Electrical Engineering

Signals and Systems

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Signals and Systems

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Signals and Systems

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Introduction to Signals and Systems

This book starts with basic and extensive chapter on signals in which continuous and discrete-time case are discussed in parallel. A variety of basic signals, functions with their mathematical description, representation and properties are incorporated. A substantial amount of examples are given for quick sketching of functions. A chapter on systems is discussed separately which deals with classification of systems, both in continuous and discrete domain and more emphasize is given to LTI systems and analytical as well as graphical approach is used to understand convolution operation. These two chapters makes backbone of the subject.

Further we shall proceed to transform calculus which is important tool of signal processing. A logical and comprehensive approach is used in sequence of chapters. The continuous time Fourier series which is base to the Fourier transform, deals with periodic signal representation in terms of linear complex exponential, is discussed.

The Fourier transform is discussed before Laplace transform. The sampling, a bridge between continuous-time and discrete-time, is discussed to understand discrete-time domain.

A major emphasis is given on proof of the properties so that students can understand and analyzes fundamental easily.

A point wise recapitulation of all the important points and results in every chapter proves helpful to students in summing up essential developments in the chapter which is an integral part of any competitive examination.

CHAPTER 1

Introduction to Signals

Introduction

A signal is any quantity having information associated with it. It may also be defined as a function of one or more independent variables which contain some information.
A function defines a relationship between two sets i.e. one is domain and another is range.

It means function defines mapping from one set to another and similarly a signal may also be defined as mapping from one set (domain) to another (range). e.g.

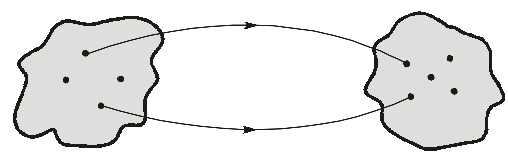


Figure-1.1

- A speech signal would be represented by acoustic pressure as a function of time.
- A monochromatic picture would be represented by brightness as a function of two spatial variables.
- A voltage signal is defined by a voltage across two points varying as function of time.
- A video signal, in which color and intensity as a function of 2-dimensional space (2D) and 1-dimensional time (i.e. hybrid variables).

NOTE: In this course of "signals and systems", we shall focus on signals having only one variable and will consider 'time' as independent variable.

1.1 Elementary Signals

These signals serve as basic building blocks for construction of somewhat more complex signals. The list of elementary signals mainly contains singularity functions and exponential functions.

These elementary signals are also known as basic signals/standard signals.

Let us discuss these basic signals one-by-one.

1.1.1 Unit Impulse Function

A continuous-time unit impulse function δ(t), also called as Dirac delta function is defined as

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

The unit-impulse function is represented by an arrow with strength of '1' which represents its 'area' or 'weight'.

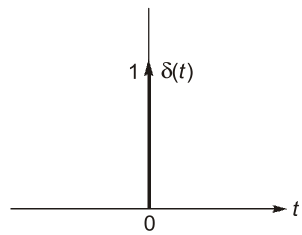


Figure-1.2

The above definition of an impulse function is more generalised and can be represented as limiting process without any regard to shape of a pulse. For example, one may define impulse function as a limiting case of rectangular pulse, triangular pulse Gaussian pulse, exponential pulse and sampling pulse as shown below:

(i) Rectangular Pulse

$$\delta(t) = \lim_{\varepsilon \to 0} p(t)$$

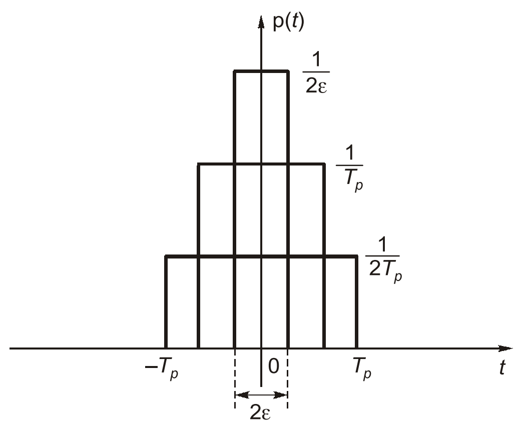


Figure-1.3

(ii) Triangular Pulse

$$\delta(t) = \begin{cases} \lim\limits_{\tau \to 0} \dfrac{1}{\tau}\left[1 - \dfrac{|t|}{\tau}\right] & ; \quad |t| < \tau \\ 0 & ; \quad |t| > \tau \end{cases}$$

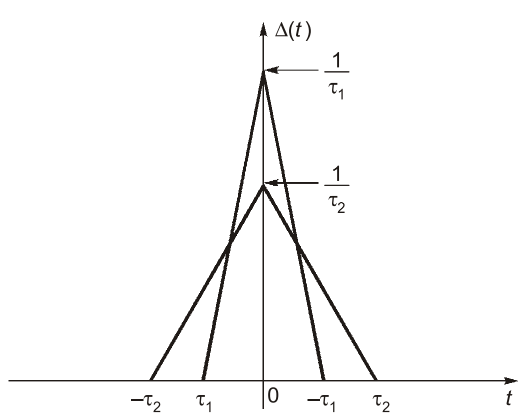


Figure-1.4

(iii) Gaussian Pulse

$$\delta(t) = \lim_{\tau \to 0} \frac{1}{\tau}\left[e^{-\pi t^2/\tau^2}\right]$$

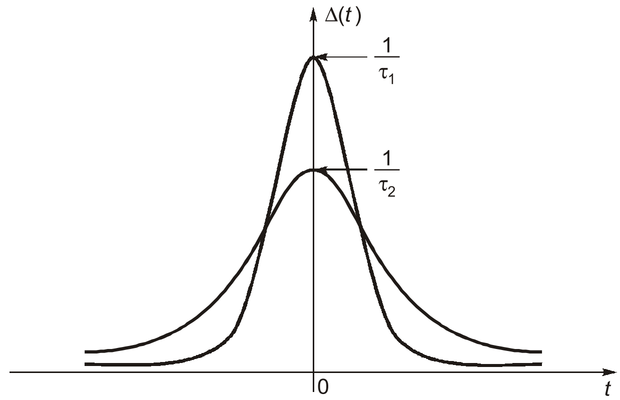


Figure-1.5

(iv) Exponential Pulse

$$\delta(t) = \lim_{\tau \to 0} \frac{1}{2\tau}\left[e^{-|t|/\tau}\right]$$

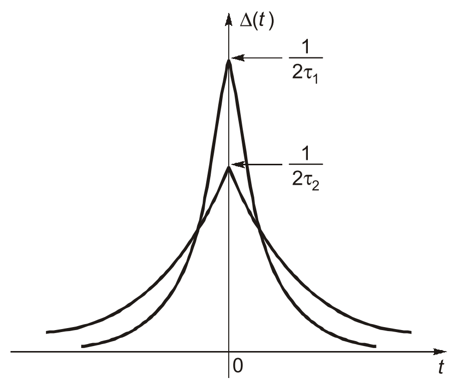


Figure-1.6

(v) Sampling Function

$$\int_{-\infty}^{\infty} \frac{k}{\pi} Sa(kt)dt = 1$$

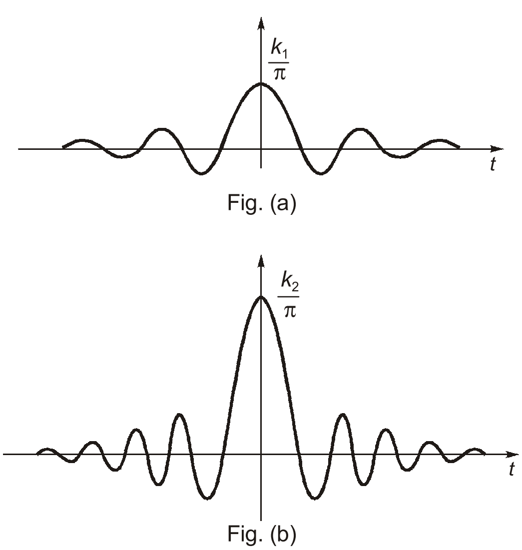


Figure-1.7

Properties of Continuous Time Unit Impulse Function

(i) Scaling property:

$$\delta(at) = \frac{1}{|a|}\delta(t) \quad ; \quad \text{'}a\text{' is a constant, postive or negative}$$

Proof:

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

Integrating above equation on both the sides with respect to 't'.

$$\int_{-\infty}^{+\infty}\delta(at)\,dt = \int_{-\infty}^{+\infty}\frac{1}{|a|}\delta(t)\,dt$$

Let $at = \tau$

$a \cdot dt = d\tau$; 'a' is a constant, positive or negative

or $|a| \cdot dt = d\tau$

Now, $$\int_{-\infty}^{+\infty}\delta(at)\,dt = \int_{-\infty}^{+\infty}\delta(\tau)\cdot\frac{d\tau}{|a|} = \int_{-\infty}^{+\infty}\frac{1}{|a|}\delta(t)\cdot dt$$

By definition, $$\int_{-\infty}^{+\infty}\delta(t)dt = \int_{-\infty}^{+\infty}\delta(\tau)d\tau = 1$$

Important Expressions

- $\delta(at \pm b) = \frac{1}{|a|}\delta\left(t \pm \frac{b}{a}\right)$
- $\delta(-t) = \delta(t)$ $\quad\cdots$ $\delta(t)$ is an even function of time.

(ii) Product property/multiplication property:

$$x(t)\delta(t - t_o) = x(t_o)\,\delta(t - t_o)$$

Proof:

The function $\delta(t - t_o)$ exists only at $t = t_o$. Let the signal $x(t)$ be continuous at $t = t_o$.

Therefore, $$x(t)\,\delta(t - t_o) = x(t)\big|_{t = t_o}\cdot\delta(t - t_o)$$

$$= x(t_o)\,\delta(t - t_o)$$

Important Expressions

- $x(t)\,\delta(t) = x(0)\,\delta(t)$

(iii) Sampling property:

$$\int_{-\infty}^{+\infty}x(t)\,\delta(t - t_o)\,dt = x(t_o)$$

Proof :

Using product property of impulse function

$$x(t)\,\delta(t-t_o) = x(t_o)\,\delta(t-t_o)$$

Integrating above equation on both the sides with respect to 't'.

$$\int_{-\infty}^{+\infty} x(t)\,\delta(t-t_o)dt = \int_{-\infty}^{+\infty} x(t)\,\delta(t-t_o)dt$$

$$= x(t_o)\int_{-\infty}^{+\infty} \delta(t-t_o)dt = x(t_o)$$

Important Expressions

- $\displaystyle\int_{-\infty}^{+\infty} x(t)\delta(t)\,dt = x(0)$

(iv) The first derivative of unit step function results in unit impulse function.

$$\delta(t) = \frac{d}{dt}u(t)$$

Proof :

Let the signal $x(t)$ be continuous at t = 0.

Consider the integral $\displaystyle\int_{-\infty}^{+\infty} \frac{d}{dt}[u(t)]\,x(t)dt = [u(t)\,x(t)]_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} x'(t)\,u(t)\,dt$

$$= x(\infty) - \int_{0}^{\infty} x'(t)\,d(t)$$

$$= x(\infty) - [x(t)]_0^{\infty}$$

$$= x(0) \quad \text{...(i)}$$

We know from sampling property $\displaystyle x(0) = \int_{-\infty}^{+\infty} x(t)\,\delta(t)\,dt$ ...(ii)

From equations (i) and (ii), we get

$$\int_{-\infty}^{+\infty} \frac{d}{dt}[u(t)]\,x(t)dt = \int_{-\infty}^{+\infty} x(t)\delta(t)\,dt$$

On comparing, we get $\displaystyle\delta(t) = \frac{d}{dt}u(t)$

(v) Derivative property:

$$\int_{t_1}^{t_2} x(t)\delta^n(t-t_o)dt = (-1)^n x^n(t)\Big|_{t=t_0} \;;\quad t_1 < t_0 < t_2 \text{ and suffix } n \text{ means } n^{th} \text{ derivative}$$

Proof:

Let the signal $x(t)$ be continuous at $t = t_0$ where $t_1 < t_0 < t_2$.

Consider the derivative $$\frac{d}{dt}[x(t)\,\delta(t-t_0)] = x(t)\,\delta'(t-t_0) + x'(t)\,\delta(t-t_0)$$

Integrating above equation on both the sides with respect to 't'.

$$\int_{t_1}^{t_2} \frac{d}{dt}[x(t)\,\delta(t-t_0)]dt = \int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt + \int_{t_1}^{t_2} x'(t)\,\delta(t-t_0)dt$$

$$[x(t)\,\delta(t-t_0)]_{t_1}^{t_2} = \int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt + \int_{t_1}^{t_2} x'(t)\,\delta(t-t_0)dt$$

$$[x(t_2)\,\delta(t_2-t_0) - x(t_1)\,\delta(t_1-t_0)] = \int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt + \int_{t_1}^{t_2} x'(t)\,\delta(t-t_0)dt$$

Here, $\delta(t_1 - t_0) = 0$ and $\delta(t_2 - t_0) = 0$ because $t_0 \neq t_1$ or $t_0 \neq t_2$

So, $$0 = \int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt + \int_{t_1}^{t_2} x'(t)\delta(t-t_0)dt$$

$$\int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt = (-1)\int_{t_1}^{t_2} x'(t)\delta(t-t_0)dt \qquad (\because \text{ using sampling property})$$

$\Rightarrow$ $$= (-1)\,x'(t_0)$$

Hence, $$\int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt = (-1)^1\,x'(t_0)$$

If same procedure is repeated for second derivative, we get

$$\int_{t_1}^{t_2} x(t)\,\delta''(t-t_0)dt = (-1)^2\,x''(t_0)$$

On generalising aforementioned results, we get

$$\int_{t_1}^{t_2} x(t)\,\delta^n(t-t_0)dt = (-1)^n\,x^n(t_0)$$

(vi) Shifting Property:

According to shifting property, any signal can be produced as combination of weighted and shifted impulses.

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

Proof:

Using product property

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

Replacing t_0 by τ

$$x(t)\,\delta(t-\tau) = x(\tau)\,\delta(t-\tau)$$

Integrating above equation on both the sides with respect to 'τ'.

$$\int_{-\infty}^{+\infty} x(t)\,\delta(t-\tau)d\tau = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)d\tau$$

$$x(t)\int_{-\infty}^{+\infty} \delta(t-\tau)d\tau = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)d\tau$$

$$x(t)\cdot 1 = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)d\tau$$

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

(vii) The derivative of impulse function is known as ***doublet*** function.

$$\delta'(t) = \frac{d}{dt}\delta(t)$$

Graphically,

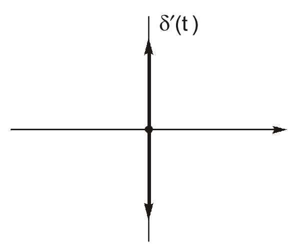


Figure-1.8

Area under the ***doublet*** function is always zero.

Discrete-Time Case

The discrete time unit impulse function δ[n], also called unit sample sequence or delta sequence is defined as

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

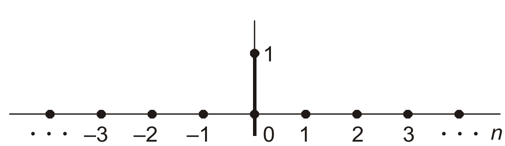


Figure-1.9

It is also known as *Kronecker delta*.

Properties of Discrete Time Unit Impulse Sequence

(i) Scaling property:

$$\delta[kn] = \delta[n]\ ;\ k \text{ is an integer}$$

Proof:
By definition of unit impulse sequence

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Similarly,
$$\delta[kn] = \begin{cases} 1, & kn = 0 \\ 0, & kn \neq 0 \end{cases}$$

$$= \begin{cases} 1, & n = \dfrac{0}{k} = 0 \\ 0, & n \neq \dfrac{0}{k} \neq 0 \end{cases}$$

$$= \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} = \delta[n]$$

(ii) Product property:

$$x[n]\,\delta[n - n_0] = x[n_0]\,\delta[n - n_0]$$

From definition,

$$\delta[n - n_0] = \begin{cases} 1, & n = n_0 \\ 0, & n \neq n_0 \end{cases}$$

We see that impulse has a non zero value only at $n = n_0$

Therefore,
$$x[n]\,\delta[n - n_0] = x[n]\big|_{n = n_0}\,\delta[n - n_0]$$

$$x[n]\,\delta[n - n_0] = x[n_0]\,\delta[n - n_0]$$

(iii) Shifting property:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n - k]$$

Proof:
From product property

$$x[n]\,\delta[n - n_0] = x[n_0]\,\delta[n - n_0]$$

Replacing n_0 by 'k'

$$x[n]\,\delta[n - k] = x[k]\,\delta[n - k]$$

$$\Rightarrow \quad \sum_{k=-\infty}^{+\infty} x[n]\,\delta[n - k] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n - k]$$

$$\Rightarrow \quad x[n] \sum_{k=-\infty}^{+\infty} \delta[n - k] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n - k]$$

$$\Rightarrow \quad x[n] \cdot 1 = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n - k]$$

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n - k]$$

(iv) The first difference of unit step sequence results in unit impulse sequence.

$$\delta[n] = u[n] - u[n-1]$$

Proof:

By definition of unit step sequence

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k] \quad \text{...(i)}$$

$$= \delta[n] + \sum_{k=1}^{\infty} \delta[n-k]$$

But,

$$u[n-1] = \sum_{k=1}^{\infty} \delta[n-k]$$

$$u[n] = \delta[n] + u[n-1]$$

We get,

$$\delta[n] = u[n] - u[n-1]$$

Graphically we can see,

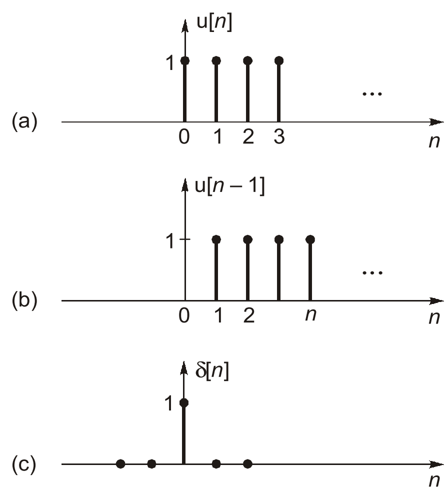


Figure-1.10

Summary Table:

S.No.	Properties of CT unit Impulse Function	Properties of DT unit impulse sequence
1.	$\delta(t)=\begin{cases}\infty, & t=0\\ 0, & \text{otherwise}\end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)dt=1$	$\delta[n]=\begin{cases}1, & \text{if } n=0;\\ 0, & \text{otherwise}\end{cases}$
2.	$x(t)\,\delta(t-t_0)=x(t_0)\delta(t-t_0)$	$x[n]\delta[n-k]=x[k]\delta[n-k]$
3.	$\delta(t)=\frac{d}{dt}u(t)$	$\delta[n]=u[n]-u[n-1]$
4.	$\int_0^{\infty}\delta(t-\tau)d\tau=u(t)$	$\sum_{k=0}^{\infty}\delta[n-k]=u[n]$
5.	$x(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)d\tau$	$x[n]=\sum_{-\infty}^{\infty}x[k]\,\delta[n-k]$
6.	$\int_{-\infty}^{\infty}x(t)\delta(t-t_0)dt=x(t_0)$	$\sum_{n=-\infty}^{\infty}x[n]\delta[n-n_0]=x[n_0]$
7.	$\delta(at)=\frac{1}{\lvert a\rvert}\delta(t)$ $\delta(at\pm b)=\frac{1}{\lvert a\rvert}\delta\left(t\pm\frac{b}{a}\right)$ $\delta(-t)=\delta(t)$	$\delta[kn]=\delta[n]$ $\delta[-n]=\delta[n]$
8.	$\int_{t_1}^{t_2}x(t)\delta(t)dt=\begin{cases}x(0), & t_1<t<t_2\\ 0, & \text{otherwise}\end{cases}$	
9.	$\int_{t_1}^{t_2}x(t)\delta^n(t-t_0)dt=(-1)^n\ x^n(t_0),\ t_1<t_0<t_2$ where suffix n mean n^{th} derivative	
10.	$\delta'(t)=\frac{d}{dt}\delta(t)$	

***Example* - 1.1** The Dirac delta function $\delta(t)$ is defined as

(a) $\delta(t)=\begin{cases}1 ; & t=0\\ 0 ; & \text{otherwise}\end{cases}$

(b) $\delta(t)=\begin{cases}\infty ; & t=0\\ 0 ; & \text{otherwise}\end{cases}$

(c) $\delta(t)=\begin{cases}1 ; & t=0\\ 0 ; & \text{otherwise}\end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)dt=1$

(d) $\delta(t)=\begin{cases}\infty ; & t=0\\ 0 ; & \text{otherwise}\end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)dt=1$

***Solution :*(d)**

Example - 1.2 The integral $\int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{6}\right) 6\sin(t)$ dt evaluate to

(a) 6 (b) 3
(c) 1.5 (d) 0

Solution :(b)

Given signal is

$$x(t) = \int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{6}\right) 6\sin t\, dt$$

By shifting property of unit impulse function

$$\int_{t_1}^{t_2} x(t)\delta(t - t_0)dt = \begin{cases} x(t_0); & t_1 < t_0 < t_2 \\ 0; & \text{elsewhere} \end{cases}$$

$$\int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{6}\right) 6\sin(t)\, dt = 6 \cdot \sin\frac{\pi}{6}$$

$$= 6 \times \frac{1}{2} = 3$$

Example - 1.3 If $y(t) + \int_{0^-}^{\infty} y(\tau)x(t-\tau)d\tau = \delta(t) + x(t)$, then $y(t)$ is

(a) $u(t)$ (b) $\delta(t)$
(c) $r(t)$ (d) 1

Solution :(b)

Let $y(t) = \delta(t)$

$$y(t) + \int_{0^-}^{\infty} y(\tau)\, x(t-\tau)d\tau = \delta(t) + \int_{0^-}^{\infty} \delta(\tau)\, x(t-\tau)d\tau$$

$$= \delta(t) + x(t)$$

So, $y(t) = \delta(t)$ satisfies the given equation.

Example - 1.4 Which of the following is NOT a property of impulse function?

(a) $x(t)\,\delta(t - t_o) = x(t_o)\,\delta(t - t_o)$ (b) $x(t) * \delta(t - t_o) = x(t - t_o)$

(c) $\int_{t_1}^{t_2} x(t)\,\delta(t - t_o)\, dt = x(t_o)\,;\, t_1 < t < t_2$ (d) $\int_{-\infty}^{+\infty} x(t)\,\delta^n(t - t_0)\, dt = (-1)^n \frac{d^n}{dt^n} x(t)\Big|_{t = t_0}$

Solution :(d)

By derivative property

$$\int_{-\infty}^{+\infty} x(t)\,\delta^n(t - t_0)dt = (-1)^n \frac{d^n}{dt^n} x(t)\Big|_{t=t_0}$$

Student's Assignments 1

Objective Questions

Q.1 Which one of the following relations is **not** correct?

(a) $f(t)\,\delta(t) = f(0)\,\delta(t)$ (b) $\int_{-\infty}^{\infty} f(t)\,\delta(\tau)\,d\tau = 1$

(c) $\int_{-\infty}^{\infty} \delta(\tau)\,d(\tau) = 1$ (d) $f(t)\,\delta(t-\tau) = f(\tau)\,\delta(t-\tau)$

Q.2 The odd component of the signal $x(t) = e^{-2t}\cos t$ is

(a) $\cosh(2t)\cos t$ (b) $-\sinh(2t)\cos t$

(c) $-\cosh(2t)\cos t$ (d) $\sinh(2t)\cos t$

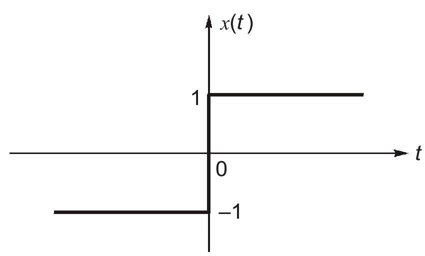
Q.3 The value of

$$\int_{-2}^{2}\left[(t-3)\,\delta(2t+2) + 8\cos\pi t\;\delta'(t-0.5)\right]dt \text{ is}$$

(a) 23.13 (b) 13.56

(c) 6.39 (d) 7.85

Q.4 Function $x(t)$ is shown in the figure.



The $x(t)$ in terms of unit step function is _____ and the odd part of unit step function is ____ respectively.

(a) $2u(t) + 1;\ \frac{1}{2}\text{sgn}(t)$

(b) $2u(t) - 1;\ \frac{1}{2}\text{sgn}(t)$

(c) $2u(t) - 1;\ \frac{1}{2}$

(d) $2u(-t) - 1;\ \frac{1}{2}$

Q.5 An LTI system has the input signal $x[n]$. The correct sequence of operation to get output $y[n] = x[n - M/L]$; $M > 1$, $L > 1$ is

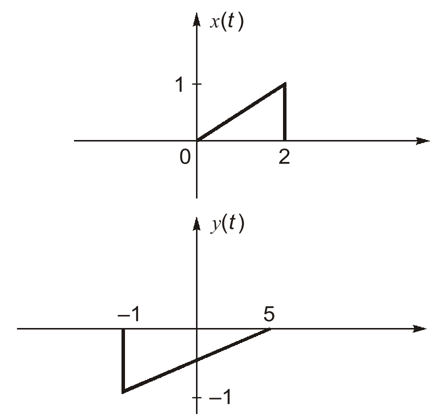
(a) Interpolation by L, Delay by M, Decimation by L

(b) Delay by M, Interpolation by L, Decimation by M

(c) Decimation by L, Delay by M, Interpolation by L

(d) Interpolation by L, Decimation by L, Delay by M

Q.6 Two signals $x(t)$ and $y(t)$ are shown below.



then $x(t)$ in terms of $y(t)$ can be written as

(a) $-x\left(\frac{t-5}{3}\right)$ (b) $-x\left(\frac{t+5}{3}\right)$

(c) $-x\left(\frac{-(t+5)}{3}\right)$ (d) $-x\left(\frac{-(t-5)}{3}\right)$

Q.7 Fundamental frequency of periodic signal $e^{j\omega_o n}$ is given as

(where m is integer and N is the period of the signal)

(a) $m\left(\frac{N}{2\pi}\right)$ (b) $N\left(\frac{2\pi}{m}\right)$

(c) $m\left(\frac{2\pi}{N}\right)$ (d) None of these

Q.8 A discrete time system is given as:

$$x[n] = \cos\left(\frac{n}{4}\right) \cdot \sin\left(\frac{\pi n}{4}\right)$$

The signal is

(a) periodic with 8 (b) periodic with $8(\pi + 1)$
(c) periodic with 4 (d) non-periodic

Numerical Questions

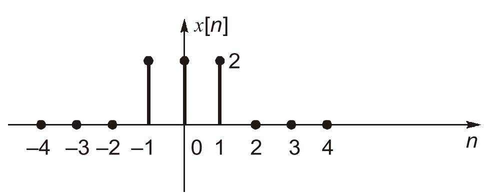
Q.9 The power of signal $x[n] = (-1)^n\, u[n]$ is _____ W.

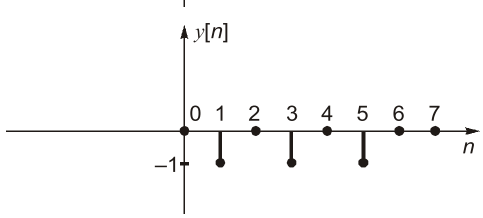
Q.10 A discrete time signal is given as

$$x[n] = \cos\left(\frac{\pi n}{3}\right) \cdot \left(u[n] - u[n-6]\right)$$

The energy of the signal is __________ J.

Q.11 Two functions $x[n]$ and $y[n]$ are shown in following figures.





If $y[n] = \alpha\, x\left[\frac{n - n_0}{k}\right]$ then value of $n_0 + \alpha + k$ is _____.

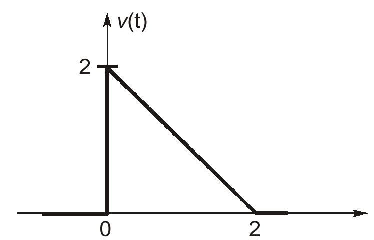
Answers :

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(b)
5.	(a)	**6.**	(d)	**7.**	(c)	**8.**	(d)
9.	(0.5)	**10.**	(3)	**11.**	(4.5)		

Student's Assignments 2

Q.1 With sketches of waveforms, explain the four class of signals.

Q.2 For the non-recurring waveform shown below, express $v(t)$ in terms of steps, ramps and related functions as needed.



Ans. $[v(t) = 2u(t) - r(t) + r(t-2)]$

Q.3 Show that,

(i) $\int_{-\infty}^{+\infty} \delta(t)\, e^{-j\omega t} dt = 1$

(ii) $\int_{-\infty}^{+\infty} \delta(t-2) \cos\left(\frac{\pi t}{4}\right) dt = 0$

(iii) $\int_{-\infty}^{+\infty} e^{-2(x-t)} \delta(2-t) dt = e^{-2(x-2)}$

Q.4 Calculate the energy of following signal

$$y(t) = \int_{-\infty}^{t} [\delta(\tau + 2) - \delta(\tau - 2)]\, d\tau$$

Q.5 Prove that shifting a signal does not affect its energy.

■■■■