MADE EASY PUBLICATIONS

POSTAL BOOK PACKAGE 2024

CONTENTS

ELECTRICAL ENGINEERING

Objective Practice Sets

Electromagnetic Theory

Note: This book contains copyright subject matter to MADE EASY Publications, New Delhi. No part of this book may be reproduced, stored in a retrieval system or transmitted in any form or by any means. Violators are liable to be legally prosecuted.

1
CHAPTER

Vector Analysis

MCQ and NAT Questions

Q.1 If $\vec{G} = 15 r\hat{a}_\phi$ then $\oint \vec{G}\cdot\vec{dl}$ over the circular path $r = 2$ m, $\theta = 30°$, $0 < \phi < 2\pi$ is
(a) 120π (b) 120
(c) 60π (d) 60

Q.2 Which of the following is true?
(a) $\text{Curl}(\vec{A}\cdot\vec{B}) = \text{Curl } \vec{A} + \text{Curl } \vec{B}$
(b) $\text{Div } (\vec{A}\cdot\vec{B}) = \text{Div } \vec{A}\cdot\text{Div } \vec{B}$
(c) $\text{Div } (\text{Curl } \vec{A}) = 0$
(d) $\text{Div}(\text{Curl } \vec{A}) = \Delta\cdot\vec{A}$

Q.3 Which of the following equations is correct?
1. $\hat{a}_x \times \hat{a}_x = |a_x|^2$
2. $(\hat{a}_x \times \hat{a}_y) + (\hat{a}_y \times \hat{a}_x) = 0$
3. $\hat{a}_x \times (\hat{a}_y \times \hat{a}_z) = \hat{a}_x \times (\hat{a}_z \times \hat{a}_y)$
4. $\hat{a}_r \cdot \hat{a}_\theta + \hat{a}_\theta \cdot \hat{a}_r = 0$

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 2 and 4 only

Q.4 Match **List-I** (Term) with **List-II** (Type) and select the correct answer:

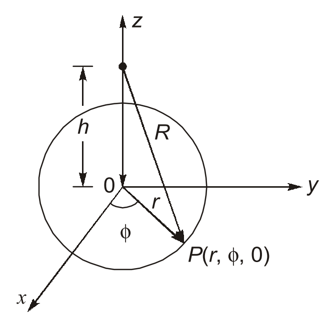
List-I	List-II
A. Curl $(\bar{F}) = 0$	**1.** Laplace equation
B. Div $(\bar{F}) = 0$	**2.** Irrotational
C. Div Grad $(\phi) = 0$	**3.** Solenoidal
D. Div Div $(\phi) = 0$	**4.** Not defined

Codes:

	A	B	C	D
(a)	2	3	1	4
(b)	4	1	3	2
(c)	2	1	3	4
(d)	4	3	1	2

Q.5 Laplacian of a scalar function V is
(a) Gradient of V
(b) Divergence of V
(c) Gradient of the gradient of V
(d) Divergence of the gradient of V

Q.6 The unit vector $\vec{a}_R$ which points from $z = h$ on the z-axis towards $(r, \phi, 0)$ in cylindrical co-ordinates as shown below is given by



(a) $\dfrac{h\vec{a}_r - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ (b) $\dfrac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$

(c) $\dfrac{h\vec{a}_\phi - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ (d) $\dfrac{r\vec{a}_z - h\vec{a}_\phi}{\sqrt{r^2 + h^2}}$

Q.7 Match **List-I (Vector Identities)** with **List-II (Equivalent expression)** and select the correct answer using the codes given below the lists:

List-I
A. $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D})$
B. $\vec{A}\times(\vec{B}\times\vec{C})$
C. $(\vec{A}\times\vec{B})\times(\vec{C}\times\vec{D})$

List-II
1. $(\vec{A}\cdot\vec{C}\cdot\vec{D})\vec{B} - (\vec{B}\cdot\vec{C}\cdot\vec{D})\vec{A}$
2. $[(\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D}) - (\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})]$
3. $(\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$

Codes:

	A	B	C
(a)	1	3	2
(b)	3	1	2
(c)	2	1	3
(d)	2	3	1

Q.8 If $\vec{P} = x^2y^2\vec{i} + (x-y)\vec{k}$, $\vec{Q} = zx\vec{i}$ and $\phi = xy^2z^3$, then match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

	List-I		List-II
A.	Div. $\vec{Q}$	1.	$y^2z^3\vec{i} + 2yxz^3\vec{j} + 3z^2y^2x\vec{k}$
B.	Grad ϕ	2.	$-\vec{i} + \vec{k}x^2$
C.	Curl $\vec{P}$	3.	z

Codes:

	A	B	C
(a)	1	2	3
(b)	2	1	3
(c)	3	1	2
(d)	3	2	1

Q.9 Which of the following statements is not true of a phasor?
(a) It may be a scalar or a vector.
(b) It is a time dependent quantity.
(c) It is a complex quantity.
(d) All are true.

Q.10 The maximum space rate of change of the function which is in increasing direction of the function is known as
(a) curl of the vector function
(b) gradient of the scalar function
(c) divergence of the vector function
(d) Stokes theorem

Q.11 **Assertion (A):** Divergence of a vector function $\vec{A}$ at each point gives the rate per unit volume at which the physical entity is issuing from that point.
Reason (R): If some physical entity is generated or absorbed within a certain region of the field, then that region is known as source or sink respectively and if there are no sources or sinks in the field, the net outflow of the incompressible physical entity over any part of the region is zero. However, the net outflow is said to be positive, if the total strength of the sources are greater than the total strength of sink and vice-versa.
(a) Both A and R are true and R is a correct explanation of A.
(b) Both A and R are true but R is not a correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q.12 Given a vector field $\vec{F}$. The Stoke's theorem states that,
(a) $\oint \vec{F}\cdot\vec{dl} = \iint(\vec{\nabla}\times\vec{F})\cdot\vec{ds}$
(b) $\oint \vec{F}\times\vec{dl} = \iint(\vec{\nabla}\cdot\vec{F})\,\vec{ds}$
(c) $\int \vec{F}\cdot\vec{dl} = \iint(\vec{\nabla}\times\vec{F})\cdot\vec{ds}$
(d) $\int \vec{F}\times\vec{dl} = \iint(\vec{\nabla}\cdot\vec{F})\,\vec{ds}$

Q.13 The vector $\vec{A}$ directed from (2, – 4, 1) to (0, –2, 0) in Cartesian coordinates is given by
(a) $-2\vec{a}_x + 2\vec{a}_y + \vec{a}_z$ (b) $-2\vec{a}_x + 2\vec{a}_y - \vec{a}_z$
(c) $-\vec{a}_x - 2\vec{a}_y + 2\vec{a}_z$ (d) $\vec{a}_x - 2\vec{a}_y - \vec{a}_z$

Q.14 The vector field given by
$$\vec{A} = yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z \text{ is}$$
(a) rotational and solenoidal
(b) rotational but not solenoidal
(c) irrotational and solenoidal
(d) irrotational but not solenoidal

Q.15 If $\vec{A} = \dfrac{\vec{a}_x}{\sqrt{x^2+y^2}}$, then the value of $\nabla\cdot\vec{A}$ at (2, 2, 0) will be
(a) –0.0884 (b) 0.0264
(c) –0.0356 (d) 0.0542

Q.16 If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then the value of $\vec{i}\times(\vec{r}\times\vec{i}) + \vec{j}\times(\vec{r}\times\vec{j}) + \vec{k}\times(\vec{r}\times\vec{k})$ is
(a) $\vec{r}$ (b) $2\vec{r}$
(c) $3\vec{r}$ (d) $6\vec{r}$

Q.17 What is the value of constant b so that the vector
$$\vec{V} = (x+3y)\vec{i} + (y-2x)\vec{j} + (x+bz)\vec{k}$$
is solenoidal?
(a) 2 (b) –1
(c) 3 (d) –2

Q.18 **Assertion (A):** The Gauss's divergence theorem permits us to express certain integrals by means of surface integrals.

Reason (R): Gauss's divergence theorem states that "the surface integral of the curl of a vector field taken over any surface s is equal to the line integral of the vector field around the closed periphery (contour) of the surface.
(a) Both A and R are true and R is a correct explanation of A.
(b) Both A and R are true but R is not a correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q.19 Which of the following option is not correct?
(a) A vector field $\vec{A}$ is solenoid, if $\nabla \cdot \vec{A} = 0$
(b) A vector field $\vec{A}$ is irrotational, if $\nabla \times \vec{A} = 0$
(c) A vector field V is harmonics, if $\nabla^2 V \neq 0$
(d) All options are correct

Q.20 Which of the following statements is not true regarding vector algebra?
(a) Dot product of like unit vector is unity.
(b) Dot product of unlike unit vector is zero.
(c) Cross product of two like unit vectors is a third unit vector having positive sign for normal rotation and negative for reverse rotation.
(d) All the above statements are true.

Q.21 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I
A. Gauss's divergence theorem
B. Stroke's theorem
C. The divergence
D. The curl

List-II
1. $\nabla \cdot \vec{A}$
2. $\oint_L \vec{A} \cdot d\vec{l} = \iint_s (\nabla \times \vec{A}) \cdot d\vec{s}$
3. $\iint_s \vec{A} \cdot d\vec{s} = \iint_s \vec{A} \cdot \vec{e}\, d\vec{s}$ ($\vec{e}$ - An unit vector)
4. $\nabla \times \vec{A}$

Codes:

	A	B	C	D
(a)	3	2	4	1
(b)	2	3	1	4
(c)	3	2	1	4
(d)	2	3	4	1

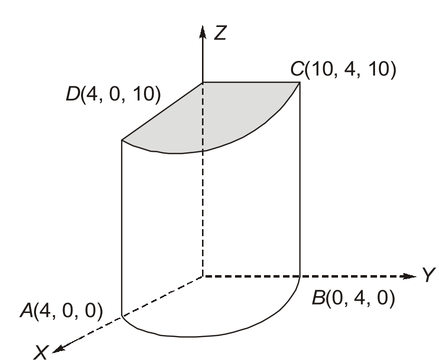
Q.22 A rigid body is rotating with an angular velocity of ω where, $\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$ and v is the line velocity. If $\vec{r}$ is the position vector given by $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then the value of curl $\vec{v}$ will be equal to
(a) $1/2\,\omega$ (b) ω
(c) $1/3\,\omega$ (d) $2\,\omega$

Q.23 Which of the following identity is not true?
(a) $\vec{A}(\vec{B} \cdot \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$
(b) $\nabla \cdot (\nabla \times \vec{A}) = 0$
(c) $\nabla \times \nabla \varphi \neq 0$
(d) None of the above

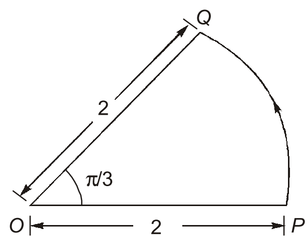
Q.24 The value of divergence of a vector quantity $\vec{A} = 4xy\,\hat{a}_x + xz\,\hat{a}_y + xyz\,\hat{a}_z$ at a point $P(1, -2, 3)$ will be
(a) 6 (b) –16
(c) –10 (d) 12

Q.25 Laplace equation in cylindrical coordinates is given by
(a) $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r \partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = 0$
(b) $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$
(c) $\nabla^2 V = \frac{-\rho}{\varepsilon}$
(d) $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r^2 \partial V}{\partial r}\right) + \left(-\frac{1}{r^2 \sin\theta}\right)\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$

Q.26 Consider the object shown in figure below calculate. The surface area $ABCD$,



Q.27 If $\vec{A} = \hat{a}_\rho + \hat{a}_\phi + \hat{a}_z$, the value of $\oint \vec{A} \cdot \vec{dl}$ around the closed circular quadrant shown in the given figure is ______.



Q.28 Given, $W = x^2y^2 + xy$, compute ∇W and the direction derivative $d\omega/dl$ in the direction,

$$\vec{A} = 3\hat{a}_x + 4\hat{a}_y + 12\hat{a}_z \text{ at } (2, -1, 0)$$

Q.29 If $\vec{E}$ is the electric field intensity then $\vec{\nabla} \times (\vec{\nabla} \cdot A)$ is equal to

(a) $\vec{E}$ (b) $|\vec{E}|$
(c) Null vector (d) zero

Q.30 Divergence of the vector field,

$V(x, y, z) = (x \sin xy)\hat{i} - (y \sin xy)\hat{i} + \sin z^2 \hat{k}$ is

Divergence = $2z \cos z^2$

(a) $2z \cos z^2$ (b) $\sin xy + 2z \cos z^2$
(c) $x \sin xy - \cos z$ (d) none of these

Q.31 The line integral of the vector field

$\vec{F} = 5x\vec{i} + 3y\hat{j} + x^2 z\hat{k}$ along a path from (0, 0, 0) to (1, 1, 1) parameterized by (t, t^2, t) is

Q.32 If the vector V given below is irrotational, then the values of a, b and c will be respectively

$$V = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$$

(a) $a = 4$, $b = 2$ and $c = -1$
(b) $a = 2$, $b = -1$ and $c = 4$
(c) $a = 4$, $b = -1$ and $c = 2$
(d) $a = 2$, $b = 4$ and $c = -1$

Q.33 What is the value of $\iint_s \vec{F} \cdot d\vec{s}$, where $\vec{F} = 4xz\vec{i}_1 - y^2\vec{i}_2 + yz\vec{i}_3$?

Here, s is the surface bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$ and $\vec{i}_1, \vec{i}_2, \vec{i}_3$ are unit vectors along x, y and z axes respectively.

(a) 1/2 (b) 5/2
(c) 2 (d) 3/2

Q.34 Given a vector $\vec{A} = 30e^{-r}\hat{a}_r - 2z\hat{a}_z$ in cylindrical co-ordinates. If a volume is enclosed by $r = 2$, $\phi = 2\pi$ and $z = 5$ then $\int (\nabla \cdot \vec{A}) dv$ = _______ .

Q.35 If $\vec{r} = x\vec{I}_x + y\vec{I}_y + z\vec{I}_z$, then which of the following relation will hold true?

(a) $\nabla \vec{r} = 3$ (b) $\nabla \times \vec{r} = 0$
(c) Both (a) and (b) (d) Neither (a) nor (b)

Q.36 If $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ is the position vector of point (x, y, z), then $\nabla(ln|r|)$ is

(a) $|r|\vec{r}$ (b) $|r|^2 \vec{r}$
(c) $\dfrac{\vec{r}}{|r|}$ (d) $\dfrac{\vec{r}}{|r|^2}$

Q.37 If $B = x^2y\,\hat{a}_x + (2x^2 + y)\hat{a}_y - (y - z)\hat{a}_z$ then $\nabla(\nabla \cdot B)$ is

(a) $2\hat{a}_x + 2xy\,\hat{a}_y$ (b) $2y\,\hat{a}_x + 2x\,\hat{a}_y$
(c) $x\,\hat{a}_x + y\,\hat{a}_y$ (d) $xy\,\hat{a}_x + xy\,\hat{a}_y$

Q.38 If $H = R\sin\theta\,\hat{a}_\phi$ (in spherical coordinates) then the magnitude of curl of the vector field H at the origin is ______.

Q.39 An iron ring has a mean circumference of 120 cm and a cross-sectional area of 8 sq cm. It is wound with 480 turns. When it is carrying 2 A, the flux is found to be 1 mWb. The permeability of the iron at this flux density is _________ .

Q.40 The divergence of the vector $\vec{A}$ which is given as follows,

$\vec{A} = 2r\cos\theta \cdot \cos\phi\hat{a}_r + r^{1/2}\hat{a}_\phi$ at point $\left(1, \dfrac{\pi}{4}, \dfrac{\pi}{3}\right)$ is

(a) 1.52 (b) 2.12
(c) 3.45 (d) 2.75

Q.41 If a general vector is given by $\vec{A} = (\sin 2\phi)\hat{a}_\phi$ in cylindrical co-ordinate system, then the curl of vector $\vec{A}$ at (4, π/6, 0) will be

(a) $\frac{\sqrt{3}}{8}\hat{a}_z$ (b) $-0.5\hat{a}_z$

(c) $\frac{\sqrt{3}}{4}\hat{a}_z$ (d) $0.5\hat{a}_z$

Q.42 The values of constants *a*, *b* and *c* so that

$\vec{V} = (x+2y+az)\hat{a}_x + (bx-3y-z)\hat{a}_y + (4x+cy+2z)\hat{a}_z$

is irrotational, then sum of value of constant *a*, *b* and *c* will be _______ .

Q.43 If a vector field $\vec{A}$ is said to be solenoidal, then which one of the following relations is true?

(a) $\oint_L \vec{A}\cdot\vec{dL} = 0$ (b) $\nabla\times\vec{A} \neq 0$

(c) $\oint_s \vec{A}\cdot\vec{ds} = 0$ (d) $\nabla\times\vec{A} = 0$

Q.44 A vector $\vec{P}$ is given by

$$\vec{P} = x^3 y\vec{a}_x - x^2y^2\vec{a}_y - x^2yz\,\vec{a}_z$$

Which of the following statements is **TRUE**?

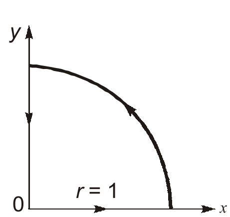
(a) $\vec{P}$ is solenoidal, but not irrotational

(b) $\vec{P}$ is irrotational, but not solensoidal

(c) $\vec{P}$ is neither solenoidal nor irrotational

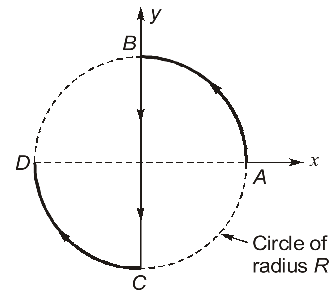
(d) $\vec{P}$ is both solenoidal and irrotational

Q.45 Given a vector field $\bar{A} = 2r\cos\phi\,T_r$ in cylindrical coordinates. For the contour as shown below, $\oint \bar{A}\cdot d\bar{l}$ is



(a) 1 (b) 1 – (π/2)
(c) 1 + (π/2) (d) –1

Q.46 What is the value of the integral $\int_c d\vec{l}$ along the curve *c* (*c* is the curve *ABCD* in the direction of the arrow) ?



(a) $2R(\hat{a}_x+\hat{a}_y)/\sqrt{2}$ (b) $-2R(\hat{a}_x+\hat{a}_y)/\sqrt{2}$

(c) $2R\,\hat{a}_x$ (d) $-2R\,\hat{a}_y$

Q.47 If $uF = \nabla v$, where *u* and *v* are scalar fields and *F* is a vector field, then *F*. curl *F* is equal to

(a) zero (b) $\frac{\nabla^2 v}{u^2}$

(c) $\frac{(\nabla v\cdot\nabla)\,v}{u^2}$ (d) not defind

Q.48 **Assertion (A):** The laplacian operator of a scalar function ϕ can be defined as "Gradient of the divergence of the scalar ϕ".

Reason (R): Laplacian operator may be a "scalar laplacian" or a "vector laplacian" depending upon whether it is operated with a scalar function or a vector, respectively.

(a) Both A and R are true and R is a correct explanation of A.
(b) Both A and R are true but R is not a correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

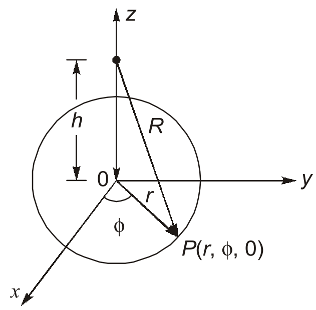
Q.49 Match **List-I (Physical quantities)** with **List-II (Dimensions)** and select the correct answer using the codes given below the lists:

	List-I		List-II
A.	Electric potential	**1.**	$MT^{-2}I^{-1}$
B.	Magnetic flux	**2.**	$ML^2T^{-3}I^{-1}$
C.	Magnetic field intensity	**3.**	IL^{-1}
D.	Magnetic flux density	**4.**	$ML^2T^{-2}I^{-1}$

Codes:

	A	B	C	D
(a)	2	4	3	1
(b)	4	2	3	1
(c)	1	2	1	3
(d)	4	2	1	3

Q.50 The unit vector $\vec{a}_R$ which points from $z = h$ on the z-axis towards $(r, \phi, 0)$ in cylindrical co-ordinates as shown below is given by

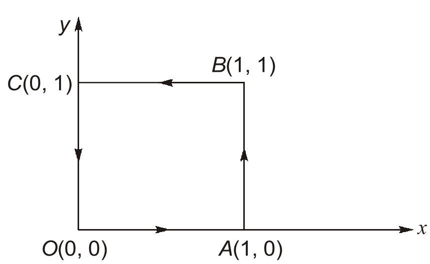


(a) $\dfrac{h\vec{a}_r - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ (b) $\dfrac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$

(c) $\dfrac{h\vec{a}_\phi - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ (d) $\dfrac{r\vec{a}_z - h\vec{a}_\phi}{\sqrt{r^2 + h^2}}$

Q.51 If $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ and $|\vec{r}| = r$ then find div $(r^2\nabla(ln r))$

Q.52 Given vector $\vec{A} = x^2y\,\hat{a}_x + 2xy^2\hat{a}_y$, find circulation of $\vec{A}$ along a closed path OABC as shown in figure below.



Q.53 For a vector field
$\vec{A} = xyz^3\hat{a}_x + xy^3z\hat{a}_y + x^3yz\hat{a}_z$. Evaluate the surface integral for a surface of unit cube by $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.

Q.54 Verify the above question using divergence theorem.

Multiple Select Questions (MSQ)

Q.55 $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ represents a position vector and $\|\vec{r}\|$ represents the normal of vector $\vec{r}$, then which of the below statements is/are true?

(a) Divergence of $\vec{r}$ is 3.

(b) Gradient of $\|\vec{r}\|^2$ is $3\vec{r}$

(c) Curl of $\vec{r}$ is 0

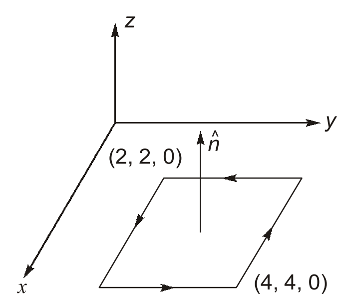
(d) Laplacian of $\|\vec{r}\|^2$ is 6.

Q.56 If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$, then which of the below relations are correct?

(a) $\nabla(\log r) = \dfrac{\vec{r}}{r}$ (b) $\nabla\left(\dfrac{1}{r}\right) = \dfrac{-\vec{r}}{r^3}$

(c) $\nabla \cdot \left(\dfrac{\vec{r}}{r^3}\right) = 1$ (d) $\nabla \cdot (3\vec{r}) = 9$

Q.57 Let $\vec{F} = xy^2\hat{a}_x + y^3\hat{a}_y + x^2y\hat{a}_z$ and the surface S consists of a square of length 2 lying in the xy plane as shown below:



Which of the following options is/are correct?

(a) $\iint\limits_S \vec{F} \cdot \hat{n}\,ds = 80$

(b) $\iint\limits_S (\vec{F} \times \hat{n})ds = 120\hat{a}_x - 112\hat{a}_y$

(c) $\nabla \times \vec{F} = x^2\hat{a}_x - 2xy\hat{a}_y - 2xy\,\hat{a}_z$

(d) $\iint\limits_S (\nabla \times \vec{F}) \cdot \hat{n}\,ds = -120$

Q.58 If $[\vec{a}, \vec{b}, \vec{c}]$ represents the scalar triple product of vectors $\vec{a}, \vec{b}$ and $\vec{c}$, then which of the below statements is/are true?

(a) $\left[\vec{a}, \vec{b}, \vec{c}\right] = \left[\vec{c}, \vec{b}, \vec{a}\right]$

(b) $\left[\vec{a}, \vec{b} + \vec{a}, \vec{c}\right] = 0$

(c) $\left[3\vec{b}, \vec{c}, \vec{a}\right] = 3\left[\vec{a}, \vec{b}, \vec{c}\right]$

(d) If $\left[\vec{a}, \vec{b}, \vec{c}\right] = 0$, the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

Q.59 The values of α for which the vectors $\vec{A} = \alpha\hat{a}_x + 2\hat{a}_y + 10\hat{a}_z$ and $\vec{B} = 4\alpha\hat{a}_x + 8\hat{a}_y - 2\alpha\hat{a}_z$ are perpendicular is/are

(a) 1 (b) 2
(c) 3 (d) 4

Q.60 Which of the below vector identities are true?

(a) $A \times (B \times C) = (A \times B) \times C$
(b) $A \times (B \times C) + C \times (A \times B) + B \times (C \times A) = 0$
(c) $(B \times C) \times (C \times A) = C(A \cdot B \times C)$
(d) $(A \times B) \cdot (C \times D) = (A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C)$

Q.61 For the scalar function, $\phi = x^2yz^3$, which of the below statements is/are correct?

(a) From the point (2, 1, –1) the directional derivative of ϕ is maximum in the direction represented by vector $-12\hat{i} - 4\hat{j} + 4\hat{k}$.
(b) The magnitude of greatest rate of change of ϕ from the point (2, 1, –1) is $4\sqrt{11}$.
(c) $(x - 2) + (y - 1) - 3(z + 1) = 0$ represents the tangent plane to the surface $\phi = 0$ at point (2, 1, –1).
(d) ϕ satisfies the Laplacian equation.

■■■■

Answers Vector Analysis

1. (c)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (a)
10. (b)	**11.** (a)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (a)	**16.** (b)	**17.** (d)	**18.** (c)
19. (c)	**20.** (c)	**21.** (c)	**22.** (d)	**23.** (c)	**24.** (c)	**25.** (a)	**26.** (62.83)	**27.** (5.14)
28. (–1.15)		**29.** (d)	**30.** (a)	**31.** (3.75)	**32.** (a)	**33.** (d)	**34.** (129.43)	
35. (c)	**36.** (d)	**37.** (b)	**38.** (2)	**39.** (1244)	**40.** (b)	**41.** (a)	**42.** (5)	**43.** (c)
44. (a)	**45.** (a)	**46.** (d)	**47.** (a)	**48.** (d)	**49.** (a)	**50.** (b)	**51.** (3)	**52.** (0.33)
53. (0.5)	**54.** (0.5)	**55.** (a,c,d)	**56.** (b,d)	**57.** (b,c)	**58.** (c,d)	**59.** (a,d)	**60.** (b,c,d)	**61.** (b,c)

Explanations Vector Analysis

1. (c)

For spherical coordinate systems,

$$\vec{dl} = r\sin\theta d\phi \hat{a}_\phi$$

$$\oint \vec{G} \cdot \vec{dl} = \int_0^{2\pi} 15r\hat{a}_\phi \cdot r\sin\theta d\phi\, \hat{a}_\phi$$

$$= 15 \cdot r^2 \cdot \sin\theta(2\pi)$$

$$= 15 \cdot (2)^2 \times \sin 30°(2\pi)$$

$$\oint \vec{G} \cdot \vec{dl} = 60\pi$$

2. (c)

Divergence (Curl $\vec{A}$) $= 0$

3. (d)

$$(\hat{a}_x \times \hat{a}_x) = 0$$

Since cross product with same vector is zero because $\theta = 0$ so $\sin\theta = 0$

$$\hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$\hat{a}_y \times \hat{a}_x = -\hat{a}_z$$

$$(\hat{a}_x \times \hat{a}_y) + (\hat{a}_y \times \hat{a}_x) = \hat{a}_z + (-\hat{a}_z) = 0$$

4. (a)

curl $\vec{F} = 0$ that means vector $\vec{F}$ is irrotational

div $\vec{F} = 0$ that means vector $\vec{F}$ is solenoidal since flux coming out from a solenoid is zero.

div(grad ϕ) = $\nabla \cdot (\nabla\phi) = 0 \rightarrow$ Laplace equation

div(divϕ) = not defined because div$\phi \rightarrow$ scalar quantity and div of a scalar quantity is not defined.

5. (d)

$$\nabla^2 V = \bar{\nabla} \cdot (\bar{\nabla} V)$$

= divergence of gradient of V

6. (b)

Let the unit vector be given by $\vec{a}_R$.

Now, $\vec{R}$ = Difference of two vectors

$$= r\vec{a}_r - h\vec{a}_z$$

$$\therefore \text{ Unit vector, } \vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$$

7. (d)

- $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D})$ is called "product of four vectors".
- $\vec{A}\times(\vec{B}\times\vec{C})$ is called "vector triple product".
- $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D})$ is called "vector product of four vectors".

8. (c)

Here, Div. $\vec{Q} = \nabla\cdot\vec{Q}$

$$= \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial y}\right)\cdot(\vec{i}\,zx)$$

$$= (\vec{i}\cdot\vec{i})\left(\frac{\partial}{\partial x}\cdot zx\right) = 1\cdot z = z$$

Also, Grad $\phi = \nabla\phi$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)(xy^2z^3)$$

$$= (y^2z^3\,\vec{i} + 2yxz^3\,\vec{j} + 3z^2y^2x\vec{k})$$

and, Curl $\vec{P} = \nabla\times\vec{P}$

$$= \begin{vmatrix} \vec{i} & 0 & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2y & 0 & (x-y) \end{vmatrix}$$

$$= \vec{i}\left[\frac{\partial}{\partial y}(x-y) - 0\right] + \vec{k}\left[0 - \frac{\partial}{\partial y}(x^2y)\right]$$

$$= \vec{i}\,[-1] + \vec{k}\,[x^2] = -\vec{i} + x^2\,\vec{k}$$

9. (a)

A phasor is always a vector quantity.

10. (b)

Gradient of a scalar;

∇A = maximum rate of change of scalar A with respect to given coordinates system.

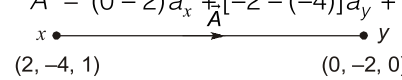
11. (a)

Both assertion and reason are true and reason is the correct explanation of assertion. Reason is the physical interpretation of divergence.

13. (b)

The vector $\vec{A}$ is given as

$$\vec{A} = (0-2)\vec{a}_x + [-2-(-4)]\vec{a}_y + (0-1)\vec{a}_z$$



$$= -2\vec{a}_x + 2\vec{a}_y - \vec{a}_z$$

14. (c)

The vector field $\vec{A}$ will be irrotational, if $\nabla\times\vec{A} = 0$.

Now, $$\nabla\times\vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(xz)\right]\vec{a}_x + \left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz)\right]\vec{a}_y + \left[\frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(yz)\right]\vec{a}_z$$

$$= [x-x]\,\vec{a}_x + [y-y]\,\vec{a}_y + [z-z]\,\vec{a}_z$$
$$= 0$$

Hence, $\vec{A}$ is irrotational.

The vector field $\vec{A}$ will be solenoidal, if $\nabla\cdot\vec{A} = 0$

Here,

$$\nabla\cdot\vec{A} = \left(\vec{a}_x\frac{\partial}{\partial x} + \vec{a}_y\frac{\partial}{\partial y} + \vec{a}_z\frac{\partial}{\partial z}\right)\cdot(yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z)$$

$$= \vec{a}_x\cdot\vec{a}_x\frac{\partial}{\partial x}(yz) + \vec{a}_y\cdot\vec{a}_y\frac{\partial}{\partial y}(xz) + \vec{a}_z\cdot\vec{a}_z\frac{\partial}{\partial z}(xy)$$

$$= 0 + 0 + 0 = 0$$

Hence, $\vec{A}$ is solenoidal.

15. (a)

Given, $$\vec{A} = \frac{1}{\sqrt{x^2+y^2}}\vec{a}_x$$

$$\therefore \quad \nabla\cdot\vec{A} = \frac{\partial}{\partial x}(A_x) + \frac{\partial}{\partial y}(A_y) + \frac{\partial}{\partial z}(A_z)$$

$$= \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{x^2+y^2}}\right) + 0 + 0$$