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MADE EASY PUBLICATIONS

ELECTRONICS ENGINEERING

CONVENTIONAL Practice Sets

CONTENTS

CONTROL SYSTEMS

1 CHAPTER Introduction

Q.1 (a) A control system is defined by following mathematical relationship

$$\frac{d^2x}{dt^2}+\frac{6dx}{dt}+5x=12(1-e^{-2t})$$

Find the response of the system at $t \to \infty$

(b) A function $y(t)$ satisfies the following differential equation

$$\frac{dy(t)}{dt}+y(t)=\delta(t)$$

Where $\delta(t)$ is delta function. Assuming zero initial condition and denoting unit step function by $u(t)$. Find $y(t)$.

Solution:

(a) Taking LT on both sides

$$(s^2+6s+5)\,X(s) = 12\left[\frac{1}{s}-\frac{1}{s+2}\right]$$

$$(s+1)\,(s+5)\,X(s) = \frac{24}{s(s+2)}$$

$$X(s) = \frac{24}{s(s+1)\,(s+2)\,(s+5)}$$

Response at $t \to \infty$

Using final value theorem,

$$\boxed{\lim_{t\to\infty} x(t) = \lim_{s\to 0}[sX(s)]} = \lim_{s\to 0}\frac{s\times 24}{s(s+1)\,(s+2)\,(s+5)} = 2.4$$

(b) Taking Laplace transform on both sides

$$Y(s)\,[s+1] = 1$$

$$Y(s) = \frac{1}{s+1}$$

By taking inverse Laplace transform

$$y(t) = e^{-t}\,u(t)$$

Q.2 The response $h(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relaxed condition is $h(t) = e^{-t} + e^{-2t}$. Find the response of this system for a unit step input $u(t)$?

Solution:

Transfer function is given by

$$H(s) = L\{e^{-t}+e^{-2t}\} = \frac{1}{s+1}+\frac{1}{s+2}$$

$$H(s) = \frac{C(s)}{R(s)} = \frac{1}{s+1}+\frac{1}{s+2}$$

$$R(s) = \frac{1}{s} \text{ (step input)} \qquad [\because\ r(t) = u(t)]$$

$$C(s) = R(s)\cdot H(s) = \frac{1}{s}\left[\frac{1}{s+1}+\frac{1}{s+2}\right] = \frac{1}{s(s+1)}+\frac{1}{(s+2)(s)}$$

$$= \frac{1}{s}-\frac{1}{s+1}+\frac{1}{2}\left[\frac{1}{s}-\frac{1}{s+2}\right]$$

$$= \frac{1.5}{s}-\frac{1}{s+1}-\frac{0.5}{s+2}$$

Response will be $c(t) = L^{-1}\{C(s)\}$

$$c(t) = (1.5 - e^{-t} - 0.5e^{-2t})\, u(t)$$

Q.3 A system is represented by a relation given below:

$$X(s) = R(s)\cdot\frac{100}{s^2+2s+50}$$

if $r(t)$ = 1.0 unit, find the value of $x(t)$ when $t \to \infty$.

Solution:

Since, $r(t) = 1$

Taking Laplace transform,

$\therefore$ $R(s) = \frac{1}{s}$

Applying final value theorem,

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

$$= \lim_{s\to 0} s\cdot\frac{1}{s}\cdot\frac{100}{s^2+2s+50} = 2.0 \text{ units}$$

Q.4 (a) The Laplace equation for the charging current, $i(t)$ of a capacitor arranged in series with a resistance is given by

$$I(s) = \frac{sC}{1+sRC}\cdot E(s)$$

The circuit is connected to a supply voltage of E. If E = 100 V, R = 2 MΩ, C = 1 μF. Calculate the initial value of the charging current.

(b) A series circuit consisting of resistance R and an inductance of L is connected to a d.c. supply voltage of E. Derive an expression for the steady-state value of the current flowing in the circuit using final value theorem.

Solution:

(a) Since, $E = 100\, v(t)$

Taking Laplace Transform, $E = 100\,(t)$ volts,

$\therefore$ $E(s) = \frac{100}{s}$

Substituting the given values,

$$I(s) = \frac{1\times10^{-6}s}{(2\times10^{6}\times1\times10^{-6}s+1)}\cdot\frac{100}{s} = \frac{10^{-6}s}{2s+1}\cdot\frac{100}{s}$$

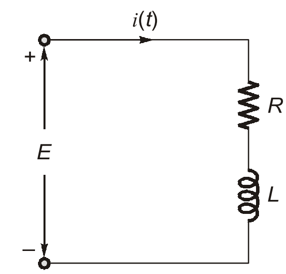
Applying the initial value theorem,

$$\boxed{i(0^+) = \lim_{t\to 0} i(t) = \lim_{s\to\infty} s\,I(s)}$$

$$i(0^+) = \lim_{s\to\infty} s\cdot\frac{10^{-4}}{1+2s} = \lim_{s\to\infty}\cdot\frac{10^{-4}}{\frac{1}{s}+2} = 50\ \mu\text{A}$$

(b) The differential equation relating the current $i(t)$ flowing in the circuit and the input voltage E is given by

$$E = R\,i(t) + L\frac{di(t)}{dt}$$



Taking Laplace transform of the equation yields,

$$E(s) = R\,I(s) + L[(sI(s) - i(0^+))]$$

Assume, $\quad i(0^+) = 0$

$\therefore \quad E(s) = R\,I(s) + Ls\,I(s)$

$\because$ E is constant (d.c. voltage)

$$E(s) = \frac{E}{s} = RI(s) + Ls\,I(s)$$

$$I(s) = \frac{E}{s(R+sL)}$$

Applying the final value theorem,

$$i_{ss} = \lim_{t\to\infty} i(t) = \lim_{s\to 0} sI(s) = \lim_{s\to 0}\frac{sE}{s(R+sL)}$$

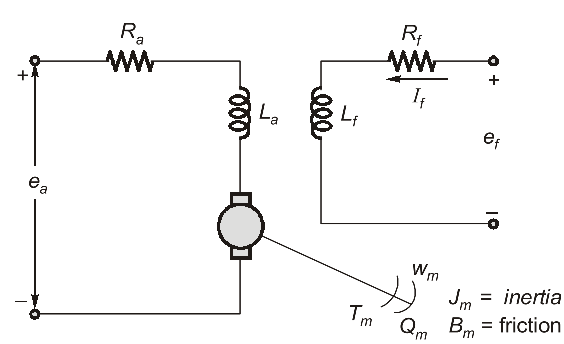
$$i_{ss} = \frac{E}{R}$$

Q.5 Determine the mechanical time constant of rotor of an electrical machine in terms of its moment of inertia J kg-m^2 and windage cum friction coefficient f N-m/rad/s. Also explain the method to determine mechanical time constant experimentally in laboratory.

Solution:

Consider a field controlled separately excited DC motor.

Constant armature in field into the motor,



$$\phi_f \propto I_f$$
$$\phi_f = k_f I_f$$
$$T_m \propto \phi_f I_a$$
$$T_m = K'\phi_f I_a$$
$$T_m = K' k_f I_f I_a$$
$$T_m = k_m k_f I_f$$

where, $\quad k_m = K'I_a$ = constant

$$e_f = L_f\frac{di_f}{dt} + R_f I_f$$

$$T_m = J_m\frac{d^2\theta_m}{dt^2} + B_m\frac{d\theta_m}{dt}$$

$$T_m(s) = J_m s^2\theta_m(s) + B_m\,s\,\theta_m(s)$$
$$T_m(s) = (J_m s^2 + B_m s)\theta_m(s)$$
$$E_f(s) = (sL_f + R_f)\,I_f(s)$$
$$= (sL_f + R_f)\frac{T_m(s)}{k_f k_m}$$

$$E_f(s) = \frac{(sL_f + R_f)(J_m s^2 + B_m s)\,\theta_m(s)}{k_f k_m}$$

$$\frac{\theta_m(s)}{E_f(s)} = \frac{k_m k_f}{s(sL_f + R_f)(J_m s + B_m)} = \frac{k_m k_f}{B_m R_f s\left(1+\frac{J_m}{B_m}s\right)\left(1+\frac{sL_f}{R_f}\right)}$$

$$\frac{\theta_m(s)}{E_f(s)} = \frac{k_m k_f}{sB_m R_f(1+\tau_m s)(1+\tau_f s)}$$

τ_m = motor time constant = J_m/B_m
τ_f = field time constant = L_f/R_f

Q.6 **The impulse response of a system S_1 is given by $y_1(t) = 4e^{-2t}$. The step response of a system S_2 is given by $y_2(t) = 2(1 - e^{-3t})$. The two systems are cascaded together without any interaction. Find response of the cascaded system for unit ramp input.**

Solution:

(a) Taking the Laplace transform of the response of S_1, we get

$$Y_1(s) = \frac{4}{s+2},$$

$$X_1(s) = 1 \;\ldots\; (x(t) = \delta(t))$$

Therefore, $$G_1(s) = \frac{Y_1(s)}{X_1(s)} = \frac{4}{s+2} \qquad [\because Y_1(s) = 1]$$

Taking the Laplace transform of the response of S_2, we get

$$Y_2(s) = 2\left(\frac{1}{s} - \frac{1}{s+3}\right) = \frac{6}{s(s+3)}$$

$$Y_2(s) = \frac{1}{s} \;\ldots\; (x_2(t) = u(t))$$

Thus, $$G_2(s) = \frac{Y_2(s)}{X_2(s)} = \frac{6}{s(s+3)}\cdot s = \frac{6}{s+3}$$

(b) The transfer function of the cascaded system is

$$G(s) = G_1(s)G_2(s) = \frac{24}{(s+2)(s+3)}$$

The Laplace transform of unit ramp is $R(s) = \frac{1}{s^2}$. Therefore,

$$G(s) = \frac{C(s)}{R(s)}$$

$$C(s) = \frac{24}{(s+2)(s+3)}\cdot\frac{1}{s^2}$$

$$\equiv \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+2} + \frac{D}{s+3}$$

$$A = \left.\frac{24}{(s+2)(s+3)}\right|_{s=0} = 4$$

$$B = \frac{d}{ds}\left[s^2 C(s)\right]_{s=0}$$

$$= \frac{d}{ds}\left[\frac{24}{(s+2)(s+3)}\right] = -\left.\frac{24(2s+5)}{(s+2)^2(s+3)^2}\right|_{s=0}$$

$$= -\frac{10}{3}$$

$$C = \left.\frac{24}{s^2(s+3)}\right|_{s=-2} = 6$$

$$D = \left.\frac{24}{s^2(s+2)}\right|_{s=-3} = -\frac{8}{3}$$

$$C(s) = \frac{4}{s^2} - \frac{10}{3}s + \frac{6}{s+2} - \frac{8}{3}e^{-3t}$$

Taking inverse Laplace transform.

Therefore, $$c(t) = 4t - \frac{10}{3}u(t) + 6e^{-3t} - \frac{8}{3}e^{-3t}$$

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2 CHAPTER Transfer Function

Q.1 Find the transfer function $\frac{Y(s)}{U(s)}$ for the system governed by the set of equations:

$$\frac{dx_1}{dt} = 2x_1 + x_2 + u$$

$$\frac{dx_2}{dt} = -2x_1 + u$$

$$y = 3x_1$$

Solution:

From equations,
$$\frac{dx_1}{dt} = 2x_1 + x_2 + u \quad \text{...(i)}$$

$$\frac{dx_2}{dt} = -2x_1 + u \quad \text{...(ii)}$$

From equation (i), (ii)
$$sX_1 = 2X_1 + X_2 + U$$

$$sX_2 = -2X_1 + U$$

$$X_2 = \left[\frac{-2X_1 + U}{s}\right]$$

$$sX_1 = 2X_1 + \left[\frac{-2X_1 + U}{s}\right] + U$$

$$X_1\left[s - 2 + \frac{2}{s}\right] = U\left[1 + \frac{1}{s}\right]$$

$$\frac{X_1}{U} = \frac{1 + \frac{1}{s}}{s - 2 + \frac{2}{s}}$$

$\Rightarrow$
$$\frac{Y}{U} = \frac{3X_1}{U} = \frac{3\left[1 + \frac{1}{s}\right]}{\left[s - 2 + \frac{2}{s}\right]}$$

$$\frac{Y(s)}{U(s)} = \frac{3(s+1)}{s^2 - 2s + 2}$$

Q.2 A torque T N-m is applied to a shaft having a moment of inertia J and coefficient of viscous friction of f produces an angular shift of θ radius. Obtain the transfer function in relation to θ and T.

Solution:

The equation for the system is given by

$$T = \frac{Jd^2\theta}{dt^2} + f\frac{d\theta}{dt} \quad \text{...(1)}$$

Assuming initial conditions as zero and taking the Laplace transform on both sides of equation (1), the following equations is obtained,

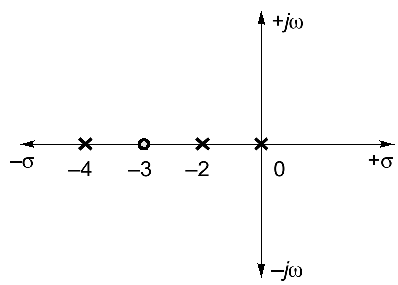
$$T(s) = Js^2\,\theta(s) + fs\,\theta(s)$$

$$T(s) = s(Js + f))\,\theta(s) \quad ...(2)$$

From equation (2), the required transfer function is obtained below,

$$\frac{\theta(s)}{T(s)} = \frac{1}{s(Js+f)}$$

Q3 The pole-zero configuration of a transfer function is given below. The value of the transfer function as $s = 1$ is found to be 3.2. Determine the transfer function and gain factor K.



Solution:

The transfer function has three poles and one zero therefore, the transfer function consists of one term in the numerator and three terms in the denominator.

Poles are located at $s = 0,\ s = -2,\ s = -4$

Zeros are located at $s = -3$

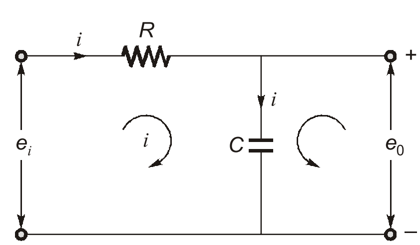
The transfer function, $$G(s) = \frac{K(s+3)}{s(s+2)(s+4)}$$

It is given that at, $s = 1,\ G(s) = 3.2$

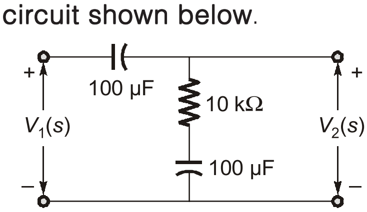
$\Rightarrow$ $$3.2 = \frac{4K}{15} \Rightarrow K = 12$$

$\therefore$ $$G(s) = \frac{12(s+3)}{s(s+2)(s+4)}$$

Q4 (a) Find the transfer function of the network given below.



(b) Find the transfer function of circuit shown below.



Solution:

(a) Applying KVL, we get $e_i = Ri + \frac{1}{C}\int i\,dt$ and $e_0 = \frac{1}{C}\int i\,dt$

Assuming initial conditions are zero and taking Laplace transform on both sides of equation (1) and (2), the following equations are obtained:

$$E_i(s) = R\,I(s) + \frac{1}{C}\frac{I(s)}{s}$$

$$E_0(s) = \frac{1}{C}\cdot\frac{I(s)}{s} \qquad \text{...(1)}$$

$$E_i(s) = \left[R + \frac{1}{Cs}\right] I(s) \qquad \text{...(2)}$$

Dividing (2) by (1), $$\frac{E_0(s)}{E_i(s)} = \frac{1}{1+sRC}$$

(b) $$\frac{V_2(s)}{V_1(s)} = \frac{R + \frac{1}{sC}}{R + \frac{1}{sC} + \frac{1}{sC}} = \frac{1+RCs}{2+RCs} = \frac{1+10\times10^3\times100\times10^{-6}s}{2+10\times10^3\times100\times10^{-6}s}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{s+1}{s+2}$$ is required transfer function.

Q.5 The transfer function of a thermocouple relating output voltage to temperature is given by $\frac{0.625\times10^{-4}}{s+0.125}$ V/°C. Put the transfer function in standard format and find the values of characterising parameters of the thermocouple. Determine the thermocouple output voltage at t = 8s, when the thermocouple kept at ambient temperature of 20°C at t = 0 s is taken to a water bath kept at 80°C.

Solution:

Standard format of transfer function is $\frac{K}{(Ts+1)}$.

So, $$\frac{V(s)}{T(s)} = \frac{0.625\times10^{-4}}{s+0.125} = \frac{5\times10^{-4}}{(8s+1)}$$

For $$T(t) = Tu(t)$$

$$T(s) = \frac{T}{s}$$

T = Temperature difference between junctions

$$\Rightarrow \quad V(s) = \frac{T}{s}\times\frac{0.625\times10^{-4}}{s+0.125} = T\times\frac{0.625\times10^{-4}}{0.125}\left(\frac{1}{s}-\frac{1}{s+0.125}\right) = 5T\times10^{-4}\left(\frac{1}{s}-\frac{1}{s+0.125}\right)$$

$$\Rightarrow \quad V(t) = 5T\times10^{-4}\,(u(t) - e^{-t/8}\,u(t))$$

$$\Rightarrow \quad V(t) = 5T\,(1-e^{-t/8})\times10^{-4}\,u(t)\text{ V}$$

Thermocouple time constant = 8 secs.

Steady state output of the thermocouple for T°C = $5T\times10^{-4}$ V

$$T = 80° - 20° = 60°\text{C}$$

Now, T = 60°C and t = 8 sec

$$\therefore \quad V(t) = 5\times60\,[1-e^{-8/8}]\times10^{-4}\text{ V} = 300\left(\frac{e-1}{e}\right)\times10^{-4} = 189.636\times10^{-4}\text{ V} = 18.963\text{ mV}$$