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ELECTRICAL ENGINEERING

CONVENTIONAL Practice Sets

CONTENTS

ELECTROMAGNETIC THEORY

1 CHAPTER Vector Analysis

Q.1 Given point, $P(9, -12, 15)$ is in Cartesian system. Express P in cylindrical and spherical systems.

Solution

$$\rho = \sqrt{x^2+y^2} = \sqrt{(9)^2+(-12)^2} = 15$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\left[\frac{(-12)}{9}\right] = -53.13°$$

P is cylindrical format will be as,

$$P = (15, -53.13°, 15)$$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{(9)^2+(-12)^2+(15)^2} = 21.213$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = \tan^{-1}\left[\frac{15}{15}\right] = 45°$$

P in spherical format will be as,

$$P = (21.213, 45°, -53.13°)$$

Q.2 Let $\vec{H} = 5\rho\sin\phi\, a_\rho - \rho z\cos\phi\hat{a}_\phi + 2\rho\hat{a}_z$ A/m. At point $P(2, 30°, -1)$ find:

(a) a unit vector along $\vec{H}$.

(b) the component of $\vec{H}$ parallel $\hat{a}_x$.

(c) the component of $\vec{H}$ normal to $\rho = 2$.

(d) the component of $\vec{H}$ tangential to $\phi = 30°$.

Solution:

At P,

$$\rho = 2,\ \phi = 30°,\ z = -1$$

$$\vec{H} = 10\sin 30°\,\hat{a}_\rho + 2\cos 30°\hat{a}_\phi - 4\hat{a}_z$$

$$= 5\hat{a}_\rho + 1.732\,\hat{a}_\phi - 4\hat{a}_z \text{ A/m}$$

(a) Unit vector along $\vec{H}$,

$$\hat{a}_H = \frac{5\hat{a}_\rho + 1.732\,\hat{a}_\phi - 4\hat{a}_z}{\sqrt{5^2+1.732^2+4^2}} = 0.7538\,\hat{a}_\rho + 0.2611\hat{a}_\phi - 0.603\hat{a}_z$$

(b)

$$H_x = H_\rho\cos\phi - H_\phi\sin\phi$$

$$= 5\rho\sin\phi\cos\phi - \rho z\cos\phi\sin\phi$$

or, P at

$$\rho = 2,\quad \phi = 30°,\ z = -1$$

$$H_x = H_x\,\hat{a}_x = (10\sin 30°\cos 30° + 2\sin 30°\cos 30°)\,\hat{a}_x$$

$$= 5.196\,\hat{a}_x \text{ A/m}$$

(c) Normal to $P = 2$ is $\vec{H}_\rho = \vec{H}_\rho\, \hat{a}_\rho$

i.e. $\vec{H}_n = 0.7538\, \hat{a}_\rho$ A/m

(d) Tangential to $\phi = 30°$

$$H_t = H_\rho\, \hat{a}_\rho + H_z\, \hat{a}_z$$

$$= 0.7538\, \hat{a}_\rho - 0.603\, \hat{a}_z \text{ A/m}$$

Q.3 E and F are vector fields given by $\vec{E} = 2x\,\hat{a}_x + \hat{a}_y + yz\hat{a}_z$ and $\vec{F} = xy\,\hat{a}_x - y^2\hat{a}_y + xyz\hat{a}_z$. Determine:

(a) $|E|$ at (1, 2, 3)

(b) The component of $\vec{E}$ along $\vec{F}$ at (1, 2, 3)

(c) A vector perpendicular to both $\vec{E}$ and $\vec{F}$ at (0, 1, –3) whose magnitude is unity.

Solution:

(a) $\vec{E} = 2x\,\hat{a}_x + \hat{a}_y + yz\,\hat{a}_z$

At point (1, 2, 3) $\Rightarrow$ $\vec{E} = 2\hat{a}_x + \hat{a}_y + 6\hat{a}_z$

$$|\vec{E}| = \sqrt{2^2 + 1^2 + 6^2} = \sqrt{41} = 6.403$$

(b) $\vec{F} = xy\,\hat{a}_x - y^2\,\hat{a}_x + xyz\,\hat{a}_z$

At (1, 2, 3), $\vec{F} = 2\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z$

$\therefore$ The component of $\vec{E}$ along $\vec{F}$

$$\vec{E}_F = (\vec{E}\cdot a_F)\hat{a}_F$$

$$= \frac{(\vec{E}\cdot\vec{F})}{|\vec{F}|}\vec{a}_F = \frac{36}{56}(2\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z) = 1.286\hat{a}_x - 2.571\hat{a}_y + 3.857\hat{a}_z$$

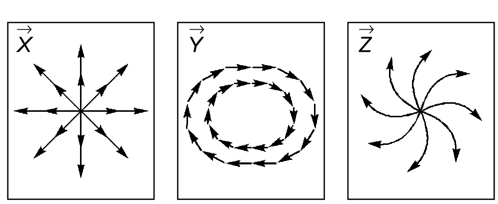
(c) At (0, 1, –3) $\vec{E} = 0\hat{a}_x + \hat{a}_y - 3\hat{a}_z$

$$\vec{F} = 0\hat{a}_x - \hat{a}_y + 0\hat{a}_z$$

$$E \times \vec{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = -3\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z$$

$$a_{E\times F} = \pm\frac{\vec{E}\times\vec{F}}{|\vec{E}\times\vec{F}|} = \pm\,\hat{a}_x$$

Q.4 The figures show diagrammatic representations of vector fields $\vec{X}$, $\vec{Y}$ and $\vec{Z}$, respectively. What can you comment about these diagrams?



Solution:

$\vec{X}$ is going away so $\vec{\nabla} \cdot \vec{X} \neq 0$.

$\vec{Y}$ is moving circulator direction so $\vec{\nabla} \times \vec{Y} \neq 0$.

$\vec{Z}$ has circular rotation so $\vec{\nabla} \times \vec{Z} \neq 0$.

Q.5 Find the divergence of vector field, $\vec{V}(x,y,z) = -(x \cos xy + x)\,\hat{i} + (y \cos xy)\,\hat{j} + (\sin z^2 + x^2 + y^2)\,\hat{k}$.

Solution:

$$\vec{V}(x, y, z) = -(x\cos xy + y)\hat{i} + (y\cos xy)\hat{j} + [\sin(z)^2 + (x^2) + (y^2)]\hat{k}$$

$$\text{Divergence} = \nabla \cdot V$$

$$= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

$$= -\cos xy + x(\sin xy)y + \cos xy - y\sin(xy)x + 2z\cos z^2$$

$$= 2z\cos z^2$$

Q.6 "A hand 'curl meter' in the form of a pin wheel is used to indicate curl of a vector field." Justify the statement.

Solution:

A pin or paddle wheel as a 'curl meter': The force is exerted on the each blade of the paddle wheel, the force being proportional to the component of the field normal to the surface of that blade. To test and field for curl we dip one paddle wheel into the field with the axis of the paddle wheel lined up with the direction of the component of curl desired. No rotation means no curl, larger angular velocities mean greater values of the curl a reverse in the direction of spin means a reversal in the sign of the curl. In order find the direction of vector curl, we should place one paddle wheel in the field and hunt around for the orientation which produces the greatest torque. The direction of the curl is then along the axis of the paddle wheel.

Q.7 Consider a function $\vec{f} = \frac{1}{r^2}\hat{r}$, where r is the direction from origin and $\hat{r}$ is unit vector in radial direction.

Find divergence of this function over a sphere of radius R, which includes origin.

Solution:

$$\vec{f} = \frac{1}{r^2} \cdot \hat{r}$$

From divergence theorem as we know,

$$\boxed{\int_{\text{vol.}} (\nabla \cdot \vec{f})\, dV = \oiint_S \vec{f} \cdot d\vec{S}}$$

$$\oiint_S \vec{f} \cdot d\vec{S} = \oiint_S \left(\frac{1}{r^2} \cdot \hat{r}\right) \cdot \left(r^2 \sin\theta \cdot d\theta \cdot d\phi \cdot \hat{r}\right)$$

$$= \oiint_S \sin\theta \cdot d\theta \cdot d\phi = 4\pi$$

Q.8 For a vector field $\vec{A}$, show explicitly that $\nabla \cdot \nabla \times \vec{A} = 0$; that is, the divergence of the curl of any vector field is zero.

Solution:

To shows $\vec{\nabla}\cdot\vec{\nabla}\times\vec{A}=0$, let us assume that $\vec{A}$ is a function of x, y, z then,

$$\vec{\nabla}\cdot\vec{\nabla}\times\vec{A} = \vec{\nabla}\cdot\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}$$

$$= \vec{\nabla}\cdot\left[\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\hat{x}-\left(\frac{\partial A_z}{\partial x}-\frac{\partial A_x}{\partial z}\right)\hat{y}+\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\hat{z}\right]$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)-\frac{\partial}{\partial y}\left(\frac{\partial A_z}{\partial x}-\frac{\partial A_x}{\partial z}\right)+\frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)$$

$$= \frac{\partial^2 A_z}{\partial x\partial y}-\frac{\partial^2 A_y}{\partial x\partial z}-\frac{\partial^2 A_z}{\partial y\partial x}+\frac{\partial^2 A_x}{\partial y\partial z}+\frac{\partial^2 A_y}{\partial z\partial x}-\frac{\partial^2 A_x}{\partial z\partial y}$$

$$= 0$$

So it is proved that divergence of curl of any vector field is always 0.

Q.9 Find the rate at which the scalar function, $V = r^2 \sin 2\phi$, in cylindrical co-ordinates, increases in the direction of the vector $\vec{A} = \hat{a}_r + \hat{a}_\phi$ at the point having co-ordinates (2, π/4, 0).

Solution:

As we know that, Gradient is a vector that represents both the magnitude and the direction of maximum space rate of the increase of the scalar function i.e.

$$\text{grad } V = \nabla V = \frac{dV}{dn}\hat{a}_n \qquad \text{...(i)}$$

But in cylindrical coordinate system, the grad V can be defined as,

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z \qquad \text{...(ii)}$$

For the case under consideration, the quantity required is,

$$\nabla V\cdot\hat{a}_A = \nabla V\cdot\frac{A}{|A|} \quad \text{at } (2, \pi/4, 0)$$

From equation (ii) we have,

$$\nabla V = \frac{\partial}{\partial r}(r^2 \sin 2\phi)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \phi}(r^2 \sin 2\phi)\hat{a}_\phi + \frac{\partial}{\partial z}(r^2 \sin 2\phi)\hat{a}_z.$$

or, $$\nabla V = 2r\sin 2\phi\ \hat{a}_r + 2r\cos 2\phi\ \hat{a}_\phi$$

Now, $$\nabla V\cdot A = (2r\sin 2\phi\ \hat{a}_r + 2r\cos 2\phi\ \hat{a}_\phi)\cdot(\hat{a}_r+\hat{a}_\phi)$$

or, $$\nabla V\cdot A = 2r\sin 2\phi + 2r\cos 2\phi \qquad \text{...(iii)}$$

Also, $$|\vec{A}| = \sqrt{1^2+1^2} = \sqrt{2}$$

$\therefore$ $$\nabla V\cdot\hat{a}_A = \frac{\nabla V\cdot A}{|A|} = \frac{1}{\sqrt{2}}[2r\sin 2\phi + 2r\cos 2\phi]$$

$$= (\sqrt{2}\,r\sin 2\phi + \sqrt{2}\,r\cos 2\phi)$$

Now, $$(\nabla V\cdot\hat{a}_A)_{\text{at}(2,\pi/4,0)} = \sqrt{2}\times 2\times\sin 2\times\frac{\pi}{4}+\sqrt{2}\times 2\times\cos 2\times\frac{\pi}{4} = 2\sqrt{2}$$