SSC-JE 2025

Staff Selection Commission
Junior Engineer Examination

Electrical Engineering

Circuit Theory

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY PUBLICATIONS

Circuit Theory

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Basic Concepts

1.1 Introduction

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics communications, and instrumentation are based on electric circuit theory. Therefore, the basic electric circuit theory course is the most important course for an electrical engineering student, and always an excellent starting point for the students of electrical engineering. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved.

We commence our study by defining some basic concepts. These concepts include charge, current, voltage, circuit elements, power and energy.

1.2 Circuit Elements

Any individual circuit component (inductor, resistor, capacitor, generator etc.) with two terminals, by which it can be connected to other electrical components.

1.3 Network and Circuit

Network is any possible inter-connection of circuit elements or branches. Circuit is a closed energized network.

NOTE:

Every circuit is a network, but not all networks are circuits.

1.4 Classification of Circuit Elements

1.4.1 Active and Passive Elements

- If we have a network element that is absorbing power i.e. energy delivered to the element $\int\limits_{-\infty}^{t} v(t)\,i(t)\,dt$ is positive, then the element is *passive element*. Eg: resistor, inductor, diode and capacitor.

- If we have a network element that is delivering power i.e. energy delivered to the element $\int\limits_{-\infty}^{t} v(t)\,i(t)dt$ is negative, then the element is *active element*. Eg: Independent sources, transistor and op-amp.

NOTE:

An active element can provide power or power gain to the circuit for infinite duration of time, that is why charged capacitor or inductor are not active elements.

1.4.2 Bilateral and Unilateral Elements

- For a *bilateral element*, the voltage current relationship is the same for current flowing in either direction. Eg: resistor, inductor and capacitor.
- For a *unilateral element*, the voltage current relationship is different for two directions of current flow. Eg: diode.

1.4.3 Lumped and Distributed Elements

- *Lumped elements* are considered as the separate elements which are very small in size. For Eg: resistor, inductor and capacitor.
- *Distributed elements* are not electrically separable. These are distributed over the entire length of the circuit. Eg: Transmission lines.

1.4.4 Linear and Non-Linear Elements

- An element that follows *additivity* and *homogeneity property* for relationship between excitation and response is called a *linear element*.
- An element that does not follow *additivity* and *homogeneity property* for relationship between excitation and response is called a *non-linear element*.

NOTE

- The *homogeneity property* requires that if excitation is multiplied by a constant, then the response gets multiplied by the same constant.
- The *additivity property* requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

Example - 1.1 The *v-i* characteristic of an element is shown in the figure given below. The element is
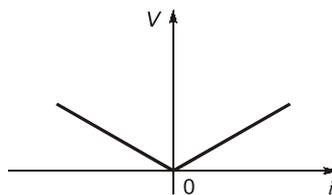
(a) Non-linear, active, non-bilateral   (b) Linear, active, non-bilateral
(c) Non-linear, passive, non-bilateral (d) Non-linear, active, bilateral   [SSC-JE : 2017]

Solution: (a)
- Non-bilateral as same *v-i* relationship doesn't exist for current flowing in either direction.
- Active as *v-i* is negative in second quadrant.
- It is a non-linear element.

NOTE ▶

- If the characteristic curve is similar in opposite quadrants, then the element is bidirectional, otherwise it is unidirectional.
- If the product of voltage and current at any point on curve is negative, then the element is active, otherwise it is passive.
- Every linear element must exhibit bidirectional property. However, the converse is not true.

1.5 Charge

- Charge is an electrical property of the atomic particles of which matter consists, measured in Coulombs (C).
- According to experimental observations, the only charges that occur in nature are integral multiple of the electronic charge $e = -1.602 \times 10^{-19}$ C.
- The Coulomb is a large unit for charges. In 1C of charge, therefore

$$\frac{1}{(1.602 \times 10^{-19})} = 6.24 \times 10^{18} \text{ electrons}$$

- The law of conservation of charge states that a charge can neither be created nor be destroyed, can be only transferred.

1.6 Current

- The phenomenon of transferring charge from one point in a circuit to another is described by the term **electric current.** An electric current may be defined as the time rate of net motion of electric charge across a cross-sectional boundary. A random motion of electrons in a metal does not constitute a current unless there is a net transfer of charge with time.
  In equation form, the current is,

$$i = \frac{dq}{dt}$$

- If the charge q is given in coulombs and the time t is measured in seconds, then current is measured in **amperes.**

1.7 Voltage

- To move the electrons from one point to other point in particular direction external force is required. In analytical circuit external force is provided by emf and it is given by,

$$v = \frac{dw}{dq}$$

  where a differential amount of charge dq is given with a differential increase in energy dw. The quantity "energy per unit charge" or identically, "work per unit charge", is given the name voltage. Thus, the voltage across a terminal pair is a measure of the work required to move the charge through the element.
- A voltage can exist between a pair of electrical terminals whether a current is flowing or not. An automobile battery, for example, has a voltage of 12 V across its terminals even if nothing whatsoever is connected to the terminals.

1.8 Power

- If potential is multiplied by the current, dq/dt, as

$$\frac{dw}{dq} \times \frac{dq}{dt} = \frac{dw}{dt} = p \qquad \text{...(A)}$$

the result is seen as to be a time rate of change of energy, which is **power p**. Thus power is the product of potential and current,

$$p = vi$$

1.9 Energy

- The capacity to do the work is called as energy. Energy as a function of power is found by integrating equation (A). Thus total energy at time t is the integral

$$w = \int_{-\infty}^{t} p\, dt$$

- The change in energy from time t_1 to time t_2 may similarly be found by integrating from t_1 to t_2.

1.10 Resistance

- The physical property of a material by virtue of which it opposes the flow of electrons through the material is known as **resistance.** Resistance is denoted by 'R' or 'r' and unit is **ohm (Ω).** Resistance is given as

$$R = \frac{\rho l}{a}$$

where, ρ = resistivity of material (reciprocal of conductivity)

l = length of material

a = area of cross-section of material

$\Rightarrow$ Resistance converts electrical energy into heat energy.

$\Rightarrow$ When circuit resistance is approaching to zero then the circuit is called as short circuit. Properties of short circuit (ideal case) are given below:

$$R = 0$$
$$V = 0$$
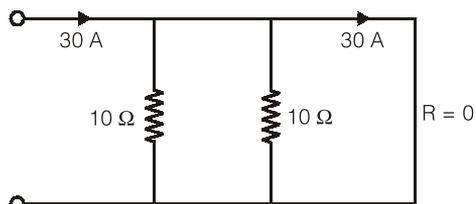
$\Rightarrow$ When circuit resistance is approaching to infinite then the circuit is called as open circuit. Properties of open circuit (ideal case) are given below:

$$I = 0$$
$$R = \infty$$

1.11 Ohm's Law

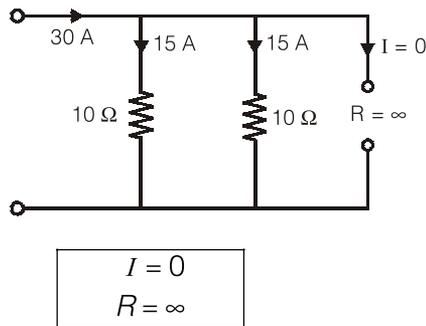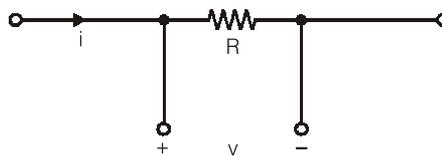At constant temperature, the potential difference *v* across the terminals of a resistor *R*, as in the figure, is directly proportional to the current *i* flowing through it. That is,

$$v = Ri$$

(Voltage current relationship of a resistor)

Ohm's law can also be expressed in terms of conductance *G* (which is reciprocal of *R*) as

$$i = Gv$$

where, *G* is conductance.   [mho or Siemens]

Field interpretation of ohm's law can be termed as given below:

⇒"At constant temperature current density is directly proportional to electric field intensity."

$$J \propto E$$

⇒ $$J = \sigma E$$

where, J = Current density (A/m^2)

σ = Conductivity [(Ω-m)$^{-1}$]

E = Electric field intensity (V/m)
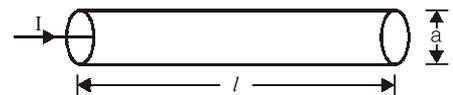
✎ Example - 1.2   Ohm's law is applicable to

(a) Constant and variable temperatures
(b) Constant temperature
(c) Variable temperature
(d) Any of the options   [SSC-JE : 2018]

Solution: (b)

Ohm's law is only applicable at constant temperature, because the resistance, which acts as a constant of proportionality is only constant at constant temperature.

1.12 Power Dissipation in a Resistor

The power dissipated in a resistor is given by

$$p = vi = (iR)i = i^2R = \frac{v^2}{R} = v^2G = \frac{i^2}{G}$$

1.13 Temperature Effects

- The ambient temperature is the temperature of the gas, liquid or solid surrounding a resistor. When the ambient temperature is varied, a change in resistance is noted. Increase in temperature causes molecular movement within the material. The drift of free electrons through the material is obstructed. This fact causes the electrical resistance of the material to rise and is called the positive temperature coefficient. On the other hand temperature increase in certain materials, particularly in semiconductors, leads to decrease in resistance and is called the negative temperature coefficient.
- The resistance values R_1 and R_2 at temperatures T_1 and T_2 respectively, can be written as

$$R_1 = R_0[1 + \alpha(T_1 - T_0)]$$

and
$$R_2 = R_0[1 + \alpha(T_2 - T_0)]$$

where R_0 is the resistance value at T_0 (= 0°C) and α is temperature coefficient.

After simplification,

$$\frac{R_1}{1 + \alpha T_1} = \frac{R_2}{1 + \alpha T_2}$$

1.13.1 Resistance Colour Codes

- Commercially wired wound resistors for domestic use or in laboratories are of two types: wire wound resistors and carbon resistor.
- Wire wound resistors are made by winding the wires of an alloy. These resistances are typically in the range of a fraction of an ohm to a few hundred ohms.
- Resistors in the higher range are mostly made from carbon. Carbon resistors are small in size and hence their values are given using a clour code.

Table: *Resistor Colour Codes*

Colour	Number	Multiplier	Tolerance
Black	0	10^0	–
Brown	1	10^1	± 1%
Red	2	10^2	± 2%
Orange	3	10^3	–
Yellow	4	10^4	± 5%
Green	5	10^5	± 0.5%
Blue	6	10^6	± 0.25%
Violet	7	10^7	± 0.1%
Gray	8	10^8	± 0.05%
White	9	10^9	–
Gold			± 5%
Silver			± 10%
No colour			± 20%

- The first two bands indicate the first two significant figures of the resistance in ohms. The third band indicates the decimal multiplier. The last band stands for tolerance or possible variation in percentage about the indicated values.

- For Eg: if the four colours are orange, blue, yellow and gold the resistance value is $36 \times 10^4 \ \Omega$ with a tolerance value of 5%.

Example - 1.3 For the graph shown in the figure, find the value of the temperature coefficient of resistance (in °C)

(a) 0.02 (b) 0.03
(c) 0.05 (d) 0.06   [SSC-JE : 2018]

Solution: (d)

Given, $R_1 = 30 \ \Omega$ at $T_1 = 0°C$ and $R_2 = 60 \ \Omega$ at $T_2 = 15°C$

$$R_2 = R_1[1 + \alpha(T_2 - T_1)]$$
$$60 = 30[1 + \alpha(15)]$$

∴ $$\alpha = \frac{1}{15} = 0.066/°C$$

Example - 1.4 What will be the colour-coding of a resistor when the resistance of the resistor is 15 Ω?

(a) Green-Brown-Black-Black   (b) Brown-Green-Brown-Brown
(c) Brown-Green-Black-Brown   (d) Brown-Green-Black-Black

Solution: (d)

Using the colour code table,

$$R = 15 \times 10^0 \pm 0\%$$

∴ Colour code is Brown-Green-Black-Black.

1.13.2 Resistivity (ρ)

- It is also known as specific resistance.
- The S.I unit of resistivity is Ω m.

- Mathematically, $$\rho = \frac{Ra}{l}$$

Thus, resistivity is the resistance of a material having unit length and unit cross-sectional area.

- Resistivity depends on
    - (a) Composition of the material of the conductor.
    - (b) Temperature.

NOTE▶

- Resistivity of an alloy is generally higher than that of its constituent metals.
- Resistivity increases with increase in temperature of the metal.
- Resistivity is independent of the conductor dimension.

Example - 1.5 The wires *A* and *B* of the same material but of different lengths *L* and 2*L* have the radius *r* and 2*r* respectively. The ratio of specific resistance will be

(a) 1 : 4   (b) 1 : 8

(c) 1 : 1   (d) 1 : 2   [SSC-JE : 2012]

Solution:(c)

As the wires of same material, specific resistance of both wires *A* and *B* will be same. Specific resistance is independent of dimension.

1.14 Inductance

If the current '*i*' flowing in an element of figure (a) changes with time, the magnetic flux 'ϕ' produced by the current also changes, which causes a voltage to be induced in the circuit, equal to the rate of flux linkages. That is,
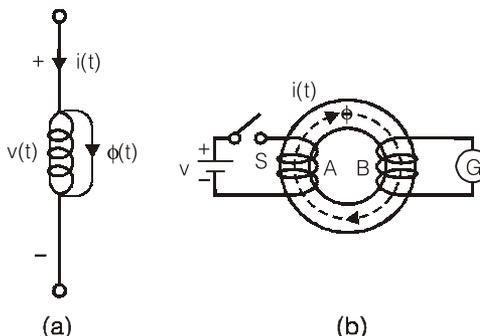
$$v = \frac{d\phi}{dt}$$

Now,

$$v \propto \frac{di}{dt}$$

i.e.

$$\boxed{v = L\frac{di}{dt}}$$

where, *L* is constant of proportionality and is called **self inductance.**

(a) Inductive circuit, (b) Mutually coupled circuit