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POSTAL BOOK PACKAGE 2024

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ELECTRONICS ENGINEERING

Objective Practice Sets

Electromagnetics

1 CHAPTER Vector Analysis

MCQ and NAT Questions

Q.1 For a given vector field $\vec{A} = 5x^2\left(\sin\frac{\pi x}{2}\right)\hat{a}_x$. The divergence $\vec{A}$ at $x = 2$ is __________.

Q.2 A field $\vec{A} = 3x^2yz\hat{a}_x + x^3z\hat{a}_y + (x^3y - 2z)\hat{a}_z$ can be termed as
(a) Irrotational (b) Divergence less
(c) Solenoidal (d) Rotational

Q.3 The angle θ_{AB} between the vectors
$A = 3a_x + 4a_y + a_z$ and $B = 2a_y - 5a_z$ is nearly
(a) 83.7° (b) 73.7°
(c) 63.7° (d) 53.7°

Q.4 The total length of the curve $\mu = \cos^2\theta$ (Cylindrical co-ordinates) from $\theta = 0$ to $\theta = \pi$ is
(a) 0.5π (b) 1.0
(c) 2.0 (d) π

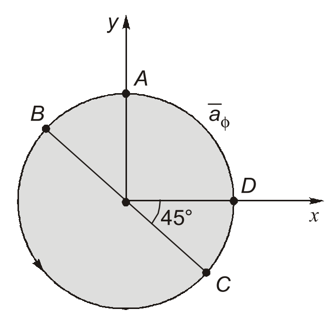
Q.5 Laplacian of a scalar function V is
(a) Gradient of V
(b) Divergence of V
(c) Gradient of the gradient of V
(d) Divergence of the gradient of V

Q.6 For a solenoidal vector field
$$\vec{F} = (x + 3y)\hat{a}_x + (5y + 2z)\hat{a}_y + (x - Qz)\hat{a}_z,$$
the value of Q must be _________.

Q.7 Divergence of a vector div D in the cylindrical coordinate system is
(a) $\frac{1}{\rho}\frac{\partial}{\partial\rho}(D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$
(b) $\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial(\phi D_\phi)}{\partial\phi} + \frac{1}{z}\frac{\partial(ZD_z)}{\partial z}$
(c) $\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$
(d) $\frac{\partial D_\rho}{\partial\rho} + \frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$

Q.8 Given a vector $\vec{A}$ in spherical coordinates as $\vec{A} = 5\sin\theta\, a_\theta + 5\sin\phi\, a_\phi$. The divergence of $\vec{A}$ i.e. $\nabla\cdot\vec{A}$ at $\left(r = 1, \theta = \frac{\pi}{2}, \phi = \frac{\pi}{3}\right)$ is ______.

Q.9 Consider points A, B, C and D on a circle of radius 2 units as in the below figure. The items in List-II are the values of $\hat{a}_\phi$ at different points on the circle. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:



List-I	List-II
A.	1. $\hat{a}_x$
B.	2. $\hat{a}_y$
C.	3. $-\hat{a}_x$
D.	4. $(\hat{a}_x + \hat{a}_y)/\sqrt{2}$
	5. $-(\hat{a}_x + \hat{a}_y)/\sqrt{2}$
	6. $(\hat{a}_x - \hat{a}_y)/\sqrt{2}$

Codes:

	A	B	C	D
(a)	3	4	5	2
(b)	1	6	5	2
(c)	1	6	2	4
(d)	3	5	4	2

Q.10 The unit vector extending from origin toward the point $G(2, -2, -1)$ is

(a) $\frac{2}{3}\hat{a}_x + \frac{2}{3}\hat{a}_y + \frac{1}{3}\hat{a}_z$ (b) $-\frac{2}{3}\hat{a}_x + \frac{2}{3}\hat{a}_y + \frac{1}{3}\hat{a}_z$

(c) $\frac{2}{3}\hat{a}_x - \frac{2}{3}\hat{a}_y - \frac{1}{3}\hat{a}_z$ (d) $-\frac{2}{3}\hat{a}_x - \frac{2}{3}\hat{a}_y - \frac{1}{3}\hat{a}_z$

Q.11 For vector field $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$,

1. $\nabla\cdot(\nabla\times\vec{r}) = 1$
2. $\nabla\times\vec{r} = 0$
3. $\nabla\cdot\vec{r} \neq 0$
4. $\nabla(\vec{r}\cdot\vec{r}) = \vec{r}$

Which of the above relations are true?

(a) 1 and 3 (b) 1 and 4
(c) 2, 3 and 4 (d) 2 and 3

Q.12 If $A = -\nabla f = (x+z)\hat{a}_x - 3z\,\hat{a}_y + (x - 3y - z)\hat{a}_z$.
Then the scalar field, f is

(a) $\frac{x^2}{2} + xz + \frac{z^2}{2}$

(b) $-\frac{x^2}{2} - 2xz + 6yz + \frac{z^2}{2}$

(c) $-xz + 3yz + \frac{z^2}{2}$

(d) $-\frac{x^2}{2} - xz + 3yz + \frac{z^2}{2}$

Q.13 If $V = \sinh x \cdot \cos ky \cdot e^{pz}$ is a solution of Laplace's equation, what will be the value of k?

(a) $\frac{1}{\sqrt{1+p^2}}$ (b) $\sqrt{1+p^2}$

(c) $\frac{1}{\sqrt{1-p^2}}$ (d) $\sqrt{1-p^2}$

Q.14 Laplace equation in cylindrical coordinates is given by

(a) $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial\phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = 0$

(b) $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

(c) $\nabla^2 V = \frac{-\rho}{\varepsilon}$

(d) $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r^2\partial V}{\partial r}\right) + \left(-\frac{1}{r^2\sin\theta}\right)\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = 0$

Q.15 The vector R_{AB} extends from $A(1, 2, 3)$ to B. If the length of R_{AB} is 10 units and its direction is given by

$$a = 0.6\hat{a}_x + 0.64\hat{a}_y + 0.48\hat{a}_z$$

the coordinates of B will be

(a) $7\hat{a}_x + 4.8\hat{a}_y + 4.8\hat{a}_z$

(b) $6\hat{a}_x + 6.4\hat{a}_y + 4.8\hat{a}_z$

(c) $7\hat{a}_x + 8.4\hat{a}_y + 7.8\hat{a}_z$

(d) $6\hat{a}_x + 8.4\hat{a}_y + 7.8\hat{a}_z$

Q.16 Given $\vec{V} = x\cos^2 y\hat{i} + x^2e^z\hat{j} + z\sin^2 y\hat{k}$ and S the surface of unit cube with one corner at the origin and edges parallel to the coordinate axis, the value of the integral $\iint_C \vec{V}.\hat{n}dS$ is ______.

Q.17 Consider the following statements:
Stokes' theorem is valid irrespective of
1. shape of closed curve C
2. type of vector A
3. type of coordinate system
4. whether the surface is closed or open

Which of the above statements are correct?

(a) 1, 2 and 4 (b) 1, 3 and 4
(c) 2, 3 and 4 (d) 1, 2 and 3

Q.18 Consider the vector field $\vec{A} = y\hat{a}_x + x\hat{a}_y$. The scalar line integral of this vector along the parabola $x = 2y^2$ from point (2, 1, –1) to (4, 2, –1) is

(a) 7 (b) 14
(c) 5 (d) 28

Multiple Select Questions (MSQs)

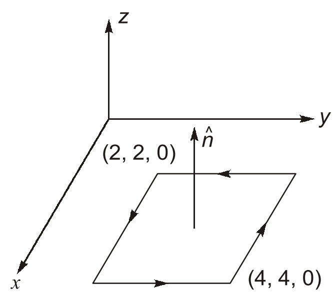
Q.19 $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ represents a position vector and $\|\vec{r}\|$ represents the normal of vector $\vec{r}$, then which of the below statements is/are true?

(a) Divergence of $\vec{r}$ is 3.
(b) Gradient of $\|\vec{r}\|^2$ is $3\vec{r}$
(c) Curl of $\vec{r}$ is 0
(d) Laplacian of $\|\vec{r}\|^2$ is 6.

Q.20 If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$, then which of the below relations are correct?

(a) $\nabla(\log r) = \frac{\vec{r}}{r}$
(b) $\nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$
(c) $\nabla \cdot \left(\frac{\vec{r}}{r^3}\right) = 1$
(d) $\nabla \cdot (3\vec{r}) = 9$

Q.21 Let $\vec{F} = xy^2\hat{a}_x + y^3\hat{a}_y + x^2y\hat{a}_z$ and the surface S consists of a square of length 2 lying in the xy plane as shown below:



Which of the following options is/are correct?

(a) $\iint_S \vec{F} \cdot \hat{n}\, ds = 80$
(b) $\iint_S (\vec{F} \times \hat{n})\, ds = 120\hat{a}_x - 112\hat{a}_y$
(c) $\nabla \times \vec{F} = x^2\hat{a}_x - 2xy\hat{a}_y - 2xy\,\hat{a}_z$
(d) $\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, ds = -120$

Q.22 If $[\vec{a}, \vec{b}, \vec{c}]$ represents the scalar triple product of vectors $\vec{a}, \vec{b}$ and $\vec{c}$, then which of the below statements is/are true?

(a) $\left[\vec{a}, \vec{b}, \vec{c}\right] = \left[\vec{c}, \vec{b}, \vec{a}\right]$
(b) $\left[\vec{a}, \vec{b} + \vec{a}, \vec{c}\right] = 0$
(c) $\left[3\vec{b}, \vec{c}, \vec{a}\right] = 3\left[\vec{a}, \vec{b}, \vec{c}\right]$
(d) If $\left[\vec{a}, \vec{b}, \vec{c}\right] = 0$, the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

Q.23 The values of α for which the vectors $\vec{A} = \alpha\hat{a}_x + 2\hat{a}_y + 10\hat{a}_z$ and $\vec{B} = 4\alpha\hat{a}_x + 8\hat{a}_y - 2\alpha\hat{a}_z$ are perpendicular is/are

(a) 1
(b) 2
(c) 3
(d) 4

Q.24 Which of the below vector identities are true?

(a) $A \times (B \times C) = (A \times B) \times C$
(b) $A \times (B \times C) + C \times (A \times B) + B \times (C \times A) = 0$
(c) $(B \times C) \times (C \times A) = C(A \cdot B \times C)$
(d) $(A \times B) \cdot (C \times D) = (A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C)$

Q.25 For the scalar function, $\phi = x^2yz^3$, which of the below statements is/are correct?

(a) From the point (2, 1, –1) the directional derivative of ϕ is maximum in the direction represented by vector $-12\hat{i} - 4\hat{j} + 4\hat{k}$.
(b) The magnitude of greatest rate of change of ϕ from the point (2, 1, –1) is $4\sqrt{11}$.
(c) $(x - 2) + (y - 1) - 3(z + 1) = 0$ represents the tangent plane to the surface $\phi = 0$ at point (2, 1, –1).
(d) ϕ satisfies the Laplacian equation.

■■■■

Answers Vector Analysis

1.	(–31.42)	2.	(a)	3.	(a)	4.	(a)	5.	(d)	6.	(6)	7.	(c)
8.	(2.5)	9.	(d)	10.	(c)	11.	(d)	12.	(d)	13.	(b)	14.	(a)
15.	(c)	16.	(1)	17.	(d)	18.	(b)	19.	(a, c, d)	20.	(b, d)	21.	(b, c)
22.	(c, d)	23.	(a, d)	24.	(b, c, d)	25.	(b, c)						

Explanations **Vector Analysis**

1. (–31.42)

$$\text{Div } \vec{A} = \frac{\partial}{\partial x}\left[5x^2\left(\sin\frac{\pi x}{2}\right)\right]$$

$$= 5x^2\left(\cos\frac{\pi x}{2}\right)\cdot\frac{\pi}{2} + 10x\sin\left(\frac{\pi x}{2}\right)$$

$$\text{Div } \vec{A}\Big|_{x=2} = \frac{\pi}{2}\times 5(2)^2\cos\pi + 10(2)\sin(\pi)$$

$$= -5\times 4\times\frac{\pi}{2} = -10\pi = -31.42$$

2. (a)

$$\vec{A} = 3x^2yz\hat{a}_x + x^3z\hat{a}_y + (x^3y - 2z)\hat{a}_z$$

$$\nabla\cdot\vec{A} = \frac{\partial}{\partial x}(3x^2yz) + \frac{\partial}{\partial y}(x^3z) + \frac{\partial}{\partial z}(x^3y - 2z)$$

$$= 6xyz - 2$$

$\nabla\cdot\vec{A} \neq 0 \Rightarrow \vec{A}$ is not a solenoidal.

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3x^2yz & x^3z & x^3y - 2z \end{vmatrix}$$

$$= (x^3 - x^3)\hat{a}_x - (3x^2y - 3x^2y)\hat{a}_y + (3x^2z - 3x^2z)\hat{a}_z$$

$$= 0$$

$\Rightarrow \vec{A}$ is irrotational.

3. (a)

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta_{AB}$$

$$|\vec{A}| = \sqrt{3^2 + 4^2 + 1^2} = \sqrt{26}$$

$$|\vec{B}| = \sqrt{2^2 + 5^2} = \sqrt{29}$$

$$\vec{A}\cdot\vec{B} = (4\times 2) - (1\times 5) = 3$$

$$\cos\theta_{AB} = \frac{3}{\sqrt{26\times 29}}$$

$$\theta_{AB} = \cos^{-1}\left[\frac{3}{\sqrt{26\times 29}}\right] = 83.7^\circ$$

4. (a)

$$\int_{\theta=0}^{\pi}\cos^2\theta\, d\theta = \int_{\theta=0}^{\pi}\left(\frac{1+\cos 2\theta}{2}\right)d\theta = \frac{\pi}{2}$$

5. (d)

$$\nabla^2 V = \vec{\nabla}\cdot(\vec{\nabla}V)$$

= divergence of gradient of V

6. (6)

The vector will be solenoidal if its divergence is zero

$$\vec{\nabla}\cdot\vec{F} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(x + 3y) + \frac{\partial}{\partial y}(5y + 2z) + \frac{\partial}{\partial z}(x - Qz) = 0$$

$$\Rightarrow 1 + 5 - Q = 0$$

$$\Rightarrow Q = 6$$

7. (c)

Divergence of vector $\vec{D}$ in different coordinates system is given by:

Cartesian coordinates:

$$\nabla\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

Cylindrical coordinates:

$$\nabla\cdot\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\cdot\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

Spherical coordinates:

$$\nabla\cdot\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(D_\theta\sin\theta) + \frac{1}{r\sin\theta}\cdot\frac{\partial D_\phi}{\partial\phi}$$

8. (2.5)

In spherical coordinates,

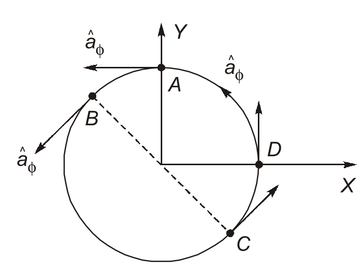
$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\theta}(A_\theta\cdot\sin\theta) + \frac{1}{r\sin\theta}\cdot\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\cdot\vec{A} = \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\theta}(5\sin^2\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(5\sin\phi)$$

$$\nabla\cdot\vec{A} = \frac{10\cos\theta}{r} + \frac{5\cos\phi}{r\sin\theta}$$

$$= \frac{5}{1}\times\frac{1}{2}\times\frac{1}{1} = 2.5$$

9. (d)



At A: $\hat{a}_\phi = -\hat{a}_x$

At B: $\hat{a}_\phi = -\dfrac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$

At C: $\hat{a}_\phi = \dfrac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$

At D: $\hat{a}_\phi = \hat{a}_y$

10. (c)

$$\overrightarrow{OG} = 2\hat{a}_x - 2\hat{a}_y - \hat{a}_z$$

$$\vec{a} = \frac{\overrightarrow{OG}}{|\overrightarrow{OG}|} = \frac{2\hat{a}_x - 2\hat{a}_y - \hat{a}_z}{\sqrt{2^2 + 2^2 + 1^2}} = \frac{2}{3}\hat{a}_x - \frac{2}{3}\hat{a}_y - \frac{1}{3}\hat{a}_z$$

11. (d)

$$\nabla \times \vec{r} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0$$

$$\nabla \cdot \vec{r} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z) = 3$$

$$\vec{r}\cdot\vec{r} = \left(x\hat{a}_x + y\hat{a}_y + z\hat{a}_z\right)\cdot\left(x\hat{a}_x + y\hat{a}_y + z\hat{a}_z\right)$$

$$= x^2 + y^2 + z^2$$

$$\nabla(\vec{r}\cdot\vec{r}) = \frac{\partial}{\partial x}(x^2+y^2+z^2)\hat{a}_x + \frac{\partial}{\partial y}(x^2+y^2+z^2)\hat{a}_y + \frac{\partial}{\partial z}(x^2+y^2+z^2)\hat{a}_z$$

$$= 2x\hat{a}_x + 2y\hat{a}_y + 2z\hat{a}_z = 2\vec{r}$$

12. (d)

$$A = -\nabla f$$

$$= -\frac{\partial f}{\partial x}\hat{a}_x - \frac{\partial f}{\partial y}\hat{a}_y - \frac{\partial f}{\partial z}\hat{a}_z$$

Comparing it with given vector,

$$\frac{\partial f}{\partial x} = -(x + z)$$

$$\Rightarrow \quad f = -\frac{x^2}{2} - xz + f_1(y,z)$$

$$\frac{\partial f}{\partial y} = 3z \Rightarrow f = 3yz + f_2(x,z)$$

$$\frac{\partial f}{\partial z} = -(x - 3y - z)$$

$$\Rightarrow \quad f = -xz + 3yz + \frac{z^2}{2} + f_3(x,y)$$

$$\therefore \quad f = -\frac{x^2}{2} - xz + 3yz + \frac{z^2}{2}$$

13. (b)

$$V = \sinh x \cdot \cos ky \cdot e^{pz}$$

Laplace equation

$$\nabla^2 V = 0$$

$$\frac{\partial^2}{\partial x^2}(V) + \frac{\partial^2}{\partial y^2}(V) + \frac{\partial^2}{\partial z^2}(V) = 0$$

$$\sinh x \cdot \cos ky \cdot e^{pz} - k^2 \sinh x \cdot \cos ky \cdot e^{pz} + p^2 \sinh x \cdot \cos ky \cdot e^{pz} = 0$$

$$\sinh x \cdot \cos ky \cdot e^{pz}(1 - k^2 + p^2) = 0$$

$$k^2 = 1 + p^2$$

$$k = \sqrt{1 + p^2}$$

14. (a)

Laplace equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad \text{(Cartesian coordinates)}$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

(Cylindrical coordinates)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

(Spherical coordinates)

15. (c)

As R_{AB} length is 10 units

$$\vec{R}_{AB} = \left|\vec{R}_{AB}\right|\vec{a}$$

$$\vec{R}_{AB} = 10\vec{a} = 6\hat{a}_x + 6.4\hat{a}_y + 4.8\hat{a}_z$$

$\vec{A}$ radial vector $= \hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$

$$\vec{R}_{AB} = \vec{B} - \vec{A}$$

$$\vec{B} = \vec{R}_{AB} + \vec{A}$$

$$\therefore \quad \vec{B} = 10\vec{a} + \vec{A} = 7\hat{a}_x + 8.4\hat{a}_y + 7.8\hat{a}_z$$

16. (1)

Using divergence theorem,

$$\oiint \vec{V}\cdot\hat{n}dS = \iiint(\nabla\cdot\vec{V})\cdot dV$$

$$\nabla\cdot\vec{V} = \frac{\partial}{\partial x}(x\cos^2 y) + \frac{\partial}{\partial y}(x^2 e^z) + \frac{\partial}{\partial z}(z\sin^2 y)$$

$$= \cos^2 y + \sin^2 y = 1$$

$$\therefore \quad \oiint \vec{V}\cdot\hat{n}dS = \iiint dV$$

$$= \text{Volume of unit cube} = 1$$