Electrical Engineering

Electrical & Electronic Measurements

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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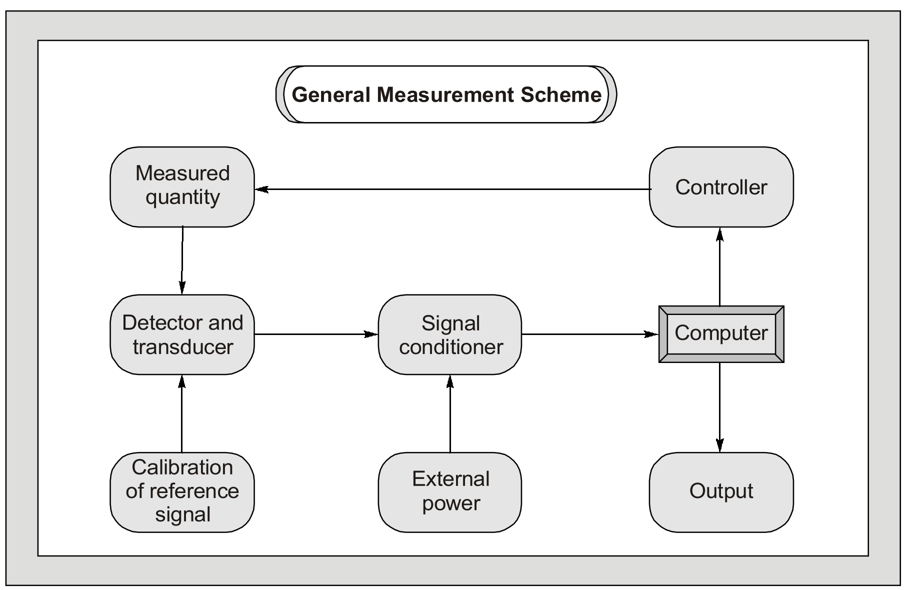
Introduction to Electrical and Electronic Measurements

Measurement and instrumentation systems have wide applications such as measurement of electrical and physical quantities like current, voltage, power, temperature, pressure, displacement etc.

The reason for measurement arises when one wants to generate data for design or when one wants to propose a theory based on a set of measurement and instrumentation for commerce.

The measurement and instrumentation systems can also be used to locate things or events. Like employees present in a building, the epicenter of an earthquake. Sometimes, measurement systems are made a part of control system. One can observe the change in the field of measurement and instrumentation due to the introduction of new standards, and sensors.



This course on instrumentation and measurement is intended to make the engineers familiar about the art of modern instrumentation and measurement systems. It is well suited for classroom courses of engineering as well as for various competitive examinations.

Equal importance has been provided to both theory as well as problems with illustrative examples after every topic. It has been tried to cover every topic so that even a beginner understands it easily to excel in the subject of measurement and instrumentation.

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CHAPTER 1

Introduction

1.1 Measurements and it's Significance

The measurement of a given quantity is essentially an act or the result of comparison between the quantity (whose magnitude is unknown) and a predefined standard. Measurement is the process by which one can convert physical parameters to meaningful numbers. The measuring process is one in which the property of an object or system under consideration is compared to an accepted standard unit, a standard defined for that particular property. For the result of the measurement to be meaningful, the standard used for comparison purposes must be accurately defined and should be commonly accepted. Also, the apparatus used and the method adopted must be provable. The importance of measurement is simply expressed in the following statement of the famous physicist "Lord Kelvin":

"I often say that when you can measure what you are speaking about and can express it in numbers, you know something about it; when you can't express it in numbers your knowledge is of a meager and unsatisfactory kind."

Method of Measurement

Direct Measurement

- In this method, the measured or the unknown quantity is directly compared against a standard.
- This method of measurement sometimes produces human errors and hence gives inaccurate results.

Indirect Measurement

- This method of measurement is more accurate and more sensitive.
- These are more preferred over direct measurement.

Mechanical, Electrical and Electronic Instruments

Mechanical

- This instruments are used for stable and static conditions:
- They are unable to respond rapidly to measurements of dynamic and transient conditions because of having moving parts that are bulky, heavy are rigid possessing high inertia.

Electrical

Electrical methods of indicating the output of detectors are more rapid than mechanical methods, but they are limited time response.

Electronic

These instruments require use of semiconductor devices. The response time of these instruments are extremely small as a very small inertia of electron is only involved. The sensitivity of these instruments are also very high. Faster response, lower weight, lower power consumption are some of the advantages of an electronic instrument.

1.2 Types of Instruments

Absolute Instruments

These instruments give the magnitude of the quantity under measurement in terms of physical constants of the instruments i.e. Tangent Galvanometer, Rayleigh's current balance.

Secondary Instruments

In these type of instruments, the quantity being measured can only be measured by observing the output indicated by the instrument. These instruments are calibrated by comparing with an absolute instrument.

1.3 Deflection and Null Type Instruments

Deflection Type

The deflection of the instrument provides a basis for determining the quantity under measurement i.e. PMMC Ammeter, Electrodynamometer and moving iron instruments. They are less accurate, less sensitive and have faster response.

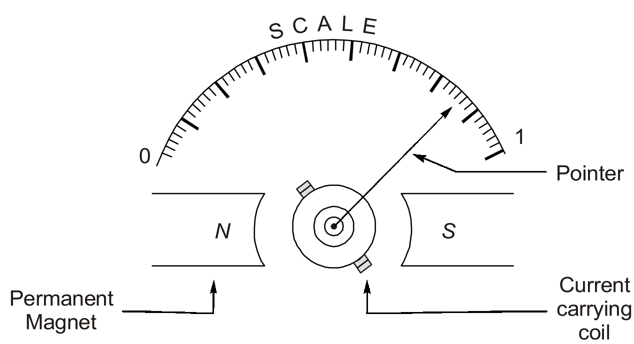


***Figure*-1.1 :** *PMMC (Deflection Type Instrument)*

Null Type Instruments

In null type instruments, a zero or null indication leads to determination of the magnitude of measured quantity. Null type instruments are more accurate, highly sensitive and are less suited for measurements under dynamic conditions than deflection type instruments.

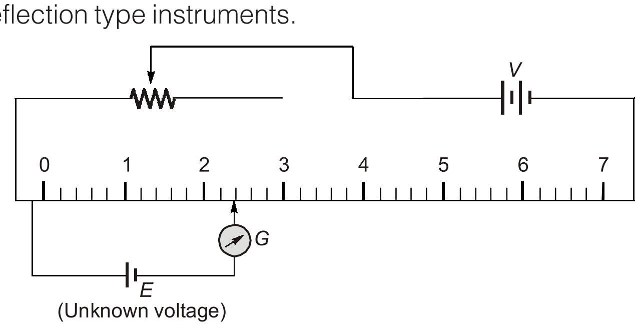


***Figure*-1.2 :** *Null Type Instrument*

Calibration

The calibration of all instruments is important since it affords the opportunity to check the instrument against a known standard and subsequently to find errors and accuracy. Calibration procedures involve a comparison of the particular instrument with a primary standard or, a secondary standard or, an instrument of known accuracy.

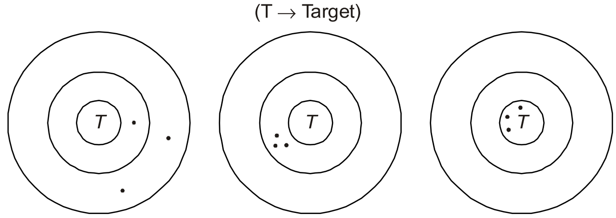
Characteristics of Instrument and Measurement Systems

Accuracy

- It is the closeness with which an instrument reading approaches the true value of the quantity being measured.
- The accuracy can be specified in terms of inaccuracy or limits of error.
- The best way to conceive the idea of accuracy is to specify it in terms of the true value of the quantity being measured.
- The accuracy of a measurement means conformity to truth.

Precision

- It is a measure of the reproducibility of the measurements i.e. given a fixed value of a variable, precision is a measure of the degree to which successive measurements differ from one another.
- The term "Precise" means clearly or sharply defined.
- Precision is used in measurements to describe the consistency or the reproducibility of results.
- Precision instruments are not guaranteed for accuracy.



- Precision depends upon number of significant figures.
- The more is significant figures the more is precision.
- Significant figures convey actual information regarding the magnitude and the measurement precision of a quantity.

Example: 302 A (Number of significant figures = 3)
302.10 V (Number of significant figures = 5)
0.000030 Ω (Number of significant figures = 6)

Example - 1.1 In calculating voltage drop, a current of 4.37 A is recorded in a resistance of 31.27 Ω. Calculate the voltage drop across the resistor to the appropriate number of significant figures.

Solution:

Current, $I = 4.37 A$ (3 significant figures)

Resistance, $R = 31.27\Omega$ (4 significant figures)

Voltage drop, $V = IR = 4.37 \times 31.27 = 136.6499$ volt

Since number of significant figures used in multiplication is 3.

So answer can be written only to a maximum of three significant figures i.e. $V = 137$

NOTE: 248 volt; 248.0 volt ⇒ More precised than other two.
⇒ 0.000248 MV

Example - 1.2 A reading is recorded as 23.90°C. The reading has
(a) three significant figures (b) five significant figures
(c) four significant figures (d) none of these

***Solution:* (c)**

Example - 1.3 Assertion (A): A precision instrument is always accurate.
Reason (R): A precision instrument is one where the degree of reproducibility of the measurements is very good.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

***Solution:* (d)**
Precision instruments are not guaranteed for accuracy. Refer to definition of precision.

Linearity

- If the output is proportional to input then, it is called linear.
- Non-linear behaviour of an instrument doesn't essentially lead to inaccuracy.
- Most of the time it is necessary that measurement system component should have linear characteristics. For example, the resistance used in a potentiometer should vary linearly with displacement of the sliding contact in order that the displacement is directly proportional to the sliding contact voltage. Any departure from linearity result in error in the read out system.

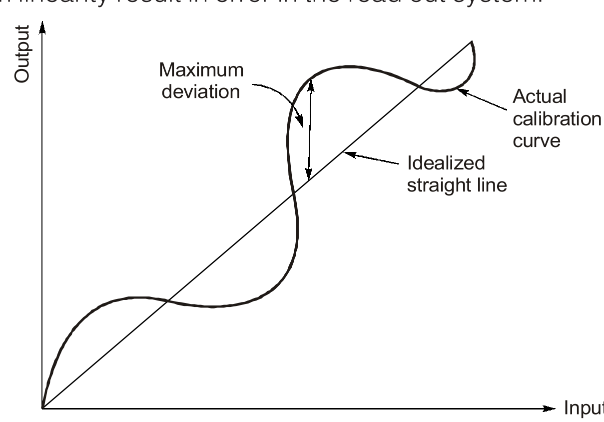


***Figure*-1.3:** *Linearity w.r.t. actual calibration curve and idealized straight line*

Reproducibility

It is the degree of closeness with which a given value may be repeatedly measured. It may be specified in terms of units for a given period of time.

Static Sensitivity

- The "static sensitivity" of an instrument is the ratio of the magnitude of the output signal or response to the magnitude of input signal or the quantity being measured.
  It's units are mm/μA; per volts etc. depending upon type of input and output.

- Sometimes the static sensitivity is expressed as the ratio of the magnitude of the measured quantity to the magnitude of the response.

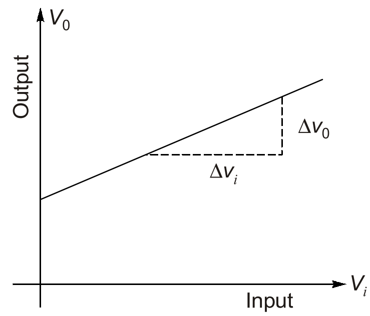


***Figure*-1.4 :** *Sensitivity*

$$\text{Static Sensitivity} = \frac{\text{Small change in output}}{\text{Small change in input}} = \frac{\Delta V_0}{\Delta V_i}$$

- The sensitivity of an instrument should be high and therefore, instrument should not have a range greatly exceeding the value to be measured.

$$\text{Deflection Factor} = \frac{1}{(\text{Static Sensitivity})}$$

Resolution or Discrimination

- The small measurable input change that can be measured by the instrument is called resolution or discrimination.
- If the input is slowly increased from some arbitrary (non-zero) input value, it will again be found that output doesn't change at all until a certain increment is exceeded. This increment is called resolution.

Example - 1.4 A digital voltmeter has a read-out range from 0 to 9,999 counts. Determine the resolution of the instrument in volt when the full scale reading is 9.999 *V*.

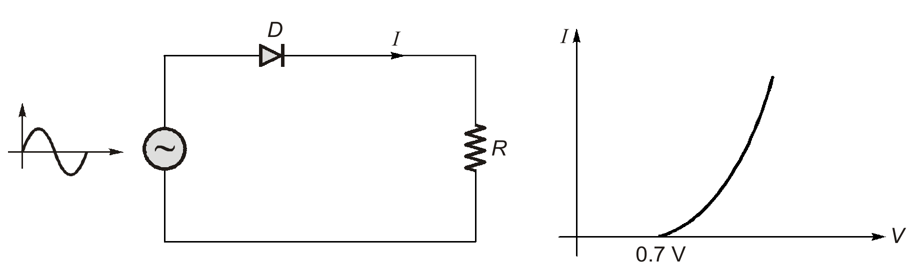
Solution:

Resolution of instrument = 1 count in 9,999

$$\text{Resolution} = \frac{1}{9999}\text{ count} = \frac{1}{9999}\times 9.999 = 10^{-3}\text{ volt} = 1\text{ mV}$$

Dead Time & Dead Zone

Dead Time: The time required for the measurement to begin to respond to the changes in the measurand is known as dead time. It is the time before which the instrument begins to respond after the measured quantity has been changed.



Dead Zone: Dead zone is the largest change of input quantity for which there is no output of the instrument.

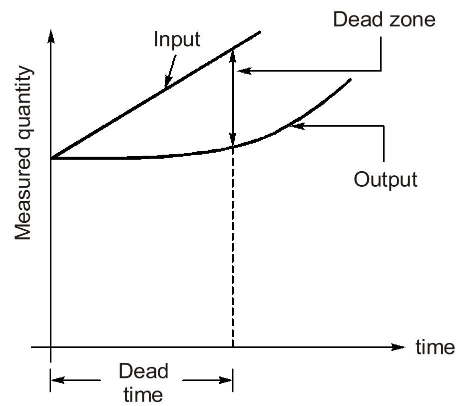


***Figure*-1.5:** *Dead Zone and Dead Time*

Signal to Noise Ratio (S/N)

- Noise is an unwanted signal superimposed upon the signal of interest thereby causing a deviation of the output from it's expected value.
- The ratio of desired to the unwanted noise is called signal to noise ratio and is expressed as

$$\boxed{\frac{S}{N} = \frac{\text{Signal Power}}{\text{Noise Power}}}$$

- In any measurement system, it is desired to have a large signal-to-noise ratio. This can be achieved by increasing the signal level without increasing the noise level or decreasing the noise level with some suitable technique.

Repeatability

It is the repetition of reading of an instrument from a given set of reading.

1.4 Errors in Measurements and their Analysis

Measurements done in a laboratory or at some other place always involve errors. No measurement is free from errors. If the precision of the equipment is adequate, no matter what it's accuracy is, a discrepancy will always be observed between two measured results.

True Value

The true value of quantity to be measured may be defined as the average of an infinite number of measured values when the average deviation due to various contributing factors tends to zero.

Guarantee Errors

The accuracy and precision of an instrument depends upon it's design, the material used and the workmanship that goes into making the instrument. Components are guaranteed to be within a certain percentage of the rated value. Thus, the manufacturer has to specify the deviations from the "nominal value" of a particular quantity. The limits of these deviations from the specified value are defined as ***"Limiting Errors"*** or ***"Guarantee Errors"***.

For example, the magnitude of a resistor is 200 Ω with a limiting error of ±10 Ω. The magnitude of the resistance will be between the limits

$$R = 200 \pm 10\ \Omega$$

or $R \geq 190\ \Omega$

and $R \leq 210\ \Omega$

Hence, the manufacturer guarantees that the value of the resistor lies between 190 Ω and 210 Ω.

Absolute (Relative) Limiting Error

The relative (fractional) error is defined as the ratio of the error to the specified (nominal) magnitude of a quantity.

$$\boxed{\text{Relative limiting error, } \varepsilon_r = \left(\frac{\text{Measured value} - \text{True value}}{\text{True value}}\right) \times 100}$$

or, $$\% \varepsilon_r = \left(\frac{\text{Actual value} - \text{Nominal value}}{\text{Nominal Value}}\right) \times 100$$

or, $$\boxed{\% \varepsilon_r = \left(\frac{A_m - A_T}{A_T}\right) \times 100} \quad \begin{cases} A_m = \text{Measured value} \\ A_T = \text{True value} \end{cases}$$

Now, $$\% \varepsilon_r = \frac{A_m - A_T}{A_T} \quad \text{or} \quad \frac{A_m}{A_T} = 1 + \varepsilon_r \quad \text{or} \quad \boxed{\frac{A_T}{A_m} = \frac{1}{1+\varepsilon_r}}$$

$$\boxed{A_T = \left(\frac{1}{1+\varepsilon_r}\right) A_m}$$

Here, $$\boxed{\frac{1}{1+\varepsilon_r} = \text{Correction factor}}$$

NOTE: Nominal value = True value and Actual value = Measured value

Example - 1.5 A resistance has nominal value of 50 Ω. When it is measured it's actual value is 60 Ω. Find the % error.

Solution:

$$\% \text{ error, } \varepsilon_r = \left(\frac{A_m - A_T}{A_T}\right) \times 100 = \left(\frac{60 - 50}{50}\right) \times 100 = 20\%$$

$$\boxed{\% \text{ error} = 20\%}$$

Example - 1.6 The measured value of a resistor is 100 Ω and it's relative error is ±10% then, it's true value and the range is?

Solution:

$$\varepsilon_r = \pm 10\% \text{ of } 100 = \pm 10\ \Omega$$

Range, $$A_T = (100 - 10) \text{ to } (100 + 10) = 90\ \Omega \text{ to } 110\ \Omega$$

Example - 1.7 The dead zone in a certain pyrometer is 0.125 percent of span. The calibration is 400°C to 1000°C. What temperature change might occur before it is detected?

(a) 0.25°C
(b) –0.50°
(c) 1.25°C
(d) 0.75°C

Solution: (b)

$$\text{Span} = 1000 - 400 = 600° \text{ C}$$

$$\therefore \quad \text{Dead zone} = 0.125\% \text{ of span} = \frac{0.125}{100} \times 600 = 0.75°\text{C}$$

Hence, a change of 0.75°C must occur before it is detected.

Combination of Quantities with Limiting Errors

When two or more quantities, each having a limiting error, are combined, it is advantageous to be able to compute the limiting error of the combination.

1. Sum or Difference of Two or more quantities

Let,
$$x_1 = a \pm \varepsilon_{r1}$$
$$x_2 = b \pm \varepsilon_{r2}$$
$$x_3 = c \pm \varepsilon_{r3}$$

$\therefore$
$$x = x_1 + x_2 + x_3$$

or,
$$x = -x_1 - x_2 - x_3$$

So,
$$x = \pm (x_1 + x_2 + x_3)$$

Relative limiting error in x is given by

$$\boxed{\varepsilon_x = \pm\left(\frac{a}{a+b+c}\cdot\varepsilon_{r1} + \frac{b}{a+b+c}\cdot\varepsilon_{r2} + \frac{c}{a+b+c}\varepsilon_{r3}\right)}$$

(ε_x = worst possible error)

Example - 1.8 Three resistances $R_1 = 10 \pm 2\%$, $R_2 = 20 \pm 5\%$, $R_3 = 50 \pm 3\%$ are connected in series. Find the % limiting error for the series combination.

Solution:

$$\varepsilon_R = \pm\left(\frac{10}{10+20+50}\times 2 + \frac{20}{10+20+50}\times 5 + \frac{50}{10+20+50}\times 3\right)$$

or,
$$\varepsilon_R = \pm 3.375\%$$

$$\boxed{\% \text{ Limiting error} = \pm 3.375\%}$$

Given,
$$R_T = 10 + 20 + 50 = 80\Omega$$

$$\boxed{R_{\text{measured}} = (80 \pm 3.375\%)\,\Omega}$$

2. Multiplication or Division Terms

Let,
$$x = \frac{x_1\, x_2}{x_3} \text{ or } \frac{x_2\, x_3}{x_1} \text{ or } x_1\, x_2\, x_3 \text{ or } \frac{x_1}{x_2\, x_3}$$

Then, relative limiting error is

$$\boxed{\varepsilon_x = \pm(\varepsilon_{r1} + \varepsilon_{r2} + \varepsilon_{r3})}$$

NOTE

When,
$$x = \frac{x_1\, x_2}{x_2 + x_3} \text{ or } \frac{x_1}{x_2 + x_3} \text{ or } \frac{x_1\, x_2}{x_2 - x_1}$$

Then, multiplication or division form is not applicable for finding relative limiting error.

Example - 1.9 In the measurement of unknown resistance by using a wheat stone bridge if $P = 20 \pm 5\%$, $Q = 50 \pm 3\%$ and $S = 30 \pm 2\%$. Find the value of the unknown resistance R and it's limiting error.

Solution:

Limiting error, $\varepsilon_R = \pm(5 + 3 + 2) = \pm 10\%$

$$R = \frac{P}{Q} \cdot S = \frac{20}{50} \times 30 = 12\,\Omega$$

So, unknown resistance, $R = 12 \pm 10\%$

3. Power of a Factor

Let, $x = x_1^m \cdot x_2^n \cdot x_3^p$ or $\dfrac{x_1^m x_2^n}{x_3^p}$ or $\dfrac{x_1^m}{x_2^n x_3^p}$

Then, Relative limiting error is $\varepsilon_r = \pm(m\varepsilon_{r1} + n\varepsilon_{r2} + p\varepsilon_{r3})$

NOTE

When x is of the form $\dfrac{x_1^m}{x_2^n + x_3^p}$ or $\dfrac{x_1^m + x_2^n}{x_3^p}$

then, above method is not applicable for finding relative limiting error.

Example - 1.10 The power is measured in a resistor by passing current through the ammeter and ammeter measures $I = (5 \pm 4\%)$ A across the resistance of $R = (10 \pm 2\%)\ \Omega$. Find the power consumed by the resistor and it's limiting error.

Solution:

Power consumed, $P = I^2R = 5^2 \times 10 = 250$ watts

and limiting error, $\varepsilon_p = \pm(2\varepsilon_I + \varepsilon_R) = \pm(2 \times 4 + 2) = 10\%$

$\therefore$ $P = (250 \pm 10\%)$ watt

4. Special Case

Resistance in parallel:

Let, $R_1 = 10 \pm 10\%$ (Range = 9 Ω to 11 Ω)

and $R_2 = 20 \pm 5\%$ (Range = 19 Ω to 21 Ω)

Equivalent resistance of parallel combination is $R = \dfrac{R_1 R_2}{R_1 + R_2}$

True value; $R = \dfrac{10 \times 20}{10 + 20} = 6.66\,\Omega = R_T$

Resistance in lower range; $R_L = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{9 \times 19}{9 + 19} = 6.107\ \Omega$ Measured value in low range = L_m

Resistance in higher range; $R_H = \dfrac{11 \times 21}{11 + 21} = 7.21875\ \Omega$ Measured value for high range = H_m

$$\text{Error in low range (for low value)} = \%\,\varepsilon_r = \left(\frac{L_m - R_T}{R_T}\right) \times 100$$

$$\text{Error in high range (for high value)} = \%\,\varepsilon_r = \left(\frac{H_m - R_T}{R_T}\right) \times 100$$

For present case,

Error in low range; $\%\varepsilon_r = \left(\frac{6.10 - 6.66}{6.66}\right) \times 100 = -8.4\ \%$

Error in high range; $\%\varepsilon_r = \left(\frac{7.2187 - 6.66}{6.66}\right) \times 100 = 8.38\ \%$

Example - 1.11 A 4-dial decade box has

Decade *a* of 10 × 1000 Ω ± 0.1%,
Decade *b* of 10 × 100 Ω ± 0.1%
Decade *c* of 10 × 10 Ω ± 0.5%
Decade *d* of 10 × 1 Ω ± 1.0%
It is set at 4639 Ω. Find the percentage limiting error and the range of resistance value.

Solution:

$$\text{Error for decade } a = \pm 4000 \times \frac{0.1}{100} = \pm 4\ \Omega$$

$$\text{Error for decade } b = \pm 600 \times \frac{0.1}{100} = \pm 0.6\ \Omega$$

$$\text{Error for decade } c = \pm 30 \times \frac{0.5}{100} = \pm 0.15\ \Omega$$

$$\text{Error for decade } d = \pm 9 \times \frac{1}{100} = \pm 0.09\ \Omega$$

∴ $$\text{Total error} = \pm(4 + 0.6 + 0.15 + 0.09) = 4.84\ \Omega$$

Relative limiting error, $\varepsilon_r = \pm\frac{4.84}{4639} = \pm 0.00104$

% limiting error; $\%\ \varepsilon_r = \pm 0.00104 \times 100 = \pm 0.104\%$

$$\text{Limiting value of resistance} = 4639\,(1 \pm 0.00104) = (4639 \pm 5)\ \Omega$$

5. Uncertainty Error

The uncertainty analysis in measurements when many variables are involved is done on the same basis as is done for error analysis when the result are expressed as standard deviations or probable errors.

Let $x = f(x_1, x_2, \ldots, x_n)$

$W_{x1}, W_{x2}, \ldots, W_{xn}$ be the uncertainties of $x_1, x_2, \ldots x_n$ respectively.

Then, uncertainty of x is given by

$$W_x = \sqrt{\left(\frac{dx}{dx_1}\right)^2 \cdot W_{x_1}^2 + \left(\frac{dx}{dx_2}\right)^2 \cdot W_{x_2}^2 + \left(\frac{dx}{dx_3}\right)^2 \cdot W_{x_3}^2 + \ldots + \left(\frac{dx}{dx_n}\right)^2 \cdot W_{x_n}^2}$$

Example - 1.12 Find the uncertainty in the measurement of power dissipated by resistor if the current flowing through the resistor is 5 A and the voltage across the resistor is 200 V and the uncertainty of the ammeter is 0.2 A and the voltmeter is 1.5 V. Find the uncertainty of the power.

Solution:

We know that, $P = VI$

$\therefore$ $$\frac{\delta P}{\delta V} = I = 5\text{ A}$$

$$W_V = 1.5\text{ V} \quad \text{(given)}$$

$$\frac{\delta P}{\delta I} = V = 200\text{ V},$$

$$W_I = 0.2\text{ A}$$

$\therefore$ Uncertainty in the measurement of power is given by

$$W_P = \sqrt{\left(\frac{\delta P}{\delta V}\right)^2 \cdot W_V^2 + \left(\frac{\delta P}{\delta I}\right)^2 \cdot W_I^2} = \sqrt{(200)^2 \times 0.2^2 + (5)^2 \times 1.5^2} = 40.69\text{ watt}$$

$\therefore$ Uncertainty in the measurement of power = 40.69 watt.

6. Standard Deviation (S.D.)

Standard deviation or the root mean square deviation is an important term in the analysis of random errors. The standard deviation of an infinite number of data is defined as the square root of the sum of the individual deviation squared, divided by the number of readings.

Let, $x = f(x_1, x_2, \ldots, x_n)$

Average, $$\bar{x} = \left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right)$$

Deviations are:

$$|d_1| = x_1 - \bar{x}$$

$$|d_2| = x_2 - \bar{x}$$

$$\vdots \quad \vdots \quad \vdots$$

$$|d_n| = x_n - \bar{x}$$

$$\text{Average deviation} = \frac{|d_1 + d_2 + \ldots + d_n|}{n}$$

Standard deviation,

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + \ldots + d_n^2}{(n-1)}} \quad (\text{for } n \le 20)$$

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + \ldots + d_n^2}{n}} \quad (\text{for } n > 20)$$

(n = No. of observations)

Variance, $V = \sigma^2 = (\text{standard deviation})^2$

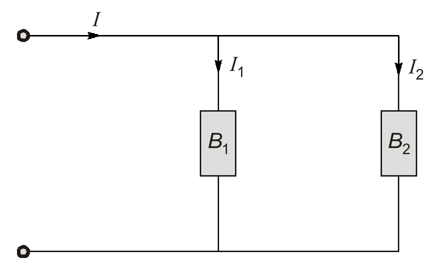
When standard deviation of $x_1, x_2, \ldots, x_n$ are $\sigma_{x1}, \sigma_{x2}, \ldots, \sigma_{xn}$ then standard deviation of x is given by:

$$\sigma_x = \sqrt{\left(\frac{dx}{dx_1}\right)^2 \cdot \sigma_{x_1}^2 + \left(\frac{dx}{dx_2}\right)^2 \cdot \sigma_{x_2}^2 + \ldots + \left(\frac{dx}{dx_n}\right)^2 \cdot \sigma_{x_n}^2}$$

Example - 1.13 A parallel circuit has two branches carries currents of I_1 = (100 ± 2)A and I_2 = (200 ± 5) A. Find the standard deviation in the measurement of total current if the errors in the currents I_1 and I_2 are due to standard deviation.

Solution:

We have



$$I = I_1 + I_2$$

$$\therefore \quad \frac{\delta I}{\delta I_1} = 1 \quad \text{and} \quad \frac{\delta I}{\delta I_2} = 1$$

$$\delta I_1 = 2$$

$$\delta I_2 = 5 \quad \text{(given)}$$

∴ Standard deviation in the measurement of total current is:

$$\sigma_I = \sqrt{\left(\frac{\delta I}{\delta I_1}\right)^2 \cdot \sigma_{I_1}^2 + \left(\frac{\delta I}{\delta I_2}\right)^2 \cdot \sigma_{I_2}^2} = \sqrt{1^2 \times 2^2 + 1^2 \times 5^2} = 5.38\,\text{A}$$

$$\boxed{\sigma_I = 5.38\,\text{A}}$$

So, $I = [(200 + 100) \pm 5.38]\,\text{A} = [(I_1 + I_2) \pm \sigma_I]$

or $I = (300 \pm 5.38)\,\text{A}$

Example - 1.14 A circuit was tuned for resonance by eight different students, and the values of resonant frequency in kHz were recorded as 532, 548, 543, 535, 546, 531, 543 and 536.

Calculate:

(a) the arithmetic mean

(b) deviations from mean

(c) the average deviation

(d) the standard deviation, and

(e) variance

Solution:

(a) The arithmetic mean of readings is given by

$$\bar{X} = \frac{\Sigma x}{n} = \left(\frac{532 + 548 + 543 + 535 + 546 + 531 + 543 + 536}{8}\right) = 539.25\text{ kHz}$$

(b) The deviations are,

$d_1 = x_1 - \bar{X} = 532 - 539.25 = -7.25$ kHz ; $d_2 = x_2 - \bar{X} = 548 - 539.25 = +8.75$ kHz

$d_3 = x_3 - \bar{X} = 543 - 539.25 = +3.75$ kHz ; $d_4 = x_4 - \bar{X} = 535 - 539.25 = -4.25$ kHz

$d_5 = x_5 - \bar{X} = 546 - 539.25 = +6.75$ kHz ; $d_6 = x_6 - \bar{X} = 531 - 539.25 = -8.25$ kHz

$d_7 = x_7 - \bar{X} = 543 - 539.25 = +3.75$ kHz ; $d_8 = x_8 - \bar{X} = 536 - 539.25 = -3.25$ kHz

(c) Average deviation is given by

$$\bar{D} = \frac{\Sigma|d|}{n} = \left(\frac{7.25 + 8.75 + 3.75 + 4.25 + 6.75 + 8.25 + 3.75 + 3.25}{8}\right)$$

$$= 5.75\text{ kHz}$$

(d) Since the number of observations are 8 which is less than 20, therefore, the standard deviation will be given by

$$S = \sqrt{\frac{\Sigma d^2}{(n-1)}}$$

$$= \sqrt{\frac{(-7.25)^2 + (+8.75)^2 + (3.75)^2 + (-4.25)^2 + (+6.75)^2 + (-8.25)^2 + (3.75)^2 + (3.25)^2}{(8-1)}}$$

$$= 6.54 \text{ kHz}$$

(e) Variance = $S^2 = 42.77\ (\text{kHz})^2$

7. Error at Desired Scale

Error at any desired scale is given by:

$$\% \varepsilon_r = \frac{\% \text{ full scale error} \times \text{Full scale value}}{\text{Desired value}}$$

Example - 1.15 An ammeter measures a full scale current of 100 *A* produces a full scale error of 5%. Find the error if the ammeter reads.

(a) 50 A (b) 25 A (c) 10 A

Solution:

(a) Desired ammeter reading = 50 A

$$\therefore \quad \% \text{ error}, \varepsilon_r = \frac{5 \times 100}{50} = 10\%$$

(b) Desired ammeter reading = 25 A

$$\therefore \quad \% \text{ error}, \varepsilon_r = \frac{5 \times 100}{25} = 20\%$$

(c) Desired ammeter reading = 10 A

$$\therefore \quad \% \text{ error}, \varepsilon_r = \frac{5 \times 100}{10} = 50\%$$

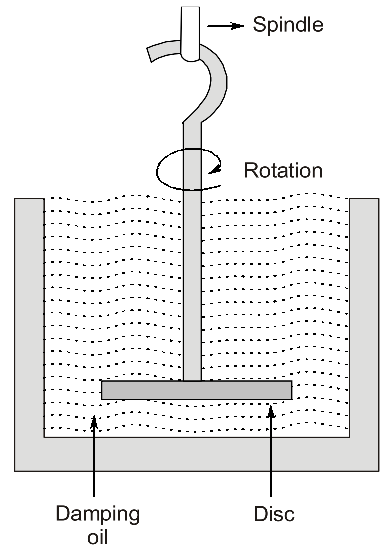
NOTE: As the desired instrument reading approaches the full scale value of measurement of the unknown quantity, error is reduced in the measurement.

Example - 1.16 A 160 ± 10% pF capacitor, an inductor of 160 ± 10% μH and a resistor of 1200 ± 11 Ω are connected in series.

(a) If all the three components are ±0% and resonant frequency is $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$, compute the resonant frequency of the combination.

(b) If all the three components are +10%, compute the expected resonant frequency of the combination and the percentage error when compared to the result of part (a).

(c) When all the three components are –10%, compute the expected resonant frequency and the percentage error when compared to the result of part (a).

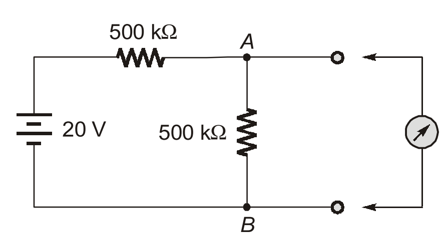


1.10 Important Prefixes and their Symbol

Factor of Multiplication	Prefix	Symbol
10^{12}	Tetra	*T*
10^{9}	Giga	*G*
10^{6}	Mega	*M*
10^{3}	kilo	*K*
10^{2}	hecto	*h*
10^{-1}	deci	*d*
10^{-2}	Centi	*c*
10^{-3}	milli	*m*
10^{-6}	micro	*u*
10^{-9}	nano	*n*
10^{-12}	pico	*p*

Student's Assignments 1

Q.1 What is the true value of voltage across the 500 kΩ resistor connected between terminals *A* and *B* as shown in given figure? What would a voltmeter with a sensitivity of 20 kΩ/V read on the following ranges : 50, 15, 5 V when connected across terminals *C* and *D*?



Q.2 The following values were obtained from the measurements of the value of a resistor: 147.2 Ω, 147.4 Ω, 147.9 Ω, 148.1 Ω, 147.1 Ω, 147.5 Ω, 147.6 Ω, 147.4 Ω, 147.6 Ω, and 147.5 Ω.

Calculate:
(i) arithmetic mean
(ii) average deviation
(iii) standard deviation, treating the data as finite
(iv) standard deviation, treating the data as population.

Q.3 The four arms of a Hay's a.c. bridge are arranged as follows:
AB is a coil of unknown impedance.
BC is a non-inductance R_1 = 1000 Ω with an error of ± 1 part in 10,000.
CD is a non-reactive resistor R_3 = 833 ± 0.25 Ω in series with no-loss capacitor C = 1.43 ± 0.001 μF.
DA is a non-reactive resistor R_2 = 16800 ± 1 part in 10,000.
The supply frequency is 50 ± 0.1 Hz. The bridge is balanced. Determine *L* and *R* of the coil and the limits or error. The balance conditions are:

$$L = \left(\frac{CR_1R_3}{1+\omega^2C^2R_3^2}\right) \text{ and}$$

$$R = \left(\frac{R_1R_2R_3C^2\omega^2}{1+\omega^2C^2R_3^2}\right)$$

Student's Assignments 1 — Answers

1. 10 V, 8 V, 5.45 V, 2.86 V
2. (i) 147.53 Ω (ii) 0.218 Ω
   (iii) 0.298 Ω (iv) 0.283 Ω
3. L = 21 ± 0.145 Hz and R = 2480 ± 29.5 Ω

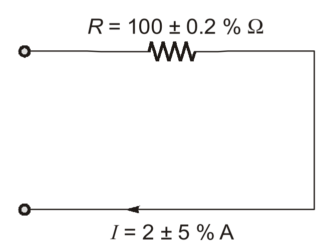
Student's Assignments 2

Q.1 Which of the following error is likely to occur in bridge method of measurement?
1. Residual error
2. Frequency and waveform error
3. Leakage and eddy current error

(a) 1 only (b) 2 only
(c) 1 and 2 only (d) 1, 2 and 3

Q.2 A utility type voltameter with an accuracy of ±3% of full scale (at 25°C) is used on 300 V scale to measure 230 V. What will be the possible percentage error and what range will the actual voltage fall within if the instrument reads 200 V?
(a) 3.9%, 200 V (b) 3.9%, 191 - 209 V
(c) 7.6%, 221 - 239 V (d) 7.6%, 200 V

Q.3 In the circuit given in the figure, the limiting error in the power dissipation 'I^2R' in the resistor *R* is



(a) 1.2% (b) 5.2%
(c) 10.2% (d) 25.2%

Q.4 A zero to 300 V voltmeter has a guaranteed accuracy of 1% full scale reading. The voltage measured by the instrument is 83 V. The percentage limiting error is
(a) 0.95 (b) 1.81
(c) 3.62 (d) 4.85

Q.5 Torque developed by an electromagnet is given as

$$F = \mu^a B^b A^c$$

where, μ = Permeability of air in the gap
B = Flux density in the air gap
A = Cross-sectional area of the gap

The values of *a*, *b* and *c* are respectively
(a) 1, 1 and 2 (b) –1, 1 and – 2
(c) 1, 2 and –1 (d) –1, 2 and 1

Q.6 A resistor of 10 kΩ with 5% tolerance is connected in series with a 5 kΩ resistor of 10% tolerance. What is the tolerance limit of the series network?
(a) 5% (b) 6.67%
(c) 10% (d) 8.33%

Q.7 Five Students made the following readings on a very accurate voltmeter which reads 2.13 V, 3.15 V, 2.97 V, 3.10 V and 2.99 V. What is the most probable value of the voltage?

(a) 3.066 V (b) 3.12 V
(c) 2.99 V (d) 2.97 V

Q.8 The power in a 3-phase, 3-wire load is measured using two 100 W full scale wattmeters W_1 and W_2. W_1 is of accuracy class ± 1% and reads 100 W. W_2 is of accuracy class ± 0.5% and reads –50 W. The uncertainly in the computation of total power will be
(a) ± 1.5% (b) ± 0.5%
(c) ± 4% (d) ± 3%

Q.9 Two resistances 100 ± 5 Ω and 150 ± 15 Ω are connected in series. If the errors are specified as standard deviations, the resultant error will be
(a) ± 10 Ω (b) ± 10.5 Ω
(c) ± 15.8 Ω (d) ± 20 Ω

Q.10 The measurement of a quantity
(a) is an act of comparison of an unknown quantity with another quantity.
(b) is an act of comparison of an unknown quantity with a known quantity whose accuracy may be known or may not be known.
(c) is an act of comparison of an unknown quantity with a predefined acceptable standard which is accurately known.
(d) none of the above

Q.11 A null type of instrument as compared to a deflection type instrument has
(a) a high accuracy (b) a lower sensitivity
(c) a faster response (d) all of these

Q.12 In measurement systems, which of the following are undesirable static characteristics?
(a) Sensitivity and accuracy
(b) Drift, static error and dead zone
(c) Reproducibility and non-linearity
(d) Drift, static error, dead zone and non-linearity

Q.13 The mean deviation $\bar{D}$ in terms of deviations from the mean value of n readings is
(a) $\frac{\Sigma|d|}{n}$ (b) $\frac{\Sigma d}{n}$
(c) $\frac{\sqrt{\Sigma d^2}}{n}$ (d) $\sqrt{\frac{\Sigma d^2}{n}}$

Q.14 A set of reading has a wide range and therefore it has:
(a) low precision (b) high precision
(c) low accuracy (d) high accuracy

Q.15 The following are the desirable dynamic characteristics of a measurement system :
(a) fast response, fidelity, measuring lag and dynamic error
(b) fast response and measuring lag
(c) fidelity and measuring lag
(d) fast response and fidelity

Q.16 The material of wires used for making resistance standards is usually
(a) Nichrome (b) Copper
(c) Manganin (d) Phosphor Bronze

Q.17 Fluid friction damping can be used in
(a) horizontally mounted instruments
(b) vertically mounted instruments
(c) both in horizontally and vertically mounted instruments
(d) none of these

Q.18 Permanent magnets used in instruments are hard core materials because
(a) they have broad hysteresis loop
(b) their energy density is high
(c) they have a high $(BH)_{max}$ product
(d) all of the above

Answer Key:

1. (d)	**2.** (b)	**3.** (b)	**4.** (c)
5. (d)	**6.** (b)	**7.** (a)	**8.** (d)
9. (d)	**10.** (c)	**11.** (a)	**12.** (d)
13. (a)	**14.** (a)	**15.** (d)	**16.** (c)
17. (b)	**18.** (d)		

Student's Assignments 2 Explanations

1. (d)
The various errors occurring in bridge method of measurement are (which we will see in later chapters):
⇒ Frequency error

$\Rightarrow$ Waveform error
$\Rightarrow$ Eddy Current error
$\Rightarrow$ Leakage Current error
$\Rightarrow$ Residual error

2. (b)
Accuracy = ± 3% of full scale which corresponds to ± 9 V.
So, range of reading for

$$200 \text{ V} = (200 \pm 9) \text{ V} = 191–209 \text{ V}$$

3. (b)

$$V_{rms} = \sqrt{\frac{1}{4}\int_0^T 4dt} = \sqrt{2} \text{ volt}$$

4. (c)

$$1\% \text{ accuracy} = \frac{300 \times 1}{100} = 3 \text{ V}$$

$$\therefore \text{Percentage limiting error} = \frac{8}{83} \times 100 = 3.62\%$$

5. (d)
Force developed by an electromagnet is:

$$F = \frac{B^2 A}{\mu} = \mu^{-1} B^2 A^1$$

6. (b)

$$R_1 = 10^4 \pm 5\% \ \Omega$$
$$= 10^4 + \frac{5}{100} \times 10^4$$
$$= 10^4 \pm 500\Omega$$
$$R_2 = 5000 \pm 10\% \ \Omega$$
$$= 5000 \pm \frac{10}{100} \times 5000$$
$$= 5000 \pm 500\Omega$$
$$\therefore \quad R = R_1 + R_2 = 15000 \pm 1000 \ \Omega$$
$$\therefore \quad \text{Tolerance limit} = \frac{1000}{15000} \times 100$$
$$= 6.666\% \approx 6.67\%$$

7. (a)
Most probable value

$$= \left(\frac{3.12 + 3.15 + 2.97 + 3.1 + 2.99}{5}\right) = 3.066 \text{ V}$$

8. (d)

$$W_1 = (100 \pm 1)W$$
$$W_2 = (-50 \pm 0.5)W$$
$$\therefore \quad W_1 + W_2 = 50 \pm \frac{1.5}{50} \times 100$$
$$= 50 \pm 3\%$$

Hence, uncertainly in measurement of power

$$= \pm 3\%$$

9. (d)
The total resistance will be $R_{equivalent} = R_1 + R_2$
Hence, the resultant error will be ± 20 Ω

■■■■