}{3}} \text{ A}$$

$$\Rightarrow \quad I_o = 2.886 \text{ A} \qquad (\text{As } I_o > 0 \text{ A})$$

14. (b)



Power supplied by each source

6 V source :

$$P_{6\,V} = 6 \times 4 = 24 \text{ W}$$

4 V source :

$$P_{4\,V} = 4 \times 4 = 16 \text{ W}$$

–1 A source :

$$P_{-1\,V} = 10 \times -1 = -10 \text{ W}$$

(as voltage in parallel branches must be equal)

10 V source

$$P_{10\,V} = 10 \times 3 = 30 \text{ W}$$

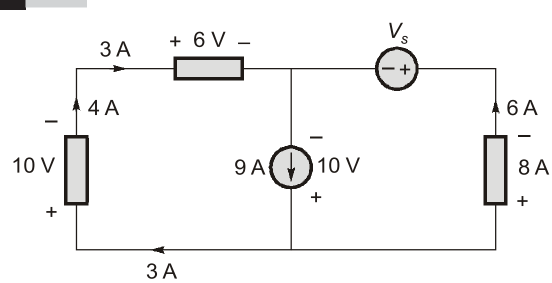
–6 A source

$$P_{-6\,A} = -6 \times 10 = -60 \text{ W}$$

So, $P_{1\,A} < 0$ and $P_{-6\,A} < 0$

Hence, –1 A and –6 A sources are being charged.

15. (a)



Let power supplied by V_s be P_s.

As the net power in a closed circuit,

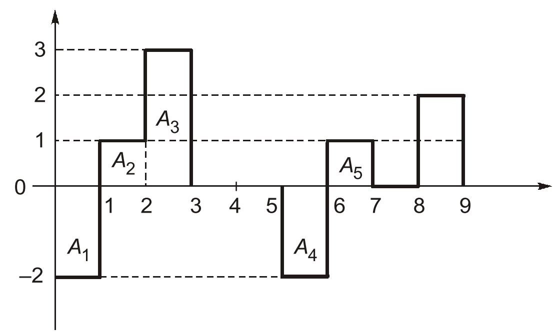
$$P_{net} = 0$$

$$\Rightarrow -10 \times 3 - 6 \times 3 + 16 \times 9 - P_s - 48 = 0$$

$$\Rightarrow \quad P_s = +48 \text{ W}$$

So, power supplied by Vs is 48 W.

16. (0)



Charge, $Q = A\int i\,dt$ = Area under the curve

for $0 < t < 6s$,

Q = Area under the curve for $t = 0$ to $t = 6s$

$$Q = A_1 + A_2 + A_3 + A_4$$

(A is area under the curve)

$$\Rightarrow \quad Q = -(2) \times 1 + 1 \times 1 + 3 \times 1 - 2 \times 1$$

$$\Rightarrow \quad Q = -4 + 4 = 0 \text{ C}$$

17. (0.5616)

Constant current,

$$I = 4 \text{ A}$$

Terminal voltage,

$$V(t) = (10 + 2t) \text{ V}$$

where t is hour.

Time, t = 3 hours

$$\text{Energy,} \quad E = \int_{t_1}^{t_2} V(t)I(t)dt$$

$$\Rightarrow \quad E = \int_0^3 (10+2t) \times 4dt = \int_0^3 (40+8t)dt$$

$$\Rightarrow \quad E = \left[40t + 4t^2\right]_0^3 = 120 + 36 = 156 \text{ Wh}$$

$$\Rightarrow \quad E = 156 \text{ Wh}$$

$$\Rightarrow \quad E = 156 \times 60 \times 60 \text{ Ws} = 156 \times 3600 \text{ J}$$

$$\Rightarrow \quad E = 561.6 \text{ J} = 0.5616 \text{ kJ}$$

■■■■

CHAPTER

2

Basic Laws

2.1 INTRODUCTION

The circuits studied in chapter 1 was interconnection of basis two-terminal circuit elements. A resistor and the various sources are two-terminal circuit elements. In this chapter, we discuss Kirchhoff's voltage law (KVL) and Kirchhoff current law (KCL), which govern the interaction of the voltage and current of interconnected circuit elements.

2.2 OHM'S LAW AND RESISTANCE

Ohm's law states that under constant temperature the apply voltage across conducting materials is directly proportional to the current flowing through the material, or

Mathematically, $V = IR$ ...(2.1)

where the constant of proportionality R is called the resistance. The unit of resistance is the ohm, which is 1 V/A and is denoted by capital omega, Ω.

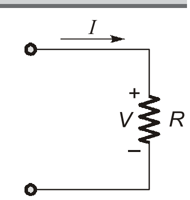


Fig. : Circuit symbol for resistance

EXAMPLE : 2.1 An electric iron draws 2 A at 120 V. Find its resistance.

Solution :

From Ohm's law, $R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$

where R is resistance of electric iron.

2.2.1 Resistance

Materials in general have a characteristic behaviour of resisting the flow of electric charge. This physical property, or ability to resist flow of current, is known as *resistance* and is represented by the symbol R. The resistance of any material with a uniform cross-sectional area A and its length l, as shown in adjoining fig. (a). We can represent resistance (as measured in the laboratory), in mathematical form,

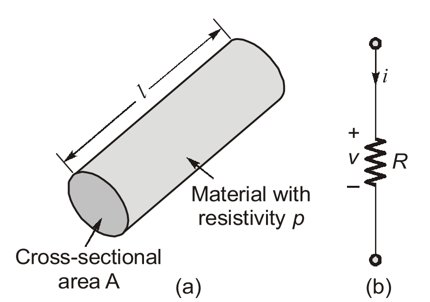


***Fig:** (a) Resistor (b) Circuit symbol for resistance*

$$R = \rho\frac{l}{A} \qquad ...(2.2)$$

where, ρ is known as the *resistivity* of the material in ohm-meters. The inverse of resistivity is called conductivity and is denoted by the symbol σ.

A resistor that obeys Ohm's law is known as a *linear* resistor. It has a constant resistance and thus its current-voltage characteristic is as illustrated in below fig. (c) its i-v graph is a straight line passing through the origin. A *non-linear* resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in below fig. (d). Examples of devices with non-linear resistance are the lightbulb and the diode. Although all practical resistors may exhibit non-linear behaviour under certain conditions, we will assume in this book that all elements actually designated as resistors are linear.

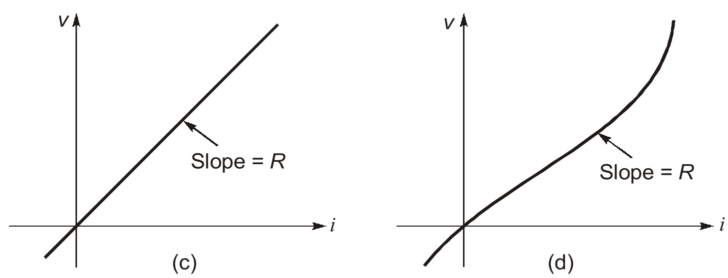


***Fig.:** The v-i characteristic of (a) A linear resistor (b) A non-linear resistor*

The conductance of a circuit element is defined as the inverse of its resistance. The symbol used to denote the conductance of an element is G, where

$$G = \frac{1}{R}$$

The unit of conductance is the Siemens denoted as S. The other notation are mho or inverted omega ℧.

2.2.2 Sign Convention

To apply Ohm's law as stated in equation (2.1), we must pay careful attention to the current direction and voltage polarity. The direction of current i and the polarity of voltage v must conform with the passive sign convention, as shown in above fig. (b). This implies that current flows from a higher potential to a lower potential in order for $v = iR$. If current flows from a lower potential to a higher potential, $v = -iR$.

2.2.3 Concept of Short Circuit and Open Circuit

Since the value of R can range from zero to infinity. So, we consider three cases:

Case-I: When $R = 0\ \Omega$ (Short circuit)

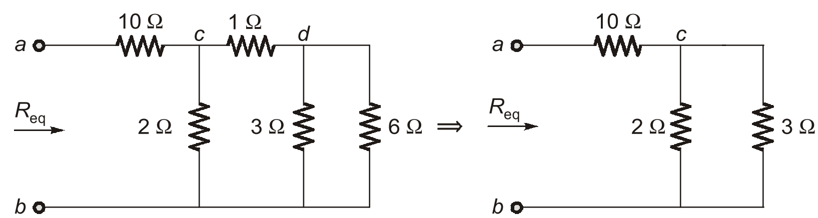
If $R = 0\ \Omega$

then, $G = \frac{1}{R} = \infty$ ⇒ V is zero
⇒ I can be anything (+ve or – ve)

Solution :

The 3 Ω and 6 Ω resistors are in parallel because they are connected to the same two nodes *c* and *b*. Their combined resistance is 2 Ω. Similarly, the 12 Ω and 4 Ω resistors are in parallel since they are connected to the same two nodes *d* and *b*. Hence their equivalent is 3 Ω. Also the 1 Ω and 5 Ω resistors are in series; hence, their equivalent resistance is 6 Ω.

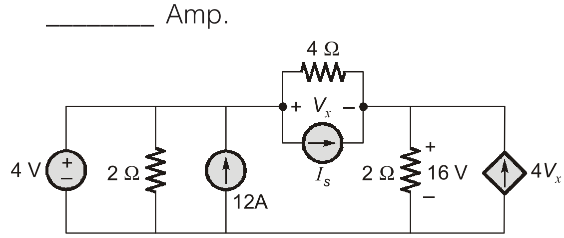
Thus, the above network can be reduced to



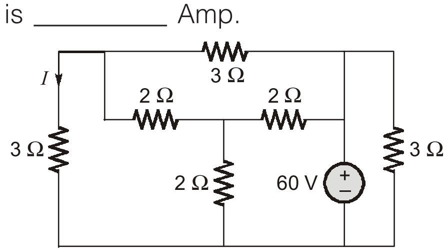
$$R_{eq} = 10 + 1.2 = 11.2\ \Omega$$

OBJECTIVE BRAIN TEASERS

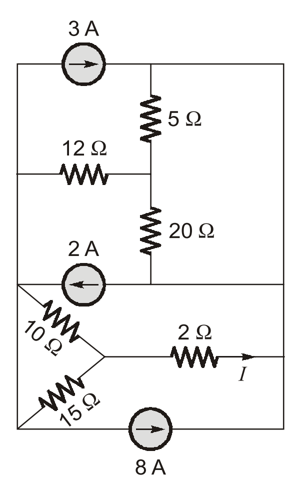
Q.1 The value of I_s in the following circuit is equal to ________ Amp.



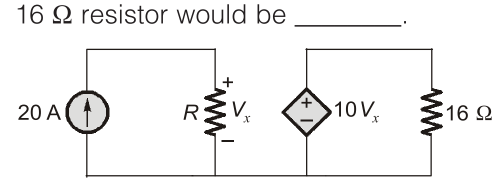
Q.2 In the circuit as shown in Figure, value of current I is __________ Amp.



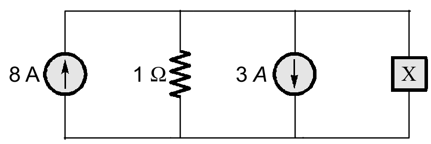
Q.3 For the given circuit, the value of current I equals to _______ Amp.



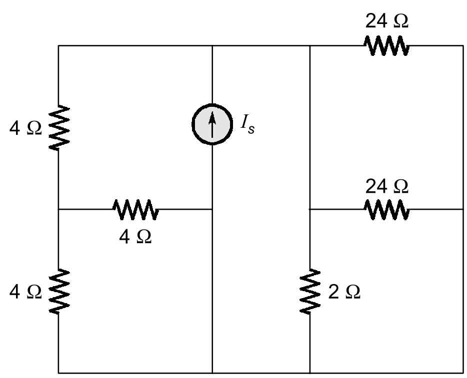
Q.4 In the following circuit, the value of R (in Ω) that is required to deliver a power of 160 kW to the 16 Ω resistor would be ________.



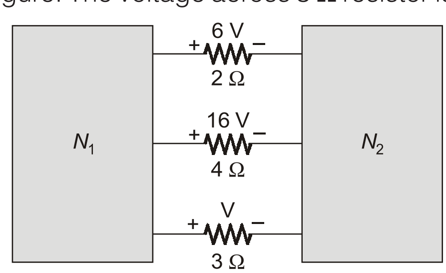
Q.5 Consider the circuit shown in figure below. Let X be a 4 Ω resistor then power absorbed by X is P_1 and when X be a 6 V independent voltage source with positive terminal at top, then power absorbed by X is P_2. The value of P_1/P_2 is _____.



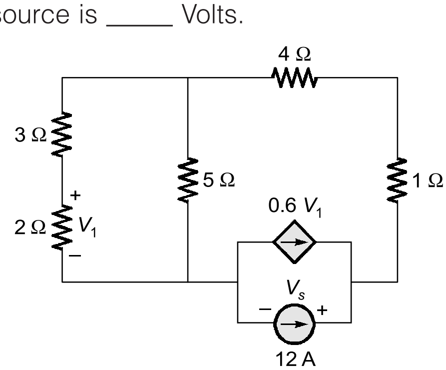
Q.6 In the following circuit, if the power dissipated by 2 Ω resistor is 32 W, then the value of current source I_s would be _____ Amp.



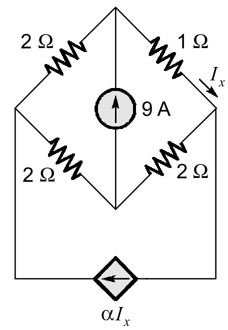
Q.7 The two electrical networks N_1 and N_2 are connected through three resistors as shown in figure. The voltage across 3 Ω resistor is ____ V.



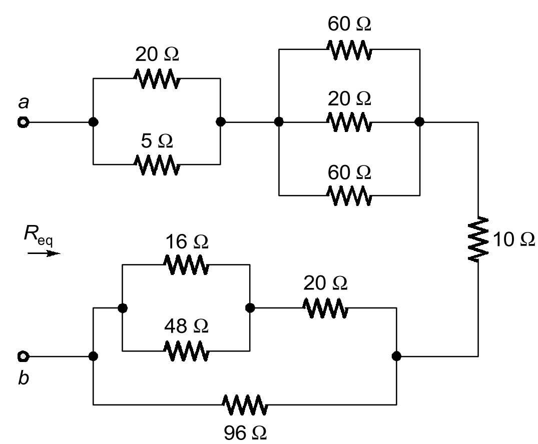
Q.8 In the given circuit voltage V_s across current source is _____ Volts.



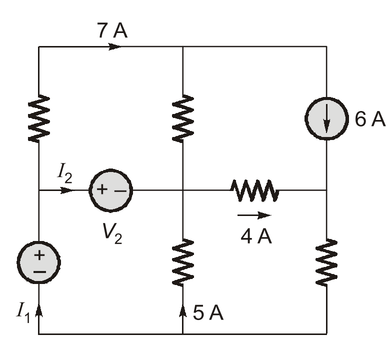
Q.9 The value of α in the network such that power supplied by 9 A source is 180 W will be equal to ____.



Q.10 The equivalent resistance R_{eq} (in Ω) looking into terminal *a-b* of the following network is _____.



Q.11 The values of current I_1 and I_2 in the circuit of following figure are respectively



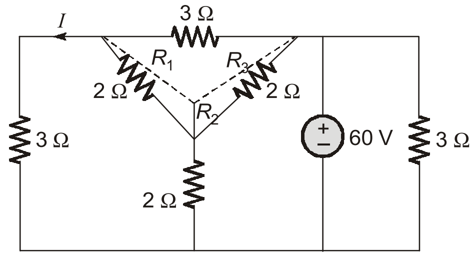
(a) 5 A, –2 A (b) 15 A, 22 A
(c) 5 A, 12 A (d) 17 A, 10 A

$\Rightarrow \quad I = 8 - 4 \times (-12) = 56\text{A}$

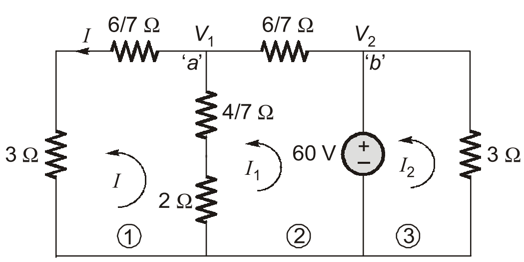
and, $\quad I = I_s + \dfrac{V_x}{4}$

$\Rightarrow \quad 56 = I_s + \dfrac{(-12)}{4} \quad \Rightarrow \quad I_s = 59 \text{ A}$

2. (10)



Δ-Y Converter



$$R_1 = \frac{3 \times 2}{3+2+2} = \frac{6}{7}\Omega$$

$$R_2 = \frac{2 \times 2}{3+2+2} = \frac{4}{7}\Omega \text{ and}$$

$$R_3 = \frac{3 \times 2}{3+2+2} = \frac{6}{7}\Omega$$

By KVL in loop 1

$$\left(3 + \frac{3}{7} + 2 + \frac{4}{7}\right)I - \left(\frac{4}{7} + 2\right)I_1 = 0$$

$\Rightarrow 45I - 18I_1 = 0$

$\Rightarrow \quad I = \dfrac{18}{45}I_1 = \dfrac{2}{5}I_1$

$\Rightarrow \quad I = 0.4I_1 \quad$ or $\quad I_1 = 2.5I \qquad$ ...(1)

By KVL in loop 2

$$\left(\frac{6}{7} + \frac{4}{7} + 2\right)I_1 - \left(\frac{4}{7} + 2\right)I = 60$$

$\Rightarrow \quad 60 = \dfrac{24}{7}I_1 - \dfrac{18}{7}I_0$

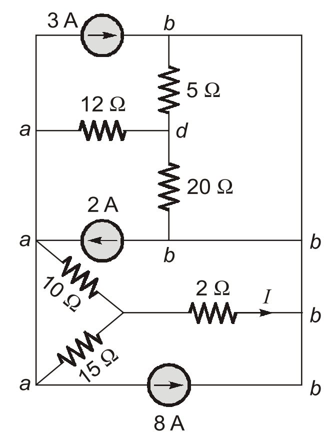
$= -18I + 24I_1 = 420 \qquad$ ...(2)

from (1) and (2) we get

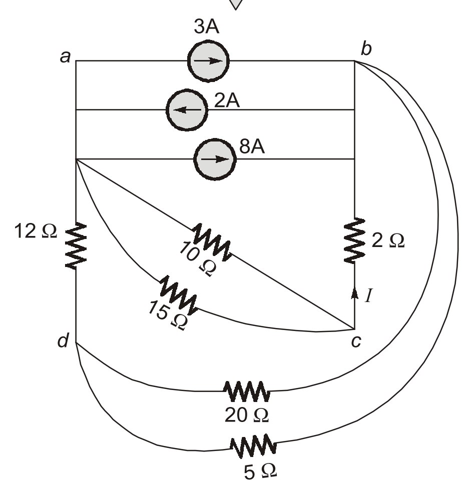
$-18I + (24 \times 2.5)I_0 = 420$

$\Rightarrow \quad 42I = 420 \quad \Rightarrow \quad I = 10$ A

3. (–6)



Circuit can be redrawn as



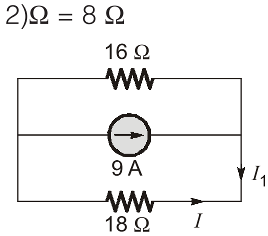
Circuit can be simplified as,

the current sources can be replaced by a single source of, 9 A

20 Ω || 5 Ω = 4 Ω and 4 Ω + 12 Ω = 16 Ω

and 10 Ω || 15 Ω = $\dfrac{150}{25}$ Ω = 6 Ω

and (6 + 2)Ω = 8 Ω



By current division,

$$I_1 = 9 \times \frac{16}{16+8} \Rightarrow I_1 = 6\text{A}$$

So, $\quad I = -I_1 \quad \Rightarrow \quad I = -6\text{ A}$

4. (8)

Power delivered to 16 Ω resistor,

$$P_{16\,\Omega} = 160\text{ kW}$$

We know that,

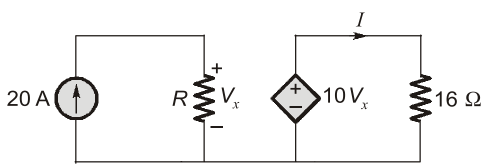
$$P_{16\,\Omega} = I^2 \times 16 = 160 \times 10^3$$

$\Rightarrow \quad I^2 = 10^4$

$\Rightarrow \quad I = \pm 100\text{ A}$

as, Power is delivered,

$$I = 100\text{ A}$$



[Considering current I as shown in figure]

and, also,

$$10\,V_x I = P_{16\,\Omega} = 160 \times 10^3$$

$$10^3\,V_x = 160 \times 10^3$$

$\Rightarrow \quad V_x = 160\text{ V}$

From Ohm's law,

$$V_x = 20R$$

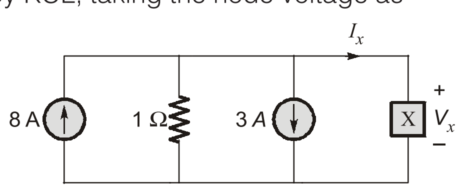
$\Rightarrow \quad 160 = 20 \times R$

$\Rightarrow \quad R = 8\ \Omega$

5. (–0.667)

For X be a 4 Ω resistor,

By KCL, taking the node voltage as



So, $\quad \dfrac{V_x}{4} + \dfrac{V_x}{1} + 3 - 8 = 0$

$\Rightarrow \quad \dfrac{5V_x}{4} = 5 \quad \Rightarrow \quad V_x = 4\text{ V}$

Power absorbed by 4 Ω resistor,

$$P_1 = \frac{V_x^2}{4} \quad \Rightarrow \quad P_1 = 4\text{ W}$$

for X be a 6 V independent voltage source

Let I_x be the current through element X, then,

By KCL, $\quad \dfrac{6}{1} + I_x = 5$

$$I_x = -1\text{ A}$$

So, power absorbed by 6 V source,

$$P_2 = +6 \times (-1)$$
$$= -6\text{ W}$$

So, $\quad \dfrac{P_1}{P_2} = \dfrac{-4}{6} = \dfrac{-2}{3}$

$\Rightarrow \quad \dfrac{P_1}{P_2} = -0.667$

6. (6)

Given: Power dissipated in 2 Ω resistor,

$$P_{2\,\Omega} = 32\text{ W}$$

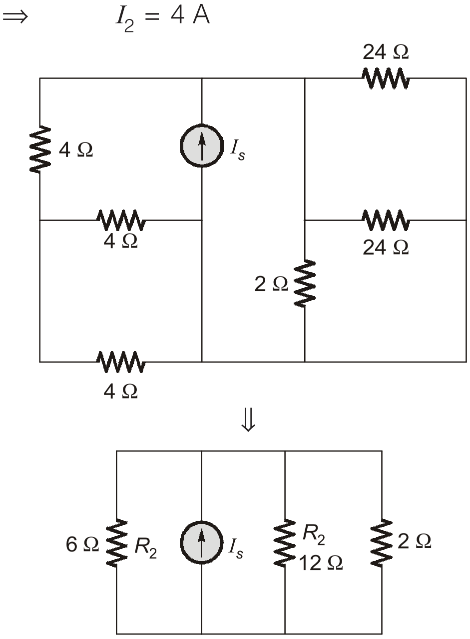
So, current in 2 Ω resistor,

$$I_2 = \sqrt{\frac{P_{2\Omega}}{2\,\Omega}}$$

or $\quad I_2^2 \times 2 = P_{2\Omega}$

$$I_2 = \sqrt{\frac{32}{2}} = 4\text{ A}$$

$\Rightarrow \quad I_2 = 4\text{ A}$



Simplified circuit

Here, 24 Ω || 24 Ω, i.e., it can be replaced by equivalent resistance of