



POSTAL BOOK PACKAGE 2024

ELECTRICAL ENGINEERING

..... CONVENTIONAL Practice Sets

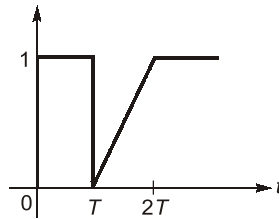
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SIGNALS & SYSTEMS

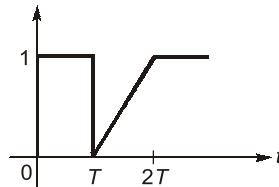
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Continuous Time Signal & System

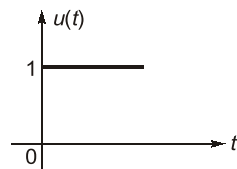
Q1 Write the equation of the function shown in the figure.



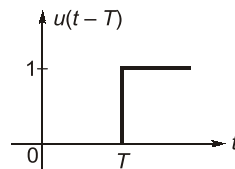
Solution:



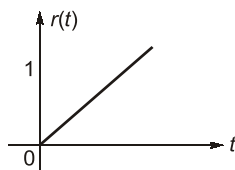
The given function can be realized as follows:



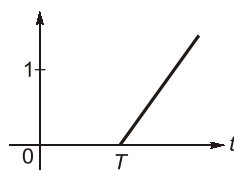
$u(t)$
(Unit step function)
(i)



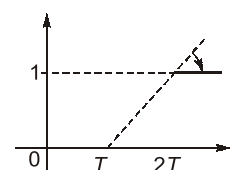
$u(t - T)$
(Unit step function right shifted)
(ii)



$r(t) = tu(t)$
(Ramp function)

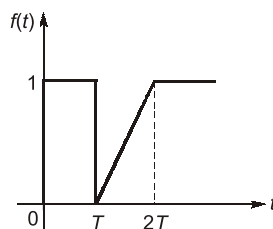


$\frac{(t - T)}{T} u(t - T)$
(Right shifted unit ramp function)
(iii)



$\frac{(t - 2T)}{T} u(t - 2T)$
(iv)

Combining the functions (i), (ii), (iii) and (iv), we get the given function



Therefore,

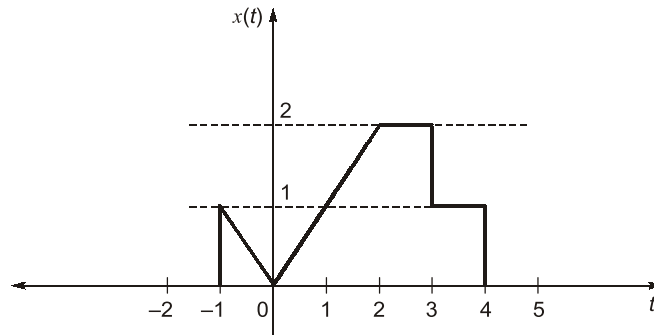
$$f(t) = u(t) - u(t - T) + \frac{(t - T)}{T} u(t - T) - u(t - T) - \frac{(t - 2T)}{T} u(t - 2T)$$

$$f(t) = u(t) - u(t - T) + \frac{1}{T} r(t - T) - \frac{1}{T} r(t - 2T)$$

or

$$f(t) = u(t) - u(t - T) + \frac{1}{T}(t - T) u(t - T) - \frac{1}{T}(t - 2T)u(t - 2T)$$

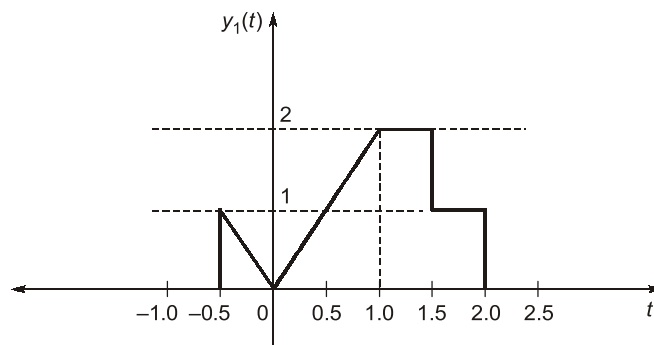
Q2 For the given signal $x(t)$ as shown below, sketch the following signals.



(a) $y_1(t) = x(2t)$ (b) $y_2(t) = x(2t + 4)$ (c) $y_3(t) = x\left(\frac{t}{2} + 2\right)$

Solution:

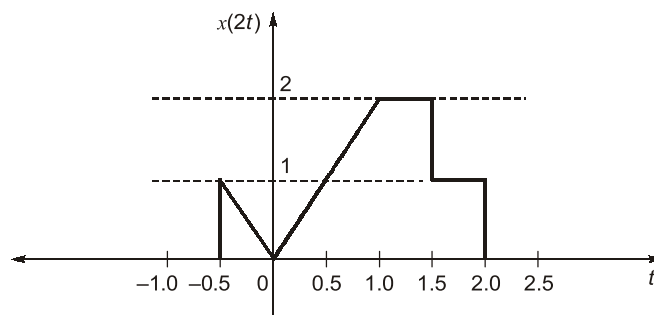
(a) We have to sketch, $y_1(t) = x(2t)$ since, $y_1(t)$ is a 2 times slowed or compressed version of $x(t)$ in time domain. So, the curve is;



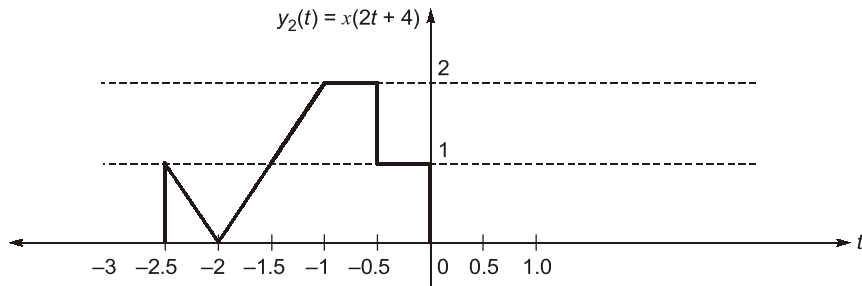
(b) We have to sketch, $y_2(t) = x(2t + 4)$ i.e. $y_2(t) = x[2(t + 2)]$.

So, we can say $y_2(t)$ is the 2 times compressed version in time of a signal which is an advance shift of 2 unit of $x(t)$.

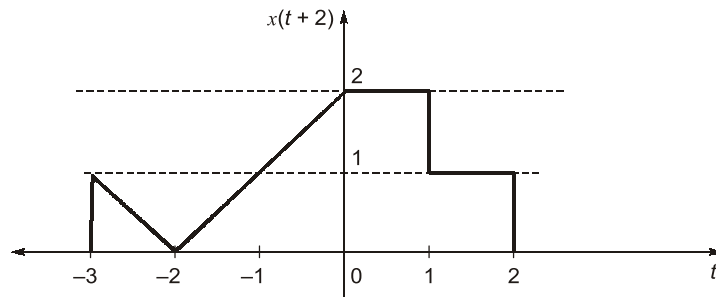
At first we sketch the following:



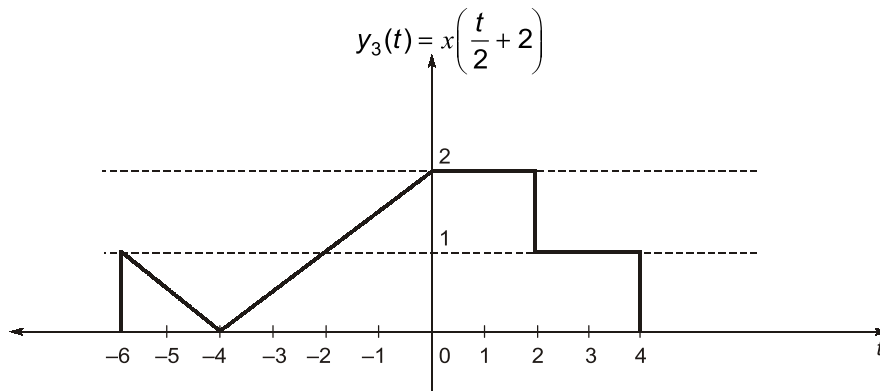
Again, we have to sketch this $x(2t)$ for an advance shift of 2 units means shift the above curve 2 unit in left side as below:



- (c) We have to sketch, $y_3(t) = x(t/2 + 2)$. For this firstly, we shift 2 unit in advance shift of $x(t)$ and then expanded this signal $x(t + 2)$, by $1/(1/2) = 2$ units of the signal.



Now finally we have to sketch, $y_3(t) = x\left(\frac{t}{2} + 2\right)$ as below:



- Q3** For the signals $y_1(t)$ and $y_2(t)$ shown below. Draw the differentiation of the signals and find the equations of differentiated signals.

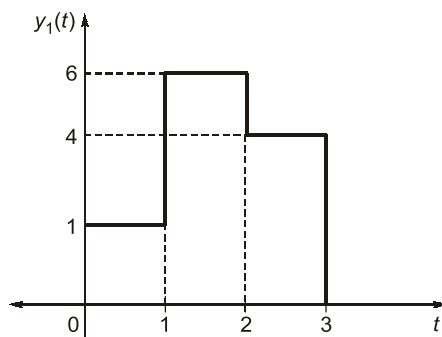


Fig. (a)

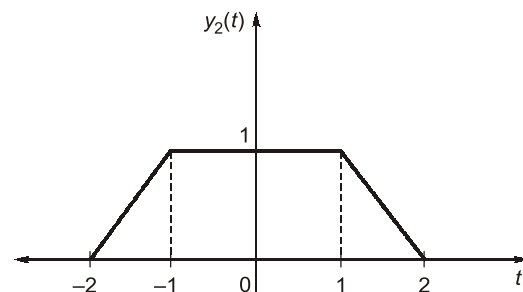


Fig. (b)

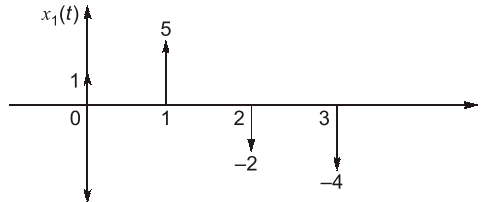
Solution:

For figure (a), we have to differentiate the signal $y_1(t)$.

Let, $\frac{dy_1(t)}{dt} = x_1(t)$... (i)

Given, $y_1(t) = u(t) + 5u(t-1) - 2u(t-2) - 4u(t-3)$

So, from equation (i), $x_1(t) = \frac{dy_1(t)}{dt} = \delta(t) + 5\delta(t-1) - 2\delta(t-2) - 4\delta(t-3)$

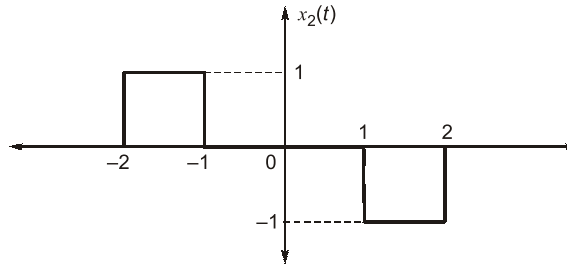


For figure (b), we have to differentiate the signal $y_2(t)$.

Let, $\frac{dy_2(t)}{dt} = x_2(t)$... (ii)

Given, $y_2(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$
(where $r(t)$ represents the ramp function)

So, from equation (ii), $x_2(t) = \frac{dy_2(t)}{dt} = u(t+2) - u(t+1) - u(t-1) + u(t-2)$



Q4 Show that the signal, $S(t) = t^{-1/4} u(t-1)$ is neither an energy nor a power signal.

Solution:

For an arbitrary continuous-time signal $s(t)$, the normalized energy content ' E ' of $S(t)$ is defined as,

$$E = \int_{-\infty}^{\infty} |S(t)|^2 dt \quad \dots (i)$$

or

$$E = \int_{-\infty}^{\infty} |t^{-1/4} u(t-1)|^2 dt$$

Since,

$$u(t-1) = \begin{cases} 1, & t > 1 \\ 0, & t < 1 \end{cases}$$

\therefore

$$E = \int_1^{\infty} |t^{-1/4}|^2 dt = \int_1^{\infty} [t]^{-1/2} dt = [2[t]^{1/2}]_1^{\infty}$$

\therefore

$$E = \infty$$

Now, the normalized average power ' P ' of $S(t)$ is defined as,

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |S(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |t^{-1/4} u(t-1)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_1^T (t^{-1/4})^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_1^T t^{-1/2} dt$$

$$\therefore P = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[2t^{1/2} \right]_1^T = \lim_{T \rightarrow \infty} \left[\frac{T^{1/2} - 1}{T} \right] = \lim_{T \rightarrow \infty} \left[\frac{1 - 1/\sqrt{T}}{\sqrt{T}} \right] = 0$$

$$\therefore P = 0$$

Here average power $P \rightarrow 0$, when total energy $E \rightarrow \infty$, which means that the condition $0 < E < \infty$ is not satisfied. Hence signal, $S(t) = t^{-1/4} u(t-1)$ is not an energy signal. Also, when $E \rightarrow \infty$, the value of $P \rightarrow 0$ which means that the condition $0 < P < \infty$ is not satisfied. Therefore, signal $S(t) = t^{-1/4} u(t-1)$ is not a power signal.

Thus, we can say that the given signal, $S(t) = t^{-1/4} u(t-1)$ is neither an energy signal nor a power signal.

Proved.

Q5 Consider a continuous-time system with input $x(t)$ and output $y(t)$ given by

$$y(t) = x(t) \cos(t)$$

Check whether the is

- (a) linear (b) time-invariant

Solution:

$$y(t) = x(t) \cos(t)$$

(a) To check linearity,

$$y_1(t) = x_1(t) \cos(t)$$

$[y_1(t)$ is output for $x_1(t)$]

$$y_2(t) = x_2(t) \cos(t)$$

$[y_2(t)$ is output for $x_2(t)$]

So the output for $(x_1(t) + x_2(t))$ will be

$$\begin{aligned} y(t) &= [x_1(t) + x_2(t)] \cos(t) \\ &= y_1(t) + y_2(t) \end{aligned}$$

So the system is linear to check time invariance.

(b) The delayed output, $y(t - t_0) = x(t - t_0) \cos(t - t_0)$

The output for delayed input,

$$y(t, t_0) = x(t - t_0) \cos(t)$$

Since,

$$y(t - t_0) \neq y(t, t_0)$$

System is time varying.

Q6 Find out whether the system is stable/causal. If the impulse response is given by, $h(t) = e^{-6|t|}$.

Solution:

As given that,

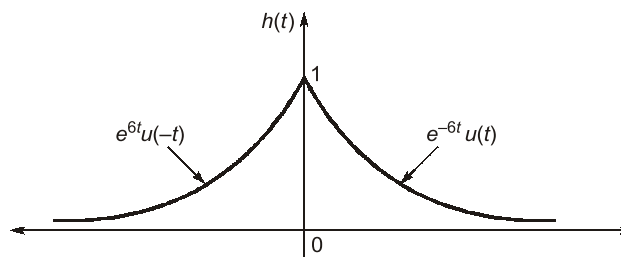
$$h(t) = e^{-6|t|}$$

\therefore

$$h(t) = e^{-6t} \cdot u(t) + e^{6t} \cdot u(-t)$$

...(i)

Here we see that, $h(t) \neq 0$ for $t < 0$ so, the given system is not **causal**.



For checking the system to be “BIBO stable”, we know that,

$$\sum_{t=-\infty}^{\infty} h(\tau) d\tau < \infty$$

...(ii)

$$\text{L.H.S of equation (ii)} = \sum_{t=-\infty}^{\infty} h(\tau) d\tau$$

$$= \sum_{t=-\infty}^{\infty} \left[e^{-6\tau} u(t) d\tau + e^{6\tau} u(-\tau) d\tau \right] = \int_{t=0}^{\infty} e^{-6\tau} d\tau + \int_{-\infty}^0 e^{6\tau} d\tau$$

$$= -\frac{1}{6} \left[e^{-6\tau} \right]_0^{\infty} + \frac{1}{6} \left[e^{6\tau} \right]_{-\infty}^0 = -\frac{1}{6}(0-1) + \frac{1}{6}(1-0)$$

$$\text{L.H.S. of equation (ii)} = \frac{1}{3} < \infty$$

So, the system is "BIBO Stable".

Q7 A continuous time LTI system is described by:

$$y(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau) d\tau$$

Find the impulse response of the system. Is the system casual?

Solution:

1st Method:

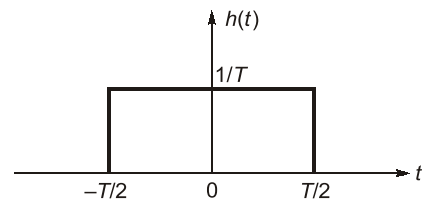
$$y(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau) d\tau \quad \dots(i)$$

Let the impulse response be $h(t)$,

$$\therefore y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \quad (ii)$$

Comparing equation (i) and (ii) we get,

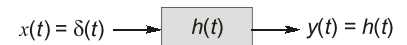
$$h(t) = \begin{cases} \frac{1}{T} & -T/2 < t < T/2 \\ 0 & \text{Otherwise} \end{cases}$$



2nd method:

$$\therefore h(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} \delta(\tau) d\tau$$

$$\Rightarrow h(t) = u\left(t + \frac{T}{2}\right) - u\left(t - \frac{T}{2}\right)$$



The system is not causal as we can clearly see from equation (i) that for calculation of $y(t)$ at any time t , we require future values of input $x(t)$.

Another way to see this is that $h(t)$ is not zero for $t < 0$, which is the basic requirement for any causal system.

Q8 Show that the following properties holds good for the derivative of $\delta(t)$.

$$(a) \int_{-\infty}^{\infty} \phi(t) \delta'(t) dt = -\phi'(0)$$

$$\text{where, } \phi'(0) = \left. \frac{d\phi(t)}{dt} \right|_{t=0}$$

$$(b) t\delta'(t) = -\delta(t)$$

Solution:

(a) We know that,
$$\int_{-\infty}^{\infty} \phi(t) g^n(t) dt = (-1)^n \int_{-\infty}^{\infty} \phi^n(t) g(t) dt \quad \dots \left(\phi^n(t) = \frac{d^n}{dt} \phi(t) \right) \quad \dots(i)$$

where, $g(t)$ is the generalized function and $\phi(t)$ is the testing function.

Let, first derivative of $\delta(t) = d'(t)$ and consider, $g(t) = \delta(t)$ and $n = 1$

Putting these values in equation (i) we get,

$$\int_{-\infty}^{\infty} \phi(t) \delta^1(t) dt = (-1)^1 \int_{-\infty}^{\infty} \phi^1(t) \delta(t) dt = - \int_{-\infty}^{\infty} \frac{d\phi(t)}{dt} \cdot \underbrace{\delta(t) dt}_{\text{defined for } t=0} = - \frac{d\phi(t)}{dt} \Big|_{t=0}$$

$$\therefore \int_{-\infty}^{\infty} \phi(t) \delta'(t) dt = \phi'(0) \quad \text{Proved.}$$

(b) Let $g(t) = t\delta'(t)$ then, from equation (i) we have,

$$\begin{aligned} \int_{-\infty}^{\infty} \phi(t) [t \delta'(t)] dt &= \int_{-\infty}^{\infty} [t \phi(t)] \delta'(t) dt \\ &= - \frac{d}{dt} [t \phi(t)] \Big|_{t=0} = - [t \phi'(t) + \phi(t)] \Big|_{t=0} \end{aligned}$$

$$\therefore \int_{-\infty}^{\infty} \phi(t) [t \delta'(t)] dt = [\phi(0)] \quad \dots(ii)$$

Again consider,
$$\int_{-\infty}^{\infty} t \phi(t) \delta'(t) dt = \int_{-\infty}^{\infty} \phi'(t) \delta(t) dt$$

$$\Rightarrow \int_{-\infty}^{\infty} \phi(t) [t \delta'(t)] dt = \int_{-\infty}^{\infty} \phi(t) [-\delta(t)] dt \quad \dots(iii)$$

Thus, from the equivalence property we conclude that,

$$t\delta'(t) = -\delta(t) \quad \text{Proved.}$$

Q.9 A system has impulse response e^{-at} . What would be the response of the system, if it is excited by a delayed unit step function (delay = T)?

Solution:

$$h(t) = e^{-at}$$

$$x(t) = u(t - T)$$

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau = \int_{-\infty}^{\infty} u(\tau - T) \cdot e^{-a(t - \tau)} d\tau$$

$$= \int_T^{\infty} e^{-at + a\tau} \cdot d\tau = e^{-at} \int_T^{\infty} e^{a\tau} d\tau = \frac{e^{-at}}{a} [e^{a(\infty)} - e^{aT}]$$

Case-I:

$$a < 0$$

$$\therefore y(t) = - \frac{e^{-at} \cdot e^{aT}}{a} = - \frac{e^{a(T-t)}}{a}$$

Case-II:

$$a > 0$$

$y(t)$ becomes unbounded in this case.