# Electronics Engineering

# Communication Systems

Comprehensive Theory

*with* **Solved Examples** and **Practice Questions**

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CHAPTER 2

# Basics of Signal and System

## Introduction

Just as a carpenter requires proper set of tools before he can sit down to make a piece of furniture, in a similar manner a communication engineer needs to know about signals before he can start the process of learning communication.

## 2.1 Signal and System

The communication technology can be conveniently broken down into three interacting parts.

- Signal processing operations performed.
- The device that performs these operations.
- The underlining physics.

Thus to study the basic form of modulation and signal processing used in the communication it will be fruitful to have a quick review of the concepts of signal and system.

### 2.1.1 Some Basic Signals

It will be very helpful to study some signals before hand, so that the analysis of the communication system becomes easier. Some important and frequently used signals and their properties are mentioned in this section.

**The Impulse Signal**

Impulse function is not a function in its strict sense. It is a distributed or generalized function. A generalized function is defined in terms of its effect on other function. The unit impulse function is generalised as any function that follow the following condition:

1. Impulse signal (Dirac delta function):

$$\delta(t) = \begin{cases} \infty\,; & t = 0 \\ 0\,; & t \neq 0 \end{cases}$$

and

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

2. Unit impulse signal:

$$\delta[n] = \begin{cases} 1\,; & n = 0 \\ 0\,; & n \neq 0 \end{cases}$$

**Properties of Impulse Function**

1. Product property

$$x(t)\,\delta(t) = x(0)\,\delta(t)$$

Similarly, $$x(t)\,\delta(t-\alpha) = x(\alpha)\,\delta(t-\alpha)$$

2. Shifting property

$$\int_{-\infty}^{\infty} x(t)\,\delta(t)dt = x(0)$$

Similarly,

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-\alpha)dt = x(\alpha)$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

3. Scaling property

$$\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$$

**Example 2.1** Find the impulse function form if $x(t) = 4t^2\,\delta(2t-4)$, where $x(t)$ is an arbitrary signal.

***Solution*:**

$$\begin{aligned} x(t) &= 4t^2\,\delta(2t-4) \\ &= 4t^2\,\delta\{2(t-2)\} \\ &= 4t^2 \cdot \frac{1}{2}\delta(t-2) \quad \text{... from scaling property} \\ &= 2t^2\,\delta(t-2) \end{aligned}$$

Now, from product property we have,

$$x(t)\,\delta(t-\alpha) = x(\alpha)\,\delta(t-\alpha)$$

So, $$x(t) = 2t^2\Big|_{t=2} \cdot \delta(t-2) = 8\,\delta(t-2)$$

**Example 2.2** Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt$ is

(a) 1 (b) −1 (c) 0 (d) $p/2$

***Solution*: (a)**

We know,

$$\int_{-\infty}^{\infty} \delta(t)\,x(t)\,dt = x(0)$$

So here, $$\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt = \cos 0 = 1$$

**Do you know?** Impulse signals do not occur naturally but they are important functions providing a mathematical frame work for the representation of various processes and signals. These come under a special class of functions known as generalized functions.

### Gate Function/Rectangular Pulse

Let us consider a rectangular pulse as shown in figure below:

$$x(t) = A\,\text{rect}(t) = \begin{cases} A, & \text{for } \frac{-1}{2} < t < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

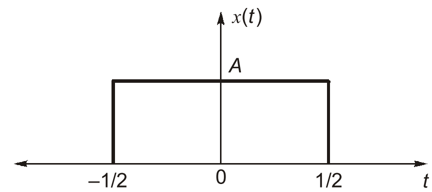


*Figure-2.1*

$$x(t) = A\,rect\left(\frac{t}{\tau}\right) = \begin{cases} A, & \text{for } \frac{-\tau}{2} < t < \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

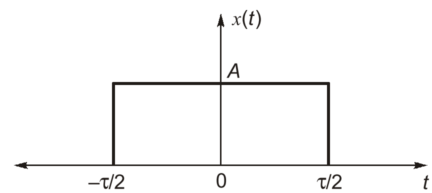


*Figure-2.2*

### Step Signal

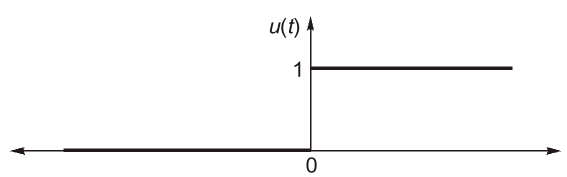


***Figure-2.3:*** *Continuous-time version of the unit-step function of unit amplitude*

The continuous-time version of the unit-step function is defined by

$$u(t) = \begin{cases} 1; & t > 0 \\ 0; & t < 0 \end{cases}$$

**NOTE**

- Figure depicts the unit-step function $u(t)$. It is said to exhibit discontinuity at $t = 0$, since the value of $u(t)$ changes instantaneously from 0 to 1 when $t = 0$. It is for this reason that we have left out the equal sign in equation; that is $u(0)$ is undefined.
- Unit step function denote sudden change in real time and a frequency or phase selectivity in frequency domain.

There is one more definition of unit step function.

$$u(t) = \begin{cases} 0 & ; t < 0 \\ 1/2 & ; t = 0 \\ 1 & ; t > 0 \end{cases}$$

### Properties of Unit-Step Function

1. $u(t - t_0) = [u(t - t_0)]^2 = u[u(t - t_0)]^k$, with $k$ being any positive integer.
2. $u(at - t_0) = u\left(t - \frac{t_0}{a}\right); a > 0$

3. $\delta(t) = \frac{d}{dt} u(t)$

4. $u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$

**Do you know?** The unit-step function $u(t)$ may also be used to construct other discontinuous waveforms. The value at $t = 0$ gives rise to Gibb's phenomenon when unit step function is constructed by sinusoidal signals.

### Sampling/Interpolating/Sinc Function

The function $\frac{\text{"}\sin \pi x\text{"}}{\pi x}$ is the **"sine over argument"** function and it is denoted by "sinc ($x$)". It is also known as **"filtering function"**.

Mathematically,

$$\text{sinc}(x) = \frac{\sin \pi x}{\pi x} = Sa(\pi x)$$

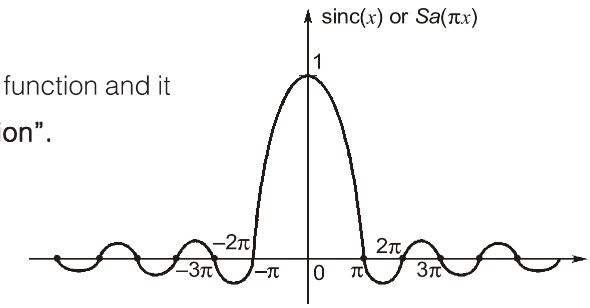



***Figure-2.4 :** Sinc Function*

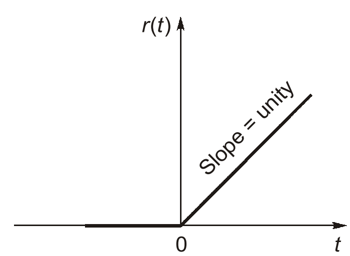
**Do you know?** Just like impulse function sinc ($x$) is also a conceptual function since it can not be realized.

### The Unit-Ramp Function

The ramp function $r(t)$ is a linearly growing function for positive values of independent variable $t$. The ramp function shown in figure is defined by

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

or $\quad r(t) = tu(t)$




The ramp function is obtained by integrating the unit step function

$$\int_{-\infty}^{t} u(\tau)\, d\tau = r(t)$$

The relationship between the impulse, step and ramp signals are represented below:

**Remember:** Relationship between impulse, step and ramp signals

$$\delta(t) \xrightarrow{\text{Integrate}} u(t) \xrightarrow{\text{Integrate}} r(t)$$

$$r(t) \xrightarrow{\text{Differentiate}} u(t) \xrightarrow{\text{Differentiate}} \delta(t)$$

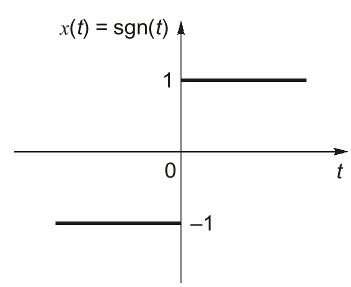
### Unit Signum Function

The unit signum function shown in figure is defined as follows

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

This function can be expressed in terms of unit step function as

$$\text{sgn}(t) = -1 + 2u(t)$$




**Unit Signum Function**

## 2.1.2 Signal-Classification

### Continuous-Time and Discrete-Time signals

The signals that are defined at each instant of time are known as continuous time signals. However, if the signals are defined only at certain time instants, it is called as discrete-time signals.

Based upon above discussion, four combinations are possible:

- Continuous time continuous amplitude signal (Analog signal)
- Continuous time discrete amplitude signal (Quantized signal)
- Discrete time continuous amplitude signal (Sampled signal)
- Discrete time discrete amplitude signal (Digital signal)

### Analog and Digital Signal

If the amplitude of the signal can take all possible values in its dynamic range, it is called as analog signal. On the other hand, a digital signal is one whose amplitude take some specific values in its dynamic range.

### Periodic and Aperiodic Signals

A signal is said to be periodic if it repeats itself after a certain time interval. For a signal to be periodic, it must satisfy the following condition.

1. It should exist for all values of '$t$'.
2. $x(t) = x(t + T)$, where $T$ is the least value after which the signal repeats itself.
3. The value of $T$ should be a fixed positive constant.

'$T$' is referred as fundamental period.

Any signal which do not follow these conditions are termed as aperiodic signal.

**NOTE**

**Periodicity of Signal $x_1(t) + x_2(t)$:**

A signal $x(t)$ that is a linear combination of two periodic signals, $x_1(t)$ with fundamental period $T_1$ and $x_2(t)$ with fundamental period $T_2$ as follows:

$$x(t) = x_1(t) + x_2(t)$$

is periodic if, $\dfrac{T_1}{T_2} = \dfrac{m}{n}$ = a rational number

Period of $x(t)$, $T = nT_1 = mT_2$

or, $T = \text{LCM}\,(T_1, T_2)$

### Deterministic and Random Signal

A signal is said to be deterministic, if they can be completely represented by a mathematical expression at any instance of time. Signals, which cannot be represented by any mathematical expression is called random signal.

**Note:** For analysis purpose random signal can also be approximated by their statistical property.

### Energy Signals and Power Signals

$x(t)$ is an energy signal if

$0 < E < \infty$ and $P = 0$

where '$E$' is the energy and '$P$' is the power of the signal $x(t)$.

For a continuous-time signal (CTS),

$$E = \int_{-\infty}^{\infty} |\, x(t)\,|^2\, dt$$

For an energy signal, energy is finite while power is zero.

**NOTE**

If $x(t) \longrightarrow E$, [where, E is energy of $x(t)$]

then $x\left(\frac{t}{\alpha}\right) \longrightarrow \alpha E$

$x(\alpha t) \longrightarrow \frac{E}{\alpha}$

$ax(t) \longrightarrow a^2E$

**$x(t)$ is a Power Signal if**

if, $0 < P < \infty$ and $E = \infty$

where $E =$ Energy of signal $x(t)$

$P =$ Power of signal $x(t)$

Almost all the practical periodic signals are **"power signals"**, since their average power is finite and non-zero. For a CTS, the average power of a signal $x(t)$ is,

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt$$

**NOTE**

- If $x(t) = A \cos \omega t$ or $A \sin \omega t$, then $P_x = A^2/2$
- If $x(t) = Ae^{\pm j\omega t} \Rightarrow P_x = A^2$
- If $x(t) = A \Rightarrow P_x = A^2$
- If $x(t) \longrightarrow P$, then $x\left(\frac{t}{\alpha}\right) \longrightarrow P$

  $x(\alpha t) \longrightarrow P$ and $ax(\alpha t) \longrightarrow a^2P$
- For an **unit step signal**, $x(t) = u(t)$ and $P_x = \frac{1}{2}$

| Energy Signal | Power Signal |
|---|---|
| 1. The total energy is obtained using $E = \lim_{T\to\infty} \int_{-T}^{T} \lvert x(t)\rvert^2\, dt$ | The average power is obtained $P = \lim_{T\to\infty} \int_{-T}^{T} \frac{1}{2T} \lvert x(t)\rvert^2\, dt$ |
| 2. For the energy signal, $0 < E < \infty$, and the average power $P = 0$ | For the power signal, $0 < P < \infty$, and the energy $E = \infty$. |
| 3. Non-periodic and finite duration signals are in general energy signals. | Periodic signals are power signals. However, all power signals need not be periodic. |
| 4. Energy signals are time limited. | Power signals exist over infinite time. |

***Table-2.1***

## 2.2 Time Domain and Frequency Domain Representation of a Signal

A signal $x(t)$ can be represented in terms of relative amplitude of various frequency components present in signal. This is possible by using exponential Fourier series. This is a frequency domain representation of the signal. The time domain representation specifies a signal value at each instant of time. This means that a signal $x(t)$ can be specified in two equivalent ways:

(i) Time domain representation; where $x(t)$ is represented as a function of time. Graphical time domain representation is termed as *waveform.*

(ii) The frequency domain representation; where the signal is represented graphically in terms of its frequency graphical frequency domain representation is termed as spectrum.

Any of the above two representations uniquely specifies the function, i.e. if the signal is specified in time domain, we can determine its spectrum. Conversely, if the spectrum is specified, we can determine the corresponding time domain signal. In order to determine the function in frequency domain, it is necessary that both amplitude spectrum and phase spectrum are specified.

**Remember**

- In many cases, the spectrum is either real or imaginary, as such, only an amplitude plot is enough as all frequency components have identical phase relation.
- We use both the conventions depending upon the problem we are studying.
- If we want to analyze the signal at our perspective, it is convenient to see signal in its time domain form, but if we want to process the signal through an LTI system, the frequency domain approach becomes much fruitful.

### 2.2.1 Decomposition of Signals

It will be fruitful for us if we can devise some technique that will allow us to break any unknown signal into some standard and known signal set. There are two methods of doing this

(1) Any signal can be broken down into an infinite set of impulse signals. This process leads to the time domain approach of signal and system.

(2) It can be broken down into an infinite set of orthogonal signals. This leads to frequency domain description of signal and system.

**Misconception:** Not any representation of signal as set of impulse or exponential function is considered as signal decomposition.

Consider a periodic signal $x(t) = \begin{cases} 1\ ; & 0 \le t \le 1 \\ -2\ ; & 1 < t < 2 \end{cases}$ with time period $T = 2$.

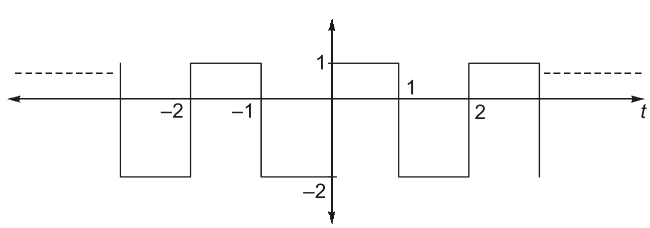



***Figure-2.5***

The derivative of this signal is related to the impulse train $g(t) = \sum_{k=-\infty}^{\infty} \left[a_k\delta(t-k) + b_k\delta(t-2k)\right]$ with period $T = 2$. [where, $a_k = 3$, $b_k = -3$]

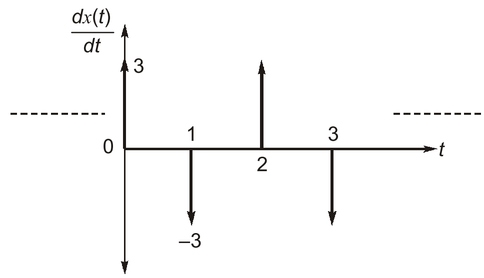



***Figure-2.6***

**NOTE:** The problem here is we have to define a differentiator in order to represent the time domain signal as set of impulses. Thus we need an alternative way of representing the signal.

## 2.3 Signals Versus Vectors

There is a strong connection between signals and vectors. Signals that are defined for only a finite number of time instants (say $N$) can be written as vectors (of dimension $N$). Thus, consider a signal $g(t)$ defined over a closed time interval $[a, b]$. Let us pick $N$ points uniformly on the time interval $[a, b]$ such that

$$t_1 = a,\ t_2 = a + \epsilon,\ \ t_3 = a + 2\epsilon,\ t_N = a + (N-1)\epsilon,\ \epsilon = \frac{b-a}{N-1}$$

Then we can write a signal vector $g$ as an N-dimensional vector

$$g = [g(t_1)\ g(t_2)\ \dots\ g(t_N)]$$

This relationship clearly shows that continuous time signals are straight forward generalizations of finite dimension vectors. Thus, basic definitions and operations in a vector space can be applied to continuous time signals as well. In a vector space, we can define the inner (dot or scalar) product of two real-valued vectors $x$ and $g$ as

$$<x, g> = \|g\| \cdot \|x\| \cos\theta$$

When $\theta$ is the angle between vectors $x$ and $g$.

By using this definition, we can express $\|x\|$, the length (norm) of a vector $x$ as

$$\|x\|^2 = <x, x>$$

**Remember:** This concept forms the basis of digital communication system.

### 2.3.1 Decomposition of a Signal and Signal Components

The concepts of vector component and orthogonality can be directly extended to continuous time signals. Consider the problem of approximating a real signal $g(t)$ in terms of another real signal $x(t)$ over an interval $[t_1, t_2]$.

$$g(t) \approx ex(t)$$

$$e = \frac{\int_{t_1}^{t_2} g(t)x(t)dt}{\int_{t_1}^{t_2} x^2(t)\,dt} = \frac{1}{E_x}\int_{t_1}^{t_2} g(t)\,x(t)dt \quad \text{(where, } E_x = \text{Energy of signals)}$$

## Student's Assignments 1

**Q.1** A modulated signal is given by,

$s(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$

where the baseband signal $m_1(t)$ and $m_2(t)$ have bandwidths of 10 kHz and 15 kHz, respectively. The bandwidth of the modulated signal, in kHz, is

(a) 10 (b) 15
(c) 25 (d) 30

**Q.2** Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt$ is

(a) 1 (b) –1
(c) 0 (d) $\frac{\pi}{2}$

**Q.3** If a signal $f(t)$ has energy $E$, the energy of the signal $f(5t)$ is equal to

(a) $E$ (b) $\frac{E}{5}$
(c) $5E$ (d) $10E$

**Q.4** The trigonometric Fourier series of an even function of time does not have

(a) the dc term (b) cosine terms
(c) sine terms (d) odd harmonic terms

**Q.5** The trigonometric Fourier series of a periodic time function can have only

(a) cosine terms
(b) sine terms
(c) cosine and sine terms
(d) dc and cosine terms

**Q.6** The expression of trigonometrical Fourier series coefficient $b_n$ in terms of exponential Fourier series coefficient $C_n$ is

(a) $j(C_n + C_{-n})$ (b) $j\left(\frac{C_n + C_{-n}}{2}\right)$
(c) $j(C_n - C_{-n})$ (d) $j\left(\frac{C_n - C_{-n}}{2}\right)$

**Q.7** Consider a real time domain signal $x(t)$ whose Fourier transform is $X(j\omega)$. Which of the following properties are true:

(i) $\text{Even}\{x(t)\} \longleftrightarrow \text{Re}\{X(j\omega)\}$

(ii) $\text{Odd}\{x(t)\} \longleftrightarrow jIm\{X(j\omega)\}$

(iii) $x^*(t) \longleftrightarrow X^*(j\omega)$

(iv) $\int_{-\infty}^{t} x(\tau)\,d\tau \longleftrightarrow \frac{X(j\omega)}{j\omega}$

(a) (i) and (ii) (b) (i), (ii) and (iii)
(c) (i) and (iii) (d) All the above are true

**Q.8** Consider two periodic signal $x_1(t)$ and $x_2(t)$, these signal can be represented in terms of linear combination of complex exponential as:

$$\text{If } x_1(t) = \sum_{k=-100}^{100} \cos(k\pi)e^{jk\left(\frac{2\pi}{50}\right)t}$$

$$\text{and } \quad x_2(t) = \sum_{k=-100}^{100} j\sin(k\pi)e^{jk\left(\frac{2\pi}{50}\right)t}$$

then which of the following option is true

(a) $x_1(t)$ is real and even
(b) $x_2(t)$ is real and even
(c) $x_1(t)$ and $x_2(t)$ are real and even
(d) $x_2(t)$ is imaginary and odd

**Q.9** If $f(t)$ is an even function, then what is its Fourier transform $F(j\omega)$?

(a) $\int_0^{\infty} f(t)\cos(2\omega t)dt$ (b) $2\int_0^{\infty} f(t)\cos(\omega t)dt$

(c) $2\int_0^{\infty} f(t)\sin(\omega t)dt$ (d) $\int_0^{\infty} f(t)\sin(2\omega t)dt$

**Q.10** If the Fourier transform of $f(t)$ is $f(j\omega)$, then what is the Fourier transform of $f(-t)$?

(a) $f(j\omega)$
(b) $f(-j\omega)$
(c) $-F(j\omega)$
(d) complete conjugate of $f(j\omega)$

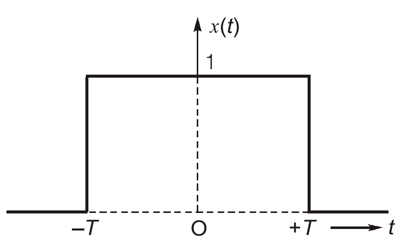
**Q.11** The trigonometric Fourier series expansion of an odd function shall have
(a) only sine terms
(b) only cosine terms
(c) odd harmonics of both sine and cosine terms
(d) none of the these

## ■ ANSWERS

| | | | | |
|---|---|---|---|---|
| 1. (d) | 2. (a) | 3. (b) | 4. (c) | 5. (c) |
| 6. (c) | 7. (a) | 8. (a) | 9. (b) | 10. (b) |
| 11. (a) | | | | |

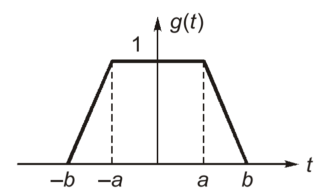
## Student's Assignments 2

**Q.1** For the rectangular pulse shown in the figure below, determine the Fourier Transform of $x(t)$ and sketch the magnitude spectrum with respect to frequency.




**Q.2** State and prove convolution theorem in Fourier transform.

**Q.3** Using time shifting and time differentiation properties, find the Fourier transform of the trapezoidal signal shown.




**Q.4** State and explain Parseval's theorem.

**Q.5** A white Gaussian noise is passed through an ideal bandpass filter with power spectral density of noise being a $\eta = \frac{N_0}{2}$. Derive the expression for the autocorrelation function of the input and output noise.

## ■ ANSWERS

1. $X(j\omega) = \dfrac{2\sin\omega T}{\omega}$

3. $G(j\omega) = \dfrac{4}{\omega^2(b-a)}\sin\dfrac{\omega(a+b)}{2}.\sin\dfrac{\omega(b-a)}{2}$

■■■■