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POSTAL BOOK PACKAGE 2024

ELECTRICAL ENGINEERING

CONVENTIONAL Practice Sets

CONTENTS

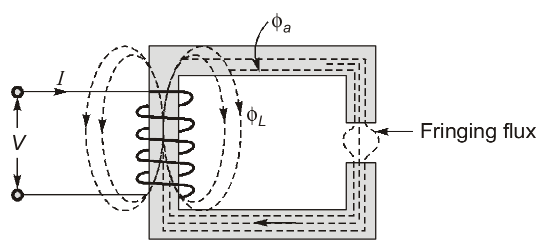
ELECTRIC MACHINES

1 CHAPTER Magnetic Circuits, Electromechanical Energy Conversion

Q.1 Explain the difference between fringing flux and leakage flux.

Solution:

Consider the magnetic circuit given below:



Of the total flux generated by the coil, some flux ϕ_L does not follow the intended path of the magnetic circuit. This flux cannot be used for any purpose. This flux is called as the leakage flux.

When the flux enters the airgap, it acquires a bulging shape. The amount of the bulging is direction proportional to the length of the airgap. Bulging increases, the effective area of the airgap and reduces flux density in it. This effect is called as fringing and the flux in the bulge is called as fringing flux.

Q.2 The flux linkage (λ) and circuit (i) is related as $\psi = (\sqrt{i})/g$. When i = 2 A and g (airgap length) = 10 cm, then determine the magnitude of mechanical force on the moving part.

Solution:

Energy in magnetic system,

$$W_f = \int_0^{\psi} i(\psi)\, d\psi$$

$$\psi = (\sqrt{i})/g \qquad \text{(Given)}$$

$$W_f = \int_0^{\psi} \psi^2 \cdot g^2 \cdot d\psi = g^2 \frac{\psi^3}{3}$$

Now, mechanical force,
$$F_f = -\frac{\partial W_f(\psi, g)}{\partial g}$$

$$F_f = -\frac{\partial}{\partial g}\left[\frac{g^2\psi^3}{3}\right] = -\frac{2}{3}\psi^3 g$$

$\because$
$$i = 2\text{ A},\ g = 10\text{ cm},\ \psi = 10\sqrt{2}$$

Hence,
$$|F_f| = \frac{2}{3}\times(10\sqrt{2})^3 \times 0.1 = 188.56\text{ N}$$

Q.3 In an electromagnetic relay, functional relation between the current in the exciting coil, the position of armature x and the flux linkage Ψ is given by

$$i = 2\Psi^3 + 3\Psi(1 - x + x^2), \quad x > 0.5$$

Find the force on the armature as a function of Ψ.

Solution:

$$i = 2\Psi^3 + 3\Psi\,(1 - x + x^2)$$

Field energy stored,

$$W_f(\Psi, x) = \int_0^{\Psi} i(\psi)\,d\Psi = \int_0^{\Psi} [2\Psi^3 + 3\Psi(1 - x + x^2)]\,d\Psi = \frac{2\Psi^4}{4} + 3\frac{\Psi^2}{4}(1 - x + x^2)$$

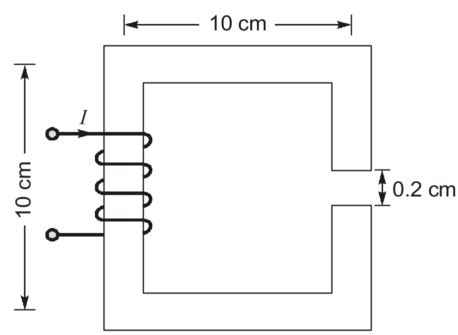
Magnetic force is given by,

$$\therefore \qquad f_e = \frac{\partial W_f(\Psi, x)}{dx} = -\frac{\partial}{x}\left[\frac{\Psi^4}{2} + \frac{3\Psi^2}{2}(1 - x + x^2)\right]$$

$$= -\left[0 + \frac{3\Psi^2}{2}(0 - 1 + 2x)\right] = \frac{3\Psi^2}{2}(1 - 2x)$$

For $x > 0.5$, f_e is negative, therefore f_e acts to decrease the field energy stored at constant flux linkage.

Q.4 The magnetic circuit shown below has uniform cross-sectional area and air gap of 0.2 cm. The mean path length of the core is 40 cm. Assume that leakage and fringing fluxes are negligible. When the core relative permeability is assumed to be infinite, the magnetic flux density computed in the air gap is 1 tesla. With same Ampere-turns, if the core relative permeability is assumed to be 1000 (linear), then determine the flux density in the air gap.



Solution:

Air gap length, $l_{ag} = 0.2$ cm,

Mean length of magnetic path, $l_m = 40$ cm

Given, $B_0 = 1$ Tesla

$$\phi = \frac{\text{mmf}}{\Re}$$

For same mmf,

$$\phi \propto \frac{1}{\Re}$$

In case-1:

$$\Re_1 = \frac{l_m}{\mu_0 A} = \frac{l_{ag}}{\mu_0 A} + \frac{l_m}{\mu_0 \mu_r A}$$

As $\mu_r = \infty$

$$\therefore \qquad \Re_1 = \frac{0.2 \times 10^{-2}}{\mu_0 A}$$

In case-2:

$$\mathfrak{R}_2 = \frac{l_{ag}}{\mu_0 A} + \frac{l_m}{\mu_0 \mu_r A}$$

$$= \frac{0.2\times 10^{-2}}{\mu_0 A} + \frac{40\times 10^{-2}}{1000\mu_0 A} = \frac{0.24\times 10^{-2}}{\mu_0 A}$$

As, flux, $\phi = BA$

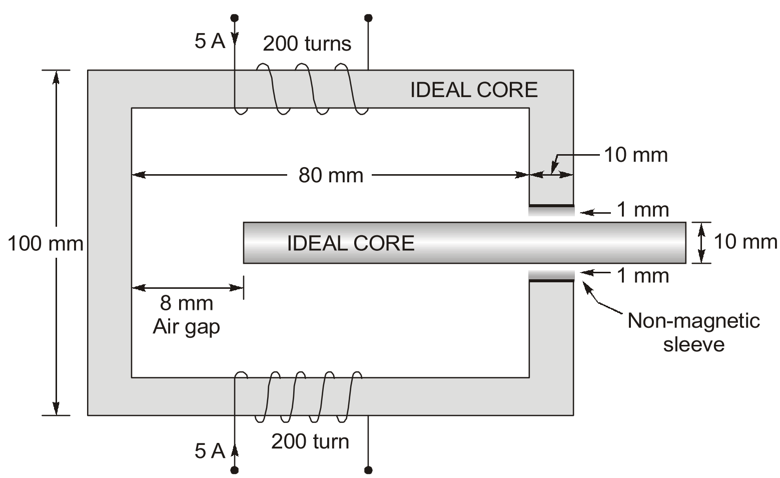
for uniform cross section area, $\phi \propto B$

Therefore, $$B \propto \frac{1}{\mathfrak{R}}$$

$\therefore$ $$B_2 = \frac{B_1 \times \mathfrak{R}_1}{\mathfrak{R}_2}$$

$$B_2 = \frac{0.2\times 10^{-2}/\mu_0 A}{0.24\times 10^{-2}/\mu_0 A} = 0.833 \text{ T}$$

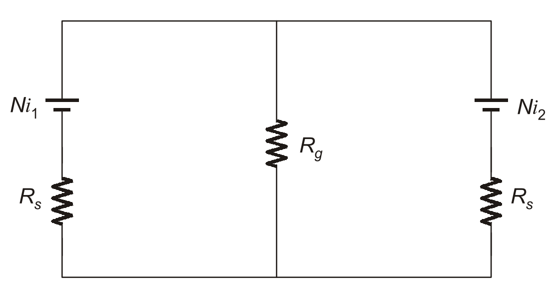
Q.5 A magnetic circuit formed by an ideal core material as shown in below figure. Determine the magnetic flux density in the air gap.



Solution:

Since, the core is ideal, therefore the reluctance of the mean magnetic path of core is zero.

The equivalent circuit is,



where R_s = reluctance of non-magnetic sleeve

R_g = reluctance of air gap

MMF, $Ni_1 = Ni_2 = 200 \times 5 = 1000$ AT

Length of non-magnetic sleeve,

$$l_s = 1 \text{ mm}$$

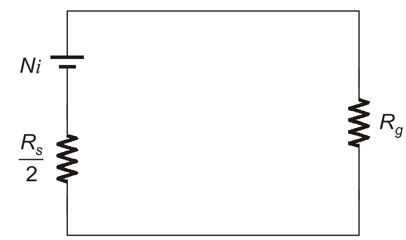
Length of air gap, $l_g = 8$ mm

Reluctance of non-magnetic sleeve,

$$\Re_s = \frac{l_s}{\mu_0 \mu_r A}$$

$$\Re_s = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times A} \quad (\mu_r = 1)$$

$$\Re_g = \frac{l_g}{\mu_0 A_g} = \frac{8 \times 10^{-3}}{4\pi \times 10^{-7} \times a}$$



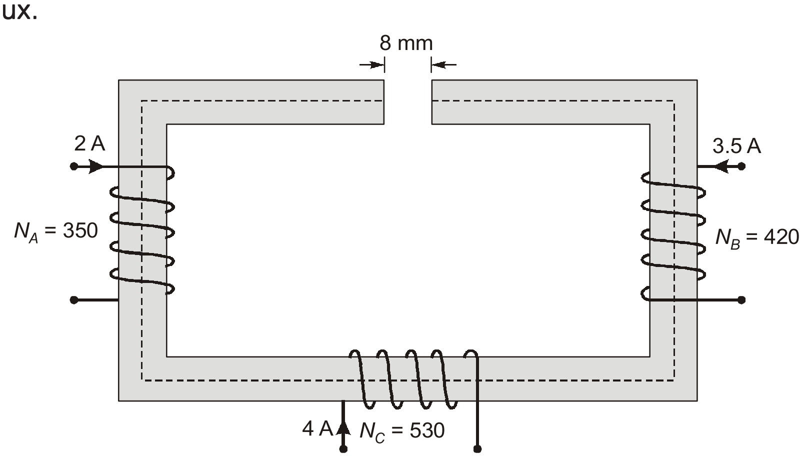
Total reluctance of magnetic circuit,

$$\Re_{eq} = \frac{\Re_s}{2} + \Re_g = \frac{1 \times 10^{-3}}{2 \times 4\pi \times 10^{-7} \times A} + \frac{8 \times 10^{-3}}{4\pi \times 10^{-7} \times A}$$

$$\Re_{eq} = \frac{8.5 \times 10^{-3}}{4\pi \times 10^{-7} \times A}$$

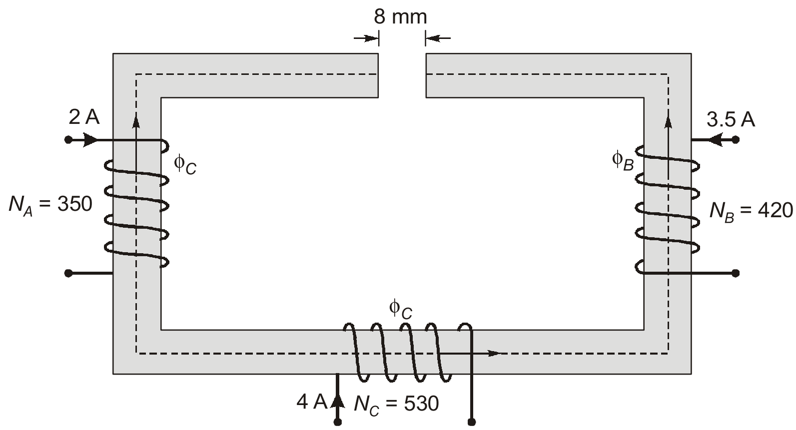
Magnetic flux density, $B = \frac{\phi}{A} = \frac{mmf}{\Re_{eq} \times A} = \frac{1000}{\frac{8.5 \times 10^{-3} \times A}{4\pi \times 10^{-7} \times A}} = 0.1478 \text{ Wb/ m}^2$

Q.6 An iron core has mean length of a magnetic circuit of 120 cm, cross-section of 4 cm × 4 cm and relative permeability of 2470. A cut of size 8 mm in the core has been made. The three coils, *A*, *B* and *C* on the core have number of turns 350, 420 and 530 respectively and the respective currents flowing are 2 A, 3.5 A and 4 A. The direction of currents is shown in figure below. Find the air gap flux. Neglecting of flux.



Solution:

As shown in figure, the flux produce by coil *B* and *C* are in the same direction but that by coil *A* is in opposite direction.



Hence net available ampere turns

$$= AT_B + AT_C - AT_A$$
$$= 420 \times 3.5 + 530 \times 4 - 350 \times 2 = 2890 \text{ AT}$$

Mean length of iron path, $l_i = 120 \text{ cm} = 1.2 \text{ m}$

Area of cross-section, $A = 4 \times 10^{-2} \times 4 \times 10^{-2} = 16 \times 10^{-4} \text{ m}^2$

Relative permeability of iron path,

$$\mu_r = 2470$$

Reluctance of iron path, $\Re_i = \dfrac{l_i}{\mu_0 \mu_r A} = \dfrac{1.2}{4\pi \times 10^{-7} \times 2470 \times 16 \times 10^{-4}} = 2.4163 \times 10^5 \text{ AT/Wb}$

Length of air gap, $l_g = 8 \times 10^{-3} \text{ m}$

Reluctance of air gap, $\Re_g = \dfrac{l_g}{\mu_0 \mu_r A} = \dfrac{8 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 16 \times 10^{-4}} = 3.978 \times 10^6 \text{ AT/Wb}$

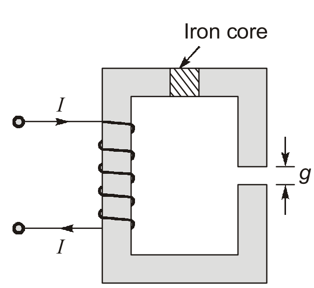
Total reluctance of the circuit

$$\Re = \Re_i + \Re_g = 4.219 \times 10^6 \text{ AT/Wb}$$

Flux in the air gap, $\phi = \dfrac{\text{Total available AT}}{\text{Total Reluctance}} = \dfrac{2890}{4.219 \times 10^6} = 684.894 \times 10^{-6} \text{ Wb}$

$$\phi = 0.685 \text{ mWb}$$

Q.7 For the magnetic circuit of figure below, length of iron path = 120 cm, g = 0.5 cm, area of cross-section of iron = 5 × 5 cm^2, μ_r = 1500, I = 2 A, N = 1000 turns.



Calculate and compare the field-energy stored and field-energy density in iron as well as in airgap. Neglect fringing and leakage flux.

Solution:

$$\text{Total reluctance} = \frac{\text{Length of iron path}}{\mu_0 \mu_r \times \text{Area}} + \frac{\text{Gap length}}{\mu_0 \times \text{Area}}$$

$$= \frac{120\times10^{-2}}{4\pi\times10^{-7}\times1500\times25\times10^{-4}} + \frac{0.5\times10^{-2}}{4\pi\times10^{-7}\times25\times10^{-4}}$$

$$= \frac{10^9}{4\pi\times25}\left[\frac{120}{1500}+\frac{0.5}{1}\right] = 1.8462\times10^6 \text{ A/Wb}$$

Flux, $$\phi = \frac{NI}{\Re} = \frac{1000\times2}{1.8462\times10^6}\text{ mWb} = 1.0833\text{ mWb}$$

Field energy stored in iron $= \frac{1}{2}\phi^2 \times$ reluctance offered by iron path

$$= \frac{1}{2}[1.0833\times10^{-3}]^2\times\frac{120\times10^{-2}}{4\pi\times10^{-7}\times1500\times25\times10^{-4}}$$

$$= 0.14942\text{ J}$$

Field energy stored in Air gap ,

$$= \frac{1}{2}\phi^2\,(\Re_{\text{airgap}})$$

$$= \frac{1}{2}(1.0833\times10^{-3})^2\times\frac{0.5\times10^{-2}}{4\pi\times10^{-7}\times25\times10^{-4}} = 0.93387\text{ J}$$

Energy density in iron core $= \frac{\text{Energy stored in iron}}{\text{Volume of iron}} = \frac{0.14942}{120\times10^{-2}\times25\times10^{-4}} = 49.81\text{ J/m}^3$

Energy density of air gap $= \frac{\text{Energy stored in air gap}}{\text{Volume of air gap}} = \frac{0.93387}{0.5\times10^{-2}\times25\times10^{-4}} = 74709.6\text{ J/m}^3$

$$\frac{\text{Energy stored in air gap}}{\text{Energy stored in iron}} = \frac{0.93387}{0.14942} = 6.25$$

$$\frac{\text{Energy density in air gap}}{\text{Energy density in iron}} = \frac{74709.6}{49.807} = 1499.98 \approx 1500$$

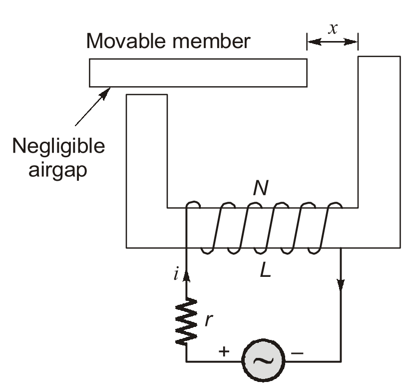
This example demonstrates that most of the field energy is stored in the air gap.

Q.8 For the electromagnetic device shown in figure, the cross-sectional area normal to the flux is A and the reluctance is offered by air gap alone. Compute the average force on the movable member in terms of N, x, A, L etc.

When,

(i) $i = I_m \cos \omega t$

(ii) $v = V_m \cos \omega t$



Solution:

(i) Reluctance,
$$\Re = \frac{x}{\mu_0 A}$$
$$L_x = \frac{N^2\mu_0 A}{x}$$
$$W_f'(i,x) = \frac{1}{2}i^2 L_x = \frac{1}{2}i^2\frac{N^2\mu_0 A}{x}$$
$$f_e = \frac{\partial W_f(i,x)}{\partial x} = \frac{-1}{2}i^2\frac{N^2\mu_0 A}{x^2} = \frac{-1}{2}\frac{N^2\mu_0 A}{x^2}(I_m\cos\omega t)^2$$
$$= \frac{-N^2\mu_0 A I_m^2}{2x^2}\cos^2\omega t = \frac{-N^2\mu_0 A I_m^2}{2x^2}\frac{(1+\cos 2\omega t)}{2}$$
$$f_{e\,avg} = \frac{-N^2\mu_0 A I_m^2}{4x^2}$$
($\because$ Average vaue of cos $2\omega t$ is zero)

(ii)
$$v = V_m\cos\omega t$$
$$v = ir + L\frac{di}{dt}$$
$$i = \frac{V_m}{\sqrt{r^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{r}\right)$$
$$f_e = -\frac{dW_f}{dx} = -\frac{d}{dx}\frac{\phi^2\Re}{2} = \frac{-\phi^2}{2}\frac{d\Re}{dx}$$
$$= \frac{-\phi^2}{2}\frac{d}{dx}\frac{x}{\mu_0 A} = \frac{-\phi^2}{2\mu_0 A} = \frac{-(Ni)^2}{2\Re^2\mu_0 A}$$
$$= \frac{-N^2}{2\mu_0 A}\cdot\frac{V_m^2}{(r^2+\omega^2L^2)}\frac{\cos^2\left(\omega t - \tan^{-1}\frac{\omega L}{r}\right)}{\left(\frac{x}{\mu_0 A}\right)^2}$$
$$= -\frac{N^2 A\mu_0 V_m^2}{2(r^2+\omega^2L^2)\,x^2}\cos^2\left(\omega t - \tan^{-1}\frac{\omega L}{r}\right)$$
$$f_{e\,avg} = -\frac{N^2 A\mu_0 v_m^2}{4(r^2+\omega^2L^2)\,x^2} = \frac{-N^2\mu_0 A V_m^2}{4\left[r^2+\omega^2\left(\frac{N^2\mu_0 A}{x}\right)^2\right]x^2} = \frac{-N^2\mu_0 A V_m^2}{4\left[r^2x^2+\omega^2N^4\mu_0^2A^2\right]}$$

Q.9 For the electromechanical system shown below, the air-gap flux density under steady-state operating condition is given by
$$B(t) = B_m \sin\omega t$$
Find the instantaneous coil voltage and current along with force of magnetic field origin: