Electrical Engineering

Communication Systems

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

Communication Systems

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Contents

Communication Systems

Chapter 5

Pulse Modulation....................................86

Chapter 6

Miscellaneous 139

■■■■

2

Amplitude Modulation

Introduction

In analog communication, message is analog and the carrier is a sine wave, which is also analog in nature. The modulation techniques in analog communication can be classified into amplitude modulation (AM) and angle modulation techniques. The amplitude of the carrier signal is varied in accordance with the message to obtain modulated signal in case of amplitude modulation. The angle modulation employs variation of angle of the carrier signal in proportion to the message. This chapter deals with the amplitude modulation techniques employed in analog communication. The next chapter deals with angle modulation techniques.

After studying the theory of amplitude modulation techniques, one will be able to know that an AM wave is made of a number of frequency components having a specific relation to one another. Based on this observation, AM can be further classified as double sideband full carrier (DSBFC), double sideband suppressed carrier (DSBSC), single sideband (SSB) and vestigial sideband (VSB) modulation techniques. This is based on how many components of the basic amplitude modulated signal are chosen for transmission. This is followed by a description of different methods for the generation of AM, DSBSC, SSB and VSB signals. To summarize, this chapter describes the basic essence of all the amplitude modulation techniques. In this chapter AM and its variants, their differences, merits and demerits are discussed. The students will also be able to calculate the frequencies present, plot the spectrum, the power or current associated with different frequency components and finally bandwidth requirements.

2.1 Amplitude Modulation

Consider a sinusoidal carrier wave $c(t)$ defined by

$$c(t) = A_c \cos(2\pi f_c t)$$

where the peak value A_c, is called the *carrier amplitude* and f_c is called the *carrier frequency*. For convenience, we have assumed that the phase of the carrier wave is zero. It is justified to make this assumption since the carrier source is always independent of the message source. We refer to $m(t)$ as the message signal which is baseband in nature. **Amplitude modulation is defined as a process in which the amplitude of the carrier wave $c(t)$ is varied linearly with the message signal $m(t)$ keeping other parameters constant.** This definition is general enough to permit different interpretations of the linearity. Correspondingly, amplitude modulation may take on different forms, depending on the frequency content of the modulated wave. In the following section, we consider the standard form of amplitude modulation.

2.1.1 Time-Domain Description

The standard form of an amplitude-modulated (*AM*) *wave* is defined by

$$x(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

where k_a is a constant called the *amplitude sensitivity* of the modulator. The modulated wave so defined is said to be a "standard" AM wave, because its frequency content is *fully* representative of amplitude modulation.

- The amplitude of the time function, multiplying $\cos(2\pi f_c t)$ is called the *envelope* of the AM wave $x(t)$. Using $a(t)$ to denote this envelope, we may thus write

$$a(t) = A_c \left| 1 + k_a m(t) \right|$$

- Here, two cases of particular interest arise, depending on the magnitude of $k_a m(t)$, compared to unity.

- For case 1, we have

$$\left| k_a m(t) \right| \leq 1, \text{ for all } t$$

Under this condition, the term $1 + k_a m(t)$ is always non-negative. We may therefore simplify the expression for the envelope of the AM wave by writing

$$a(t) = A_c[1 + k_a m(t)], \quad \text{for all } t$$

- For case 2, on the other hand, we have

$$\left| k_a m(t) \right| > 1, \text{ for all } t$$

The maximum absolute value of $k_a m(t)$ multiplied by 100 is referred as the percentage modulation. Accordingly, case 1 corresponds to a percentage modulation less than or equal to 100%, whereas case 2 corresponds to a percentage modulation in excess of 100%.

NOTE The envelope of the AM wave has a waveform that bears a *one-to-one correspondence* with that of the message signal if and only if the percentage modulation is less than or equal to 100%. This correspondence is destroyed if the percentage modulation exceeds 100%. In the later case, the modulated wave is said to suffer from **envelope distortion**, and the wave is said to be **over modulated**.

The complexity of the detector is greatly simplified if the transmitter is designed to produce an envelope $a(t)$ that has the same shape as the message signal $m(t)$. For this, two conditions are need to be satisfied.

1. The percentage modulation should be less than 100%, so as to avoid envelope distortion.
2. The message bandwidth, W, should be small as compared to the carrier frequency f_c, so that the envelope $a(t)$ may be visualized satisfactorily. Here, it is assumed that the spectral content of the message signal is negligible for frequencies outside the interval $-W \leq f \leq W$.
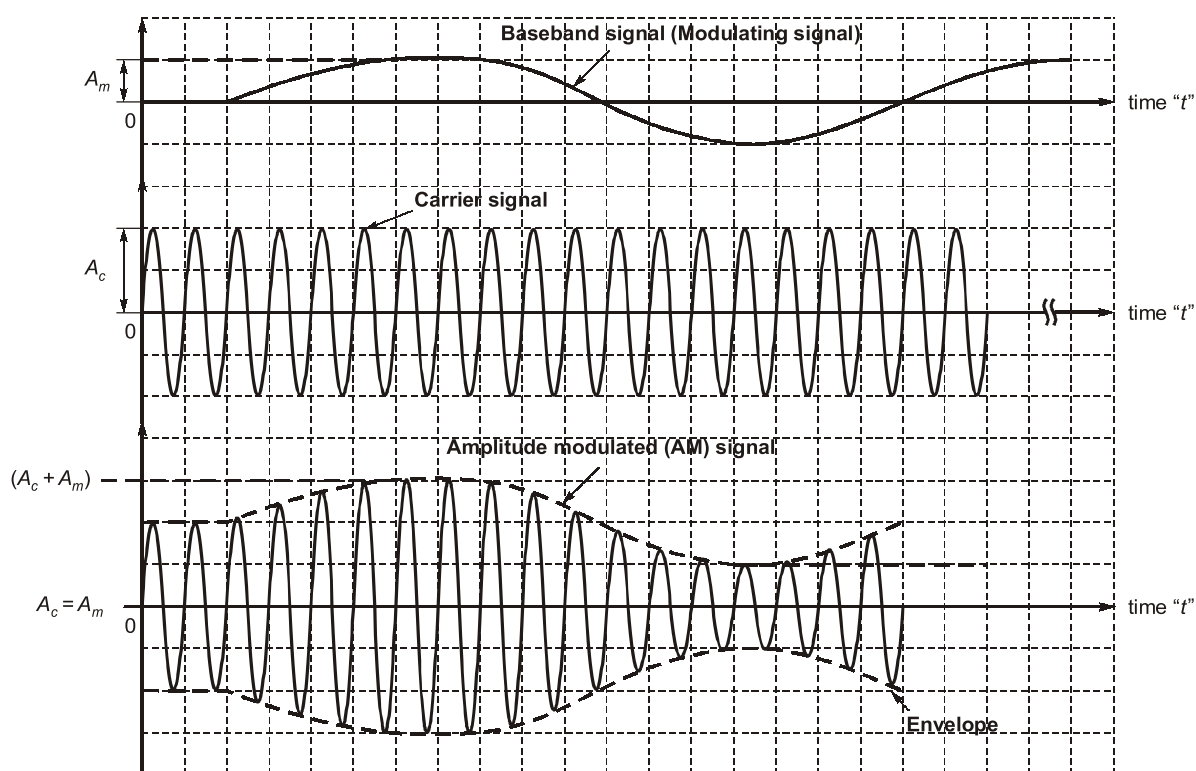
Figure-2.1 : *AM waveform for sinusoidal modulating signal*

2.1.2 Observations

1. The frequency of the sinusoidal carrier is much higher than that of the modulating signal.
2. In AM, the instantaneous amplitude of the sinusoidal high frequency carrier is changed in proportion to the instantaneous amplitude of the modulating signal. This is the principle of AM.
3. The time domain display of AM signal is as shown in Figure (2.1). This AM signal is transmitted by a transmitter. The information in the AM signal is contained in the amplitude variations of the carrier of the envelope shown by dotted lines.
4. Note that the frequency and phase of the carrier remain constant.
5. AM is used in the applications such as radio transmission, TV transmission

Example 2.1   The amplitude modulated wave form $s(t) = A_C \left[1 + K_a m(t)\right] \cos \omega_C t$ is fed to an ideal envelope detector. The maximum magnitude of $K_a m(t)$ is greater than 1. Which of the following could be the detector output?

(a) $A_c m(t)$ (b) $A_c^2 [1 + K_a m(t)]^2$ (c) $\left[A_c \left| 1 + K_a m(t) \right| \right]$ (d) $A_c \left| 1 + K_a m(t) \right|^2$

Solution: (c)

When the modulation index of AM wave is less than unity the output of the envelope detector is envelope of the AM wave but when the modulation index is greater than unity then the output of the envelope detector is not envelope but modulus of the envelope of the AM wave. Thus the detector output in given case would be $A_c \left| 1 + k_a m(t) \right|$.

2.1.3 Frequency Domain Description

To develop the frequency description of the AM wave, we take the Fourier transform of both sides. Let $S(f)$ denote the Fourier transform of $s(t)$, and $M(f)$ denote the Fourier transform of the message signal $m(t)$; we refer to $M(f)$ as the message spectrum. Accordingly, using the Fourier transform of the cosine function $A_c \cos(2\pi f_c t)$ and the frequency-shifting property of the Fourier transform, we may write

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

Let the message signal $m(t)$ be band-limited to the interval $-W \le f \le W$, (the shape of the spectrum shown in this figure is intended for the purpose of illustration only).
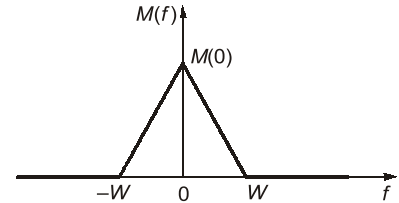
1. For positive frequencies, the portion of the spectrum of the modulated wave lying above the carrier frequency f_c is called the upper sideband, whereas the symmetric portion below f_c is called the lower sideband. For negative frequencies, the image of the upper sideband is represented by the portion of the spectrum below $-f_c$ and the image of the lower sideband by the portion above $-f_c$. The condition $f_c > W$ ensures that the sidebands do not overlap. Otherwise, the modulated wave exhibits spectral overlap and therefore frequency distortion.

Figure-2.2

2. For positive frequencies, the highest frequency component of the AM wave is $f_c + W$, and the lowest frequency component is $f_c - W$. The difference between these two frequency defines the transmission bandwidth B for an AM wave, which is exactly twice the message bandwidth W; that is

$$B = 2W$$
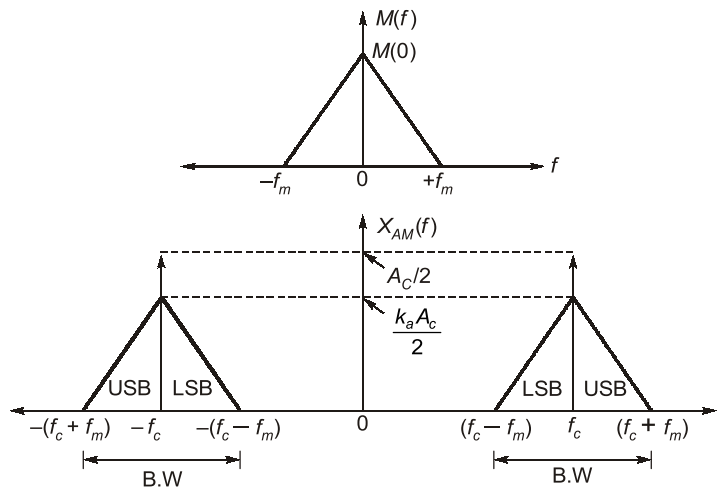
Figure-2.3

B.W $= (f_c + f_m) - (f_c - f_m)$
B.W $\approx 2f_m$ Hz or kHz
B.W $= 2\omega_m$ rad/sec

2.2 Single Tone Amplitude Modulation

$$x(t) = A_C \cos \omega_c t \dots\dots\text{carrier signal}$$
$$m(t) = A_m \cos \omega_m t \dots\dots \text{ modulating signal}$$

then after modulation, we get

$$\therefore \qquad X_{AM}(t) = [A_c + A_m \cos \omega_m t] \cos \omega_c t$$

$$X_{AM}(t) = A_c \left[1 + \frac{A_m}{A_c} \cos \omega_m t \right] \cos \omega_c t$$

$$X_{AM}(t) = A_c [1 + m_a \cos \omega_m t] \cos \omega_c t$$

where, $\quad m_a = \dfrac{A_m}{A_c}$ = Modulation Index or Depth of modulation.

The above equation can also be written as

$$X_{AM}(t) = \underbrace{A_c \cos \omega_c t}_{\text{Full carrier}} + \frac{1}{2} m_a A_c \underbrace{\cos(\omega_c + \omega_m)t}_{\text{USB}} + \frac{1}{2} m_a A_c \underbrace{\cos(\omega_c - \omega_m)t}_{\text{LSB}}$$

2.2.1 Spectrum of Sinusoidal AM signal

Figure-2.4

Figure-2.5

$$2\,A_m = V_{max} - V_{min}$$

$$\Rightarrow \qquad A_m = \frac{V_{max} - V_{min}}{2}$$

$$A_c = V_{max} - A_m = V_{max} - \frac{V_{max} - V_{min}}{2}$$
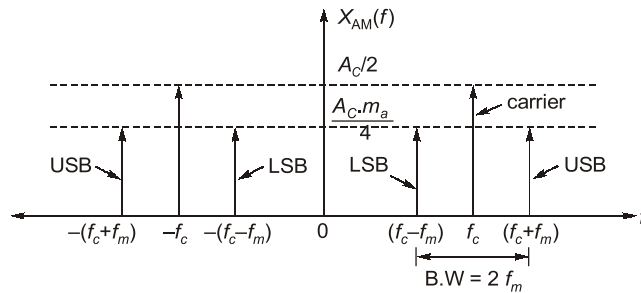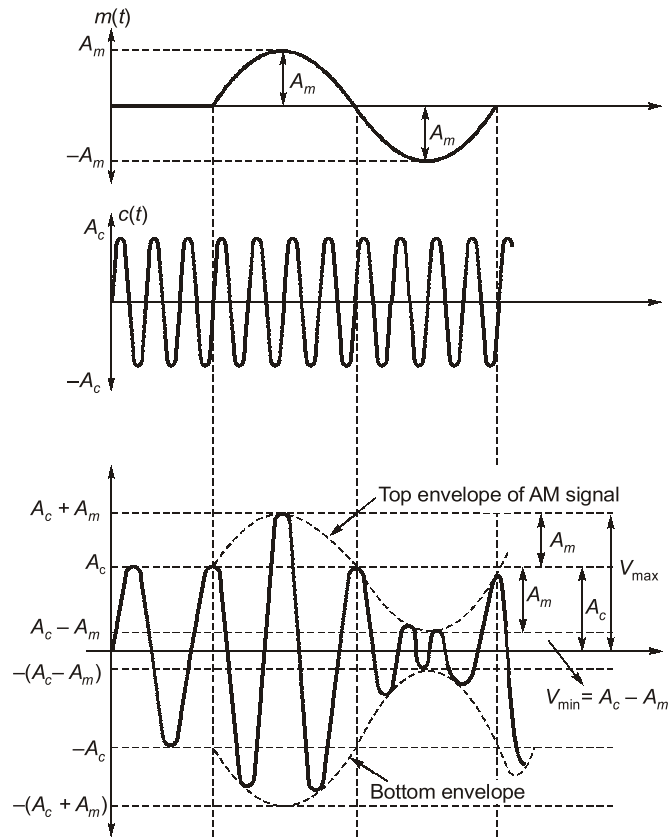
$$\Rightarrow \qquad A_c = \frac{V_{max} + V_{min}}{2}$$

Finally, we get $\qquad m_a = \dfrac{A_m}{A_C} = \dfrac{V_{max} - V_{min}}{V_{max} + V_{min}} \rightarrow$ modulation index

- % modulation = $m_a \times 100$
- Modulation index gives the depth to which the carrier signal is modulated.
- For $m(t)$ to be preserved in the envelope of AM signal, $m_a \le 1$
  i.e. $\qquad A_m \le A_c$
  so, range of m_a is, $0 \le m_a \le 1$

2.2.2 Over modulation

When $m_a > 1$ i.e. $A_m > A_C$, over modulation takes place and the signal gets distorted. Since, the negative part of waveform gets cut from the waveform leaving behind a "square wave type" of signal, which generates infinite number of harmonics. This type of distortion is known as "Non-linear distortion" or "Envelope distortion".
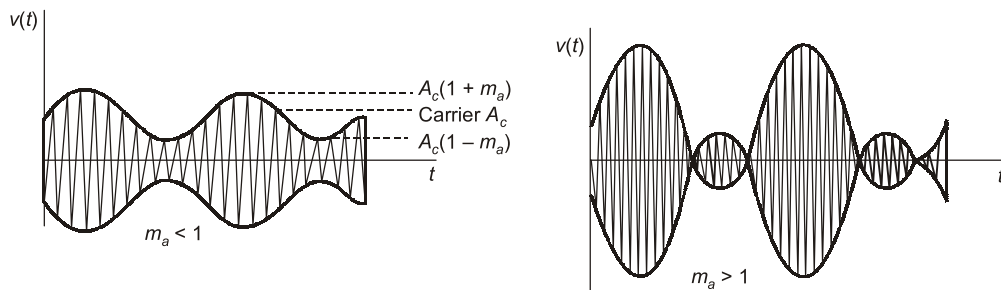
Figure-2.6 : *(a) Undermodulated AM wave (b) Over modulated AM wave*

2.3 Power Relations in AM

- In practice, the AM wave is a voltage or current wave.
- An AM wave consists of carrier and two sidebands. Hence the AM wave will contain more power than the power-contained by an unmodulated carrier.
- The amplitudes of the two sidebands are dependent on the modulation index "m". Hence the power contained in the sidebands depends on the value of m. Hence the total power in an AM wave is a function of the value of modulation index m.

2.3.1 The Total Power in AM

The total power in an AM wave is given by,

$$P_t = [\text{Carrier Power}] + [\text{Power in USB}] + [\text{Power in LSB}]$$

$$\therefore \qquad P_t = \frac{E^2}{R} + \frac{E_{USB}^2}{R} + \frac{E_{LSB}^2}{R}$$

Where E, E_{USB} and E_{LSB} are the RMS values of the carrier and sideband amplitudes and R is the characteristic resistance of antenna through which the total power is dissipated.

2.3.2 Carrier Power (P_c)

The carrier power is given by

$$P_c = \frac{E^2}{R} = \frac{\left[E_c/\sqrt{2}\right]^2}{R} = \frac{E_c^2}{2R}$$

2.3.3 Power in the Sidebands

• The power in the two sidebands is given as

$$P_{USB} = P_{LSB} = \frac{E_{SB}^2}{R}$$

• As we know the peak amplitude of each sideband is $\frac{m_a E_c}{2}$

$$P_{USB} = P_{LSB} = \frac{\left[m_a E_c/2\sqrt{2}\right]^2}{R} = \frac{m_a^2 E_c^2}{8R}$$

∴ $$P_{USB} = P_{LSB} = \frac{m_a^2}{4} \times \frac{E_c^2}{2R}$$

Hence, $$P_{USB} = P_{LSB} = \frac{m_a^2}{4} P_c$$

2.3.4 Total Power

Therefore, the total power is given by

$$P_t = P_c + P_{USB} + P_{LSB} = P_c + \frac{m_a^2}{4} P_c + \frac{m_a^2}{4} P_c$$

∴ $$P_t = \left[1 + \frac{m_a^2}{2}\right] P_c$$

or $$\frac{P_t}{P_c} = 1 + \frac{m_a^2}{2}$$

2.3.5 Modulation Index in Terms of P_t and P_c

$$\frac{P_t}{P_c} = 1 + \frac{m_a^2}{2}$$

∴ $$m_a^2 = 2\left[\frac{P_t}{P_c} - 1\right]$$

∴ $$m_a = \left[2\left(\frac{P_t}{P_c} - 1\right)\right]^{1/2}$$

2.3.6 Transmission Efficiency

• Transmission efficiency of an AM wave is the ratio of the transmitted power which contains the information (i.e. the total sideband power) to the total transmitted power.

∴ $\eta = \dfrac{P_{LSB} + P_{USB}}{P_t} = \dfrac{\left[\dfrac{m_a^2}{4}P_c + \dfrac{m_a^2}{4}P_c\right]}{\left[1 + \dfrac{m_a^2}{2}\right]P_c} = \dfrac{m_a^2/2}{1 + \dfrac{m_a^2}{2}} = \dfrac{m_a^2}{2 + m_a^2}$

• The percentage transmission efficiency is given by

$$\eta\% = \dfrac{m_a^2}{2 + m_a^2} \times 100\%$$

2.3.7 AM Power in Terms of Current

• The total power P_t of an AM wave and the carrier power P_c can be expressed in terms of currents also.

• Assume I_c to be the RMS current corresponding to the unmodulated carrier and I_t be the RMS current for AM wave.

$$P_c = I_c^2 R \quad \text{and} \quad P_t = I_t^2 R$$

∴ $\dfrac{P_t}{P_c} = \dfrac{I_t^2}{I_c^2} \times \dfrac{R}{R} = \left[\dfrac{I_t}{I_c}\right]^2$

But, $\dfrac{P_t}{P_c} = \left[1 + \dfrac{m_a^2}{2}\right]$

∴ $\left[\dfrac{I_t}{I_c}\right]^2 = 1 + \dfrac{m_a^2}{2}$

∴ $I_t = I_c\left[1 + \dfrac{m_a^2}{2}\right]^{1/2}$

Also, $1 + \dfrac{m_a^2}{2} = \left[\dfrac{I_t}{I_c}\right]^2$

∴ $m_a = \left(2\left(\left(\dfrac{I_t}{I_c}\right)^2 - 1\right)\right)^{1/2}$

Example 2.2 An AM signal with a carrier of 1 kW has 200 Watts in each sideband. What is the percentage of modulation?

Solution:

$P_c = 1000\ \text{W},$

$P_{USB} = P_{LSB} = 200\ \text{W}$

∴ Total power $P_t = 1000 + 200 + 200 = 1400\ \text{W}$

$P_t = P_c\left[1 + \dfrac{m^2}{2}\right]$

∴ $1400 = 1000\left[1 + \dfrac{m^2}{2}\right]$

∴ $m = 0.8944$

∴ Percentage modulation $= 89.44\%$

Example 2.3	The RMS antenna current of AM transmitter increases by 15% over its

unmodulated value, when sinusoidal modulation by 1 kHz is applied. Determine the modulation index.

Solution:

Let
$$I_t = \text{RMS antenna current of modulated signal.}$$
$$I_c = \text{RMS antenna current of unmodulated signal.}$$
$$\therefore \qquad I_t = 1.15\,I_c$$

We know that,

$$\left[\frac{I_t}{I_c}\right]^2 = 1+\frac{m^2}{2}$$

Substituting I_t from above equation,

$$\therefore \qquad [1.15]^2 = 1+\frac{m^2}{2}$$

$$\therefore \qquad m = 0.8$$

2.4 Modulation by Multiple Single Tone Signals

• Until now we have assumed that only one modulating signal is present. But in practice more than one modulating signals will be present. Let us see first how to express the AM wave when more than one modulating signals are simultaneously used.

• Let us assume that there are two modulating signals.
$$x_1(t) = E_{m1} \cos \omega_{m1} t$$
and
$$x_2(t) = E_{m2} \cos \omega_{m2} t$$
The total modulating signal will be the sum of these two in the time domain.

∴ The total modulating signal
$$= x_1(t) + x_2(t) = E_{m1} \cos \omega_{m1}\, t + E_{m2} \cos \omega_{m2}\, t$$

• The instantaneous value of the envelope of AM wave is
$$A = E_c + x_1(t) + x_2(t) = E_c + E_{m1} \cos \omega_{m1} + E_{m2} \cos \omega_{m2} t$$

• Substituting the value of A in this equation we get,

$$e_{AM} = E_c\left[1+\frac{E_{m1}}{E_c}\cos\omega_{m1}t + \frac{E_{m2}}{E_c}\cos\omega_{m2}t\right]\cos\omega_c t$$

$$\frac{E_{m1}}{E_c} = m_1$$

and
$$\frac{E_{m2}}{E_c} = m_2$$

• Using the following identity to simplify equation

$$\cos A \cos B = \frac{1}{2}\cos(A+B)+\frac{1}{2}\cos(A-B)$$

$$e_{AM} = E_c \cos\omega_c t + \frac{m_1 E_c}{2}\cos(\omega_c + \omega_{m1})t + \frac{m_1 E_c}{2}\cos(\omega_c - \omega_{m1})t$$

$$+\frac{m_2 E_c}{2}\cos(\omega_c + \omega_{m2})t + \frac{m_2 E_c}{2}\cos(\omega_c - \omega_{m2})t$$

2.4.1 Total Power in AM Wave

The total power is given as,

$$P_t = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$

$$P_{LSB} = P_{USB} = \frac{m_a^2}{4} P_c$$

where,

$$P_c = \frac{E_c^2}{2R}$$

Using this result here, we get

$$P_t = P_c + \frac{m_1^2}{4}P_c + \frac{m_2^2}{4}P_c + \frac{m_1^2}{4}P_c + \frac{m_2^2}{4}P_c = P_c\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2}\right]$$

Extending the concept to the AM wave with n number of modulating signals with modulating indices m_1, m_2, ..., m_n the total power is given by,

$$P_t = P_c\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + ... + \frac{m_n^2}{2}\right]$$

2.4.2 Effective Modulation Index (m_t)

We know that,

$$P_t = P_c\left[1 + \frac{m_t^2}{2}\right]$$

$$m_t = \left[m_1^2 + m_2^2 + ...m_n^2\right]^{1/2}$$

2.4.3 Trapezoidal display of AM Signal

- Modulated wave $\xrightarrow{\text{applied}}$ vertical deflection circuit of CRO.

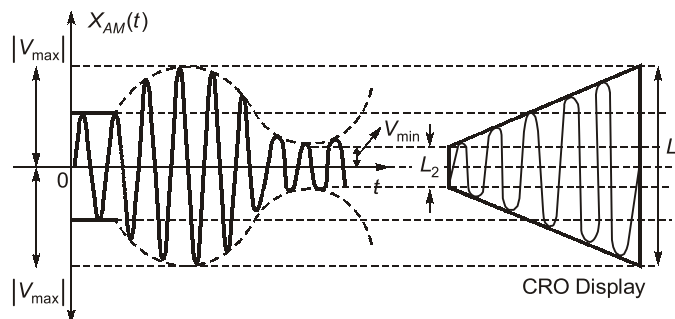- Modulating wave $\xrightarrow{\text{applied}}$ horizontal deflection circuit of CRO.

Figure-2.7

Here, $L_1 = 2 V_{max}$
and $L_2 = 2 V_{min}$
So, m_a = modulation index

$$= \frac{L_1 - L_2}{L_1 + L_2}$$

After passing from LPF with $f_c = 1$ Hz

$$\therefore \qquad y(t) = \frac{1}{2t}\left[\sin 2\pi t + \sin 2\pi t - \sin \pi t\right]$$

$$\Rightarrow \qquad y(t) = \frac{\sin 2\pi t}{2t} + \frac{1}{2t}\left[2\sin 0.5\pi t \cdot \cos 1.5\pi t\right]$$

$$\Rightarrow \qquad y(t) = \frac{\sin 2\pi t}{2t} + \frac{\sin 0.5\pi t \cos 1.5\pi t}{t}$$

| Sl. No.	Parameter	DSBFC	DSBSC	SSB	VSB
1.	Carrier suppression	N.A.	Fully	Fully	Fully
2.	Sideband suppression	N.A.	N.A.	One S.B. completely	One S.B. suppressed partially
3.	Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW < 2f_m$
4.	Transmission efficiency	Minimum	Moderate	Maximum	Moderate
5.	No. of modulating inputs	1	1	1	2
6.	Application	Radio broadcasting	Radio broadcasting	Point to point mobile communication	T.V. Video transmission
7.	Power requirement to cover same area	High	Medium	Very small	Moderate
8.	Complexity	Simple	Simple	Complex	Simpler than SSB

Table-2.1

Student's Assignments 1

Q.1 Total power saving when carrier and one of the sidebands are suppressed in an AM wave modulated to a depth of 50% is
(a) 66.67%   (b) 83.33%
(c) 94.44%   (d) 100%

Q.2 Consider the Trapezoidal pattern for AM wave shown below. Modulation index is given by

3 cm   1 cm
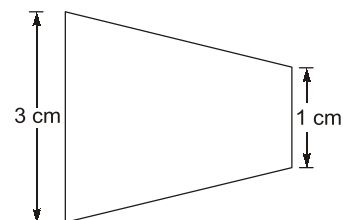
(a) 33%   (b) 50%
(c) 75%   (d) 100%

Q.3 For a message signal $m(t) = 10 \cos 100\, t$ with 60% modulation, the maximum envelope time will be
(a) 10.3 ms (b) 13.3 ms
(c) 33.3 ms (d) 10 ms

Q.4 The modulation index of an AM wave is changed from 0 to 1. The transmitted power is
(a) unchanged
(b) halved
(c) doubled
(d) increased by 50 percent

Q.5 The most commonly used filters in SSB generation are
(a) mechanical (b) RC
(c) LC (d) low-pass

Q.6 The amplitude modulated waveform $s(t) = A_C[1 + k_a\, m(t)]\cos\omega_C t$ is fed to an ideal envelope detector. The maximum magnitude of $k_a m(t)$ is greater than 1. Which of the following could be the detector output?
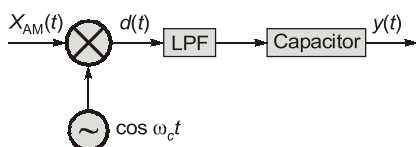
(a) $A_C\, m(t)$ (b) $A_C^2[1 + k_a\, m(t)]^2$

(c) $\left|A_C[1 + k_a\, m(t)]\right|$ (d) $A_C[1 + k_a\, m(t)]^2$

Q.7 A message signal periodic with T is applied to an AM modulator with $m = 0.5$. The modulation efficiency will be
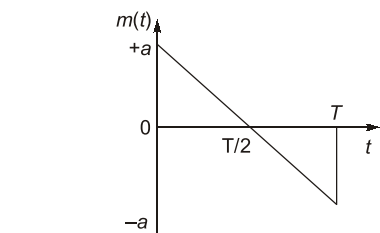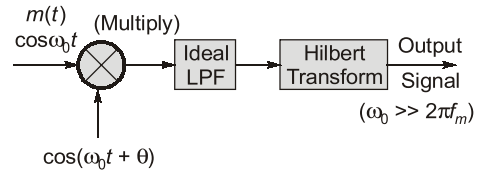

(a) 5.1% (b) 7.7%
(c) 10.2% (d) 15.4%

Q.8 For given synchronous demodulator can demodulate AM signal $X_{AM}(t) = [A + m(t)]\cos\omega_c t$. The value of $y(t)$ is

(a) $m(t)$ (b) $\dfrac{m(t)}{2}$

(c) $\dfrac{m(t)}{4}$ (d) zero

Q.9 A message $m(t)$ band limited to the frequency f_m has a power of P_m. The power of output signal is

(a) $\dfrac{P_m \cos\theta}{2}$ (b) $\dfrac{P_m}{4}$

(c) $\dfrac{P_m \sin^2\theta}{4}$ (d) $\dfrac{P_m \cos^2\theta}{4}$

■ **ANSWERS**

1.	(c)	2.	(b)	3.	(b)	4.	(d)	5.	(a)
6.	(c)	7.	(b)	8.	(b)	9.	(c)		

Student's Assignments 2

Q.1 The input to an envelope detector is a single-tone AM signal
$$x_{AM}(t) = A[1 + m_a \cos(\omega_m t)]\cos(\omega_c t)$$
where m_a is constant, $0 < m_a < 1$, and $\omega_c \gg \omega_m$.
(i) Show that if the detector output is to follow the envelope of $x_{AM}(t)$, it requires that at any time to

$$\frac{1}{RC} \geq \omega_m\left(\frac{m_a \sin\omega_m t_o}{1 + m_o \cos\omega_m t_o}\right)$$

(ii) Also prove if the detector output is to follow the envelope at all times, it is required that

$$RC \leq \frac{1}{\omega_m}\frac{\sqrt{1 - m_a^2}}{m_a}$$

Q.2 Given the SSB wave

$$s(t) = m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)$$

where f_c is carrier frequency, $m(t)$ is the message signal and $\hat{m}(t)$ is its Hilbert transformer.

The modulated wave is applied to a square-law device characterized by

$$y(t) = s^2(t)$$

Prove that the output has a time varying phase which make it impractical for detection.

Q.3 Coherent demodulation of AM signal is shown below. The LPF and message signal $m(t)$ have the same bandwidths.

$[A + m(t)]\sin\omega_c t$
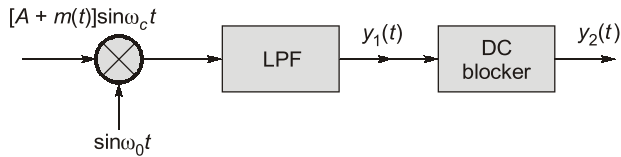
Find the expression for signal at $y_1(t)$.

Q.4 The antenna current of an AM broadcast transmitter, modulated to a depth of 40 percent by an audio sine wave, is 11 amperes. It increases to 12 amperes as a result of simultaneous modulation by another audio sine wave. What is the modulation index due to this second wave?

■ **ANSWERS** _____

3. $y_1(t) = \dfrac{A + m(t)}{2}$

4. 0.64

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