POSTAL
BOOK PACKAGE
2024

CONTENTS

ELECTRICAL ENGINEERING

Objective Practice Sets

Communication Systems

1
CHAPTER

Fourier Analysis of Signals, Energy and Power Signals

Q.1 A signal is such that $x(t) = x(t + T_0/2)$, it is also given that it is even in nature. The Fourier series expansion has
(a) absent sine term
(b) absent cos term
(c) only odd harmonics
(d) odd term of cos as $\Sigma\, a_n \cos n\omega$

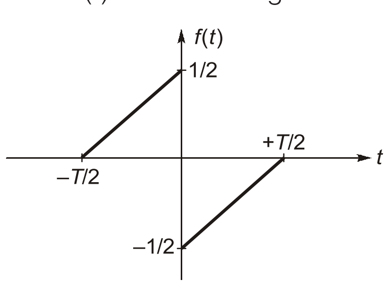
Q.2 Let $x(t)$ be a periodic signal with fundamental period T and Fourier series coefficient of Re$\{x(t)\}$ (where Re denotes the real part of signal) is
(a) $\dfrac{a_k + a_k^*}{2}$ (b) $\dfrac{a_k - a_{-k}^*}{2}$
(c) $\dfrac{a_k^* + a_{-k}}{2}$ (d) $\dfrac{a_k^* - a_k}{2}$

Q.3 If $G(t)$ represents the Fourier transform of a signal $g(t)$ which is real and odd symmetric in time then
(a) $G(f)$ is complex
(b) $G(f)$ is imaginary
(c) $G(f)$ is real
(d) $G(f)$ is real and non-negative

Q.4 The amplitude spectrum of Gaussian pulse is
(a) uniform (b) a sine function
(c) Gaussian (d) an impulse function

Q.5 A signum function is
(a) zero for t greater than zero
(b) zero for t less than zero
(c) unity for t greater than zero
(d) $2u(t) - 1$

Q.6 A function $f(t)$ is shown in figure.

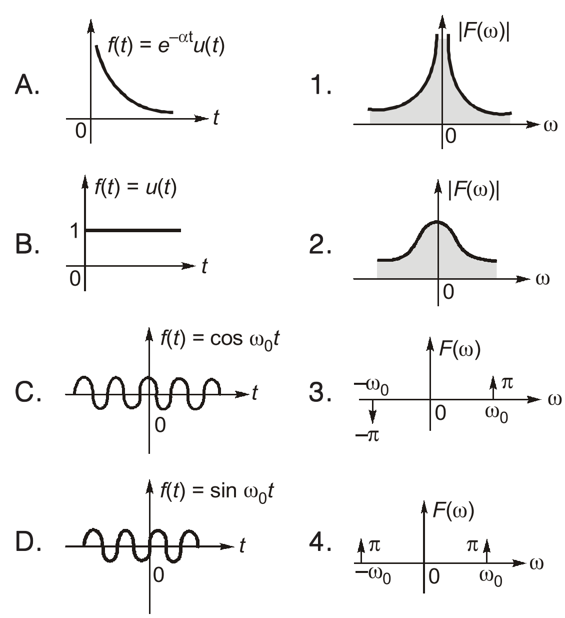


The Fourier transform $F(\omega)$ of $f(t)$ is
(a) real and even function of ω
(b) real and odd function of ω
(c) imaginary and odd function of ω
(d) imaginary and even function of ω

Q.7 What is the autocorrelation function of a rectangular pulse of duration T?
(a) A rectangular pulse of duration $2T$
(b) A rectangular pulse of duration T
(c) A triangular pulse of duration $2T$
(d) A triangular pulse of duration T

Q.8 In connection with properties of the Fourier transform, match **List-I (Function of Time)** with **List-II (Spectral Density Function)** and select the correct answer using the code given below the lists:

List-I List-II



Codes:

	A	B	C	D
(a)	1	3	2	4
(b)	2	1	4	3
(c)	1	3	4	2
(d)	2	1	3	4

Q.9 Match **List-I (Operations on $x(t)$)** with **List-II ($X(\omega)$/Fourier transform)** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Time shift	1. $\frac{1}{j\omega}X(\omega)+\pi X(0)\,\delta(\omega)$
B. Time differentiation	2. $e^{-j\omega t_0}X(\omega)$
C. Time integration	3. $X(\omega-\omega_0)$
D. Frequency shift	4. $(j\omega)^n X(\omega)$

Codes:

	A	B	C	D
(a)	2	1	4	3
(b)	2	4	1	3
(c)	3	4	1	2
(d)	3	1	4	2

Q.10 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I

A. Transmitter
B. Fourier series
C. Spectrum analyzer
D. Compression

List-II

1. For displaying signals in frequency domain.
2. Provides more gain for low-level than for higher-level signals.
3. Converts an information signal into a form suitable for propagation along a channel.
4. Represents periodic functions as a series of sinusoids.

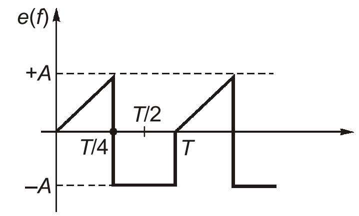
Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	3	4	1	2
(c)	3	1	4	2
(d)	1	4	2	3

Q.11 A signal has Fourier series coefficients
$C_n \Rightarrow C_{-1}=C_1=8,\ C_0=0,\ C_2=C_{-2}=2$
its power is

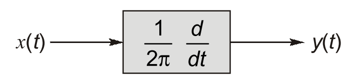
(a) 0 (b) 136
(c) 20 (d) 120

Q.12 The rms value of the periodic waveform $e(t)$ shown in figure is



(a) $\sqrt{\frac{3}{2}}A$ (b) $\sqrt{\frac{2}{3}}A$
(c) $\sqrt{\frac{1}{3}}A$ (d) $\sqrt{\frac{5}{6}}A$

Q.13 A deterministic signal $x(t)=\cos 2\pi t$ is passed through a differentiator as shown in figure.



what is its output power spectral density?

(a) $\frac{1}{4}[\delta(f-1)+\delta(f+1)]$
(b) $\frac{1}{4}[\delta(f-1)+\delta(f)]$
(c) $\frac{1}{4}[\delta(f)+\delta(f+1)]$
(d) None of the above

Q.14 The Fourier transform of $x(t)=\dfrac{2a}{a^2+t^2}$ is

(a) $2\pi e^{-a|\omega|}$ (b) $\pi e^{-2a|\omega|}$
(c) $\pi e^{-a\omega}$ (d) $\pi e^{-2a\omega}$

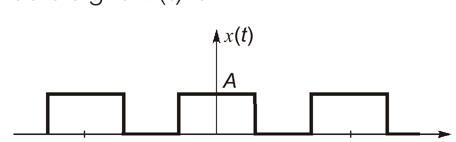
Q.15 Consider the signal defined by

$$x(t)\begin{cases} e^{j10t} & \text{for } |t|\le 1 \\ 0 & \text{for } |t|>1 \end{cases}$$

Its fourier transform is

(a) $\dfrac{2\sin(\omega-10)}{\omega-10}$ (b) $2e^{j10}\dfrac{\sin(\omega-10)}{\omega-10}$
(c) $\dfrac{2\sin\omega}{\omega-10}$ (d) $e^{j10\omega}\dfrac{2\sin\omega}{\omega}$

Q.16 Determine the Fourier series coefficients for given periodic signal $x(t)$ is



(a) $\frac{A}{j2\pi k}\sin\left(\frac{\pi}{2}k\right)$ (b) $\frac{A}{\pi jk}\cos\left(\frac{\pi}{2}k\right)$

(c) $\frac{2A}{\pi k}\sin\left(\frac{\pi}{2}k\right)$ (d) $\frac{2A}{\pi k}\cos\left(\frac{\pi}{2}k\right)$

Q.17 The Fourier series cofficients, of a periodic signal $x(t)$ expressed as $\sum_{k=-\infty}^{k=+\infty} a_k e^{j2\pi kt/T}$ are given by

$a_{-2} = 2 - j1$; $a_{-1} = 0.5 + j0.2$; $a_0 = j2$; $a_1 = 0.5 - j0.2$; $a_2 = 2 + j1$; and $a_k = 0$; for $|k| > 2$
which of the following is true?

(a) $x(t)$ has finite energy because only finitely many coefficients are non-zero
(b) $x(t)$ has zero average value because it is periodic
(c) the imaginary part of $x(t)$ is constant
(d) the real part of $x(t)$ is even

Q.18 Suppose we have given following information about a signal $x(t)$

1. $x(t)$ is real odd
2. $x(t)$ is periodic with $T = 2$
3. Fourier coefficients $C_n = 0$, $|n| > 1$
4. $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1$

The signal that satisfy these conditions

(a) $\sqrt{2}\sin\pi t$ and unique
(b) $\sqrt{2}\sin\pi t$ but not unique
(c) $2\sin\pi t$ and unique
(d) $2\sin\pi t$ but not unique

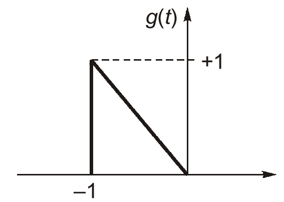
Q.19 Let $x(t)$ be a signal with its Fourier transform $X(j\omega)$ suppose we are given the following facts.

1. $x(t)$ is real.
2. $x(t) = 0$ for $t \le 0$.
3. $\frac{1}{2\pi}\int_{-\infty}^{\infty} \text{Re}\{X(j\omega)\}e^{j\omega t}d\omega = |t|e^{-|t|}$.

then a closed form expression for $x(t)$ is

(a) $2e^{-t}u(t)$ (b) $e^{-|t|}$
(c) $te^{-2t}u(t)$ (d) $2te^{-t}u(t)$

Q.20 The Fourier transform $G(\omega)$ of signal $g(t)$ in figure is given by



$$G(\omega) = \frac{1}{\omega^2}(e^{j\omega} - j\omega e^{j\omega} - 1)$$

What is the Fourier transform of $g(-t-1)$?

(a) $\frac{1}{\omega^2}[e^{-j\omega} + j\omega e^{-j\omega} - 1]$

(b) $e^{j\omega}\frac{1}{\omega^2}[e^{-j\omega} + j\omega e^{-j\omega} - 1]$

(c) $\frac{1}{\omega^2}[e^{-j\omega} + j\omega e^{-j\omega} + 1]$

(d) $\frac{2}{\omega}\left[\frac{e^{+0.5j\omega} - e^{-j0.5j\omega}}{2j}\right]$

Q.21 A signal is represented by

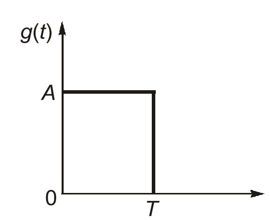
$$x(t) = \begin{cases} 1 & |t| < 1 \\ 0 & |t| > 1 \end{cases}$$

The Fourier transform of the convolved signal $y(t) = x(2t) * x(t/2)$.

(a) $\frac{4}{\omega^2}\sin\left(\frac{\omega}{2}\right)$ (b) $\frac{4}{\omega^2}\sin\left(\frac{\omega}{2}\right)\sin(2\omega)$

(c) $\frac{4}{\omega^2}\sin 2\omega$ (d) $\frac{4}{\omega^2}\sin^2\omega$

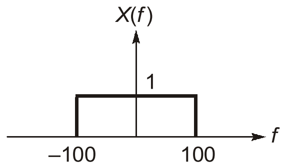
Q.22 The energy density spectrum $|G(f)|^2$ of a rectangular pulse shown in the given figure is

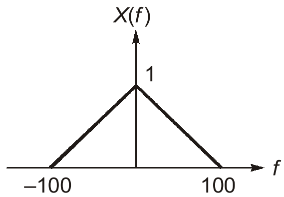


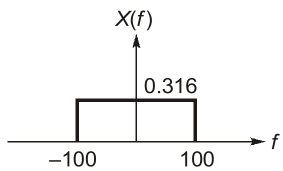
(a) $AT\left(\frac{\sin\pi fT}{\pi fT}\right)$ (b) $(AT)^2\left(\frac{\sin\pi fT}{\pi fT}\right)^2$

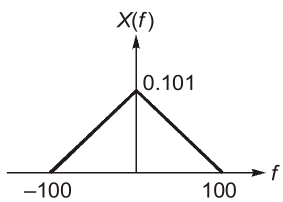
(c) $(AT)^2\left(\frac{\sin\pi fT}{\pi fT}\right)$ (d) $A^2\left(\frac{\sin\pi fT}{\pi fT}\right)$

Q.23 Frequency spectrum of signal $\frac{100}{\pi^2}\text{sin}c^2(100t)$ is

(a)


(b)


(c)


(d)


Q.24 Consider the following statements:
1. With the increase in signaling rate, the width of each pulse is reduced.
2. A signal can be band limited or time limited or both band limited and time limited simultaneously.
3. Differentiating the signal in time domain is equivalent to multiplying its Fourier transform by (j2pf).
4. Compression in time domain results in expansion of frequency spectrum and vice-versa.

Which of the above statements is/are **not** correct?
(a) 1 and 3
(b) 2 and 4
(c) 3 only
(d) 2 only

Q.25 **Assertion (A):** The Parseval's theorem implies superposition of the average powers.
Reason (R): The interpretation of the Parseval's theorem is that the total average power of the signal $x(t)$ can be found by squaring and adding the heights of the amplitude lines in the spectrum of the periodic signal $x(t)$.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is **not** the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q.26 **Assertion (A):** If two signals are orthogonal they will also be orthonormal.
Reason (R): If two signals are orthonormal they also will be orthogonal.
(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not a correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

■■■■

Answers **Fourier Analysis of Signals, Energy and Power Signals**

1. (d)	2. (a)	3. (b)	4. (c)	5. (d)	6. (c)	7. (c)	8. (b)	9. (b)
10. (b)	11. (b)	12. (d)	13. (d)	14. (a)	15. (a)	16. (c)	17. (a)	18. (b)
19. (d)	20. (b)	21. (b)	22. (b)	23. (d)	24. (d)	25. (a)	26. (d)	

Explanations Fourier Analysis of Signals, Energy and Power Signals

1. (d)

$$x(t) = x\left(t+\frac{T_0}{2}\right)$$

where T_0 is fundamental period.

$\Rightarrow x(t)$ is half wave symmetric,

it consists of only odd harmonics. ...(1)

Also, $x(t)$ is even, thus contains only cosine terms ...(2)

$\therefore$ $x(t)$ contains only odd cosine terms.

$$x(t) = \sum_{n=0}^{\infty} a_n \cos nw, \; n = \text{odd}.$$

2. (a)

$$x(t) \longleftrightarrow a_k$$

$$x(t) = R_c(x(t)) + jI_m(x(t))$$

$$x^*(t) = R_c(x(t)) - jI_m(x(t))$$

$$x(t) \longleftrightarrow a_k$$

$$x^*(t) \longleftrightarrow a_k^*(t)$$

$$x(t) + x^*(t) \longleftrightarrow a_k + a_k^*$$

$$2R_c(x(t)) \longleftrightarrow a_k + a_k^*$$

$$R_c(x(t)) \longleftrightarrow \frac{a_k + a_k^*}{2}$$

3. (b)

Function, $g(t)$	Fourier Transform, $G(f)$
Real and odd	Imaginary and odd
Real and even	Real and even
Imaginary and odd	Real and odd
Imaginary and even	Imaginary and even

4. (c)

Amplitude spectrum of Gaussian pulse is Gaussian.

5. (d)

$$Sgn(t) = \begin{bmatrix} 1 & t > 0 \\ -1 & t < 0 \end{bmatrix}$$

$$Sgn(t) = 2u(t) - 1$$

6. (c)

Signal is odd, $x(t) = -x(-t)$

Signal is half symmetric

$$x(t) = x\left(t+\frac{T_0}{2}\right)$$

$\therefore$ contains odd harmonic.

Signal $f(t)$ is real and odd,

$\therefore$ $F(\omega)$ is imaginary and odd.

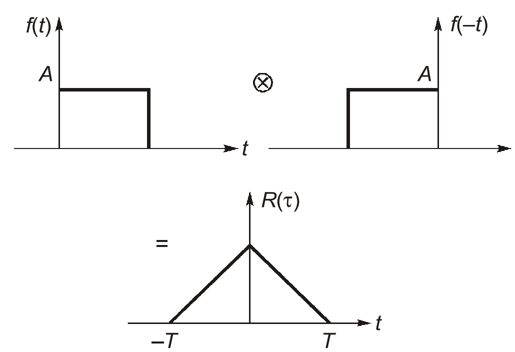
7. (c)

Autocorrelation,

$$R(\tau) = f(t) \otimes f(-t)$$

$$= \int_{-\infty}^{\infty} f(t) \cdot f(t-T)\, dt$$

i.e. convolution with the inverted version of signal itself.



8. (b)

$$F[e^{-at}u(t)] = \frac{1}{\sqrt{a^2+\omega^2}} e^{-j\tan^{-1}\frac{\omega}{a}}$$

$$F[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$F[\cos\omega_0 t] = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$$

$$F[\sin\omega_0 t] = j\pi[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$$

9. (b)

Operation	$x(t)$	$X(\omega)$/Fourier transform
• Time shift	$x(t-t_0)$	$e^{-j\omega t_0} X(\omega)$
• Time differerentiation	$\frac{d^n x(t)}{dt^n}$	$(j\omega)^n X(\omega)$
• Time integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
• Frequency shift	$x(t)e^{j\omega_0 t}$	$X(\omega-\omega_0)$

11. (b)

$$\text{Power} = \sum_{n=-\infty}^{\infty} |C_n| = \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2 dt$$

$$= |C_{-2}|^2 + |C_2|^2 + |C_1|^2 + |C_{-1}|^2 + |C_0|^2$$

$$= 2^2 + 2^2 + 8^2 + 8^2 + 0^2 = 136$$

12. (d)

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} (f(t))^2 dt}$$

$$V_{\text{rms}}^2 = \frac{1}{T}\cdot\left[\int_0^{T/4}\left(\frac{4A}{T}\cdot t\right)^2 dt + \int_{+T/4}^{T} (-A)^2 dt\right]$$

$$V_{ms}^2 = \frac{1}{T}\cdot\left[\frac{3A^2T}{4} + \frac{16A^2}{T^2}\cdot\frac{1}{3}\frac{T^3}{16\times 4}\right]$$

$$V_{ms}^2 = A^2\left[\frac{3}{4}+\frac{1}{12}\right] = A^2\cdot\frac{10}{12}$$

$$V_{ms} = A\sqrt{\frac{5}{6}}$$

13. (d)

$$S_0 = S_i(\omega)\cdot|H(\omega)|^2$$

$$H(w) = \frac{j\omega}{2\pi} = \frac{j2\pi f}{2\pi} = jf$$

$$|H(\omega)|^2 = \frac{\omega^2}{4\pi^2} = f^2$$

$$S_i(f) = \frac{1}{2}\cdot[\delta(f-1)+\delta(f+1)]$$

$$S_0(f) = \frac{f^2}{2}\cdot[\delta(f-1)+\delta(f+1)]$$

We know,

$$f^2\cdot\delta(f-1) = (+1)^2\cdot\delta(f-1)$$

$$f^2\cdot\delta(f+1) = (-1)^2\cdot\delta(f+1)$$

$$S_0(f) = \frac{1}{2}(\delta(f-1)+\delta(f+1))$$

14. (a)

$$e^{-a|t|} \longleftrightarrow \frac{2a}{a^2+\omega^2}$$

Using duality property

$$\frac{2a}{a^2+t^2} \longleftrightarrow 2\pi e^{-a|-\omega|} = 2\pi e^{-a|\omega|}$$

15. (a)

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \begin{cases} e^{j10t} & |t|<1 \\ 0 & |t|>1 \end{cases}$$

$$\Rightarrow \quad X(\omega) = \int_{-1}^{1} e^{j10t}\cdot e^{-j\omega t}dt$$

$$X(\omega) = \frac{e^{-j(\omega-10)} - e^{j(\omega-10)}}{-j(\omega-10)}$$

$$X(\omega) = \frac{2[e^{j(\omega-10)} - e^{-j(\omega-10)}]}{2j(\omega-10)}$$

$$X(\omega) = \frac{2\sin(\omega-10)}{\omega-10}$$

16. (c)

$$a_k = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cdot\cos k\omega_0 t\, dt$$

$$b_k = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cdot\sin k\omega_0 t\, dt$$

$$\Rightarrow \quad a_k = \frac{2}{T}\int_{-T/4}^{T/4} A\cos k\omega_0 t\, dt$$

$$= \frac{2A}{T}\cdot 2\int_0^{T/4}\cos k\omega_0 t\, dt$$

$$= \frac{4A}{T}\cdot\frac{1}{k_{\omega_0}}\sin k\omega_0 t$$

Also, $\omega = \dfrac{2\pi}{T}$

$$\Rightarrow \quad a_k = \frac{4A}{T}\cdot\frac{1}{k\cdot\frac{2\pi}{T}}\sin\left(k\cdot\frac{2\pi}{T}\cdot\frac{T}{4}\right)$$

$$\Rightarrow \quad a_k = \frac{A}{\pi k}\sin\left(\frac{\pi}{2}k\right)$$

$$b_k = \frac{2}{T}\int_{-T/4}^{T/4} A\sin k\omega_0 t = 0$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} A\, dt$$

$$a_0 = \frac{1}{T}\int_{-T/4}^{T/4} A\, dt = \frac{A}{T}\cdot\frac{t}{2} = \frac{A}{2}$$

$$x(t) = \frac{A}{2} + \sum_{n=0}^{\infty}\frac{2A}{\pi k}\sin\left(\frac{\pi}{2}k\right)$$

Fourier coefficient,

$$a_k = \frac{2A}{k\pi}\sin\left(\frac{\pi}{2}k\right)$$

17. (a)

$$a_{-2} = 2 - j1$$
$$a_2 = 2 + j1$$
$$\Rightarrow \quad a_2 = a_{-2}^*$$
$$a_1 = 0.5 - j0.2$$
$$a_{-1} = 0.5 + j0.2$$
$$\Rightarrow \quad a_1 = a_{-1}^*$$

Thus, $x(t)$ is real.

Energy,

$$\int |x(t)|^2\, dt = T\cdot\sum |C_n|^2$$
$$= T\sum |a_k|^2$$
$$= T(5 + 5 + 4 + 0.29 + 0.29)$$
$$= 14.58\,T$$

For periodic signal, T is fixed,

Hence energy is finite.

18. (b)

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jnw_0 t}$$

Since, $\quad c_n = 0$

$|n| > 1,\ n > 1$ and $n < -1$.

$$x(t) = C_0 + C_1 e^{jw_0 t} + c_{-1}e^{-jw_0 t}$$

Also, $x(t)$ is periodic with period '2'.

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

$x(t)$ is odd, $\quad c_1 = -c_{-1},\ c_0 = 0$

Also, $\quad$ power $= 1$

As per parsvell's theorem,

$$\frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt = \sum |C_n|^2 = 1$$

$$|c_1|^2 + |c_{-1}|^2 = 1$$

$$\Rightarrow \quad c_1 = \frac{1}{\sqrt{2}}$$

Thus, $\quad x(t) = 2c_1 \sin \pi t$

$$x(t) = 2\times\frac{1}{\sqrt{2}}\sin\pi t$$

$$\Rightarrow \quad x(t) = \sqrt{2}\sin\pi t$$

The signal that satisfies these conditions is not unique, as $x(t) = \sqrt{2}\sin\pi t$ will also satisfy these conditions.

19. (d)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

$$\Rightarrow \quad \text{Real}\,(x(t)) = 2|t|e^{-|t|}$$

Since, $\quad x(t) = 0, \qquad t \le 0$

$\Rightarrow \quad x(t) = 2te^{-t} \qquad t > 0$

$-2te^{t} = 0, \qquad t < 0$

$\Rightarrow \quad x(t) = 2t\,e^{-t}\,u(t)$

20. (b)

$$g(t) \longleftrightarrow G(\omega)$$

$$g(-t) \longleftrightarrow G(-\omega)$$

As, $t \to t + 1$

$$g(-t-1) \longleftrightarrow G(-\omega)\,e^{j\omega}$$

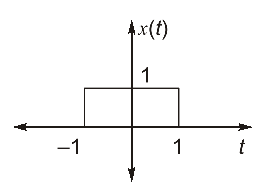
We know, $\quad G(\omega) = \dfrac{1}{\omega^2}\left(e^{j\omega} - j\omega e^{j\omega} - 1\right)$

$$G(-\omega)e^{j\omega} = \frac{1}{\omega^2}(1 + j\omega - e^{j\omega})$$

$$= \frac{e^{j\omega}}{\omega^2}\left(e^{-j\omega} + j\omega e^{-j\omega} - 1\right)$$

21. (b)

$$X(t) \longleftrightarrow X(\omega)$$



$\longleftrightarrow X(\omega) = \dfrac{2\sin\omega}{\omega}$

$$x(t) \longleftrightarrow X(\omega)$$

$$x(2t) \longleftrightarrow \frac{1}{2}\times\left(\frac{\omega}{2}\right)$$

$$x\left(\frac{t}{2}\right) \longleftrightarrow 2\times(2\,\omega)$$

$$y(t) = x(2t)\otimes x\left(\frac{t}{2}\right) \longleftrightarrow \frac{1}{2}\times\left(\frac{\omega}{2}\right)\cdot 2\times(2\omega)$$

$$Y(\omega) = X\left(\frac{\omega}{2}\right)\cdot X(2\omega)$$