



POSTAL BOOK PACKAGE 2025

ELECTRONICS ENGINEERING

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CONVENTIONAL Practice Sets

CONTENTS

CONTROL SYSTEMS

1. Introduction	2 - 4
2. Transfer Function	5 - 8
3. Block Diagram	9 - 16
4. Signal Flow Graph	17 - 23
5. Feedback Characteristics	24 - 28
6. Modelling of Control Systems	29 - 32
7. Time Domain Analysis of Control Systems	33 - 47
8. Stability Analysis of Linear Control Systems	48 - 52
9. The Root Locus Technique	53 - 63
10. Frequency Domain Analysis of Control Systems	64 - 82
11. Controllers and Compensators	83 - 86
12. State Variable Analysis	87 - 100

Introduction

Q1 (a) A control system is defined by following mathematical relationship

$$\frac{d^2x}{dt^2} + \frac{6dx}{dt} + 5x = 12(1 - e^{-2t})$$

Find the response of the system at $t \rightarrow \infty$

(b) A function $y(t)$ satisfies the following differential equation

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

Where $\delta(t)$ is delta function. Assuming zero initial condition and denoting unit step function by $u(t)$. Find $y(t)$.

Solution:

(a) Taking LT on both sides

$$(s^2 + 6s + 5) X(s) = 12 \left[\frac{1}{s} - \frac{1}{s+2} \right]$$

$$(s+1)(s+5) X(s) = \frac{24}{s(s+2)}$$

$$X(s) = \frac{24}{s(s+1)(s+2)(s+5)}$$

Response at $t \rightarrow \infty$

Using final value theorem,

$$\boxed{\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} [sX(s)]} = \lim_{s \rightarrow 0} \frac{s \times 24}{s(s+1)(s+2)(s+5)} = 2.4$$

(b) Taking Laplace transform on both sides

$$Y(s)[s+1] = 1$$

$$Y(s) = \frac{1}{s+1}$$

By taking inverse Laplace transform

$$y(t) = e^{-t} u(t)$$

Q2 (a) The Laplace equation for the charging current, $i(t)$ of a capacitor arranged in series with a resistance is given by

$$I(s) = \frac{sC}{1 + sRC} \cdot E(s)$$

The circuit is connected to a supply voltage of E . If $E = 100 \text{ V}$, $R = 2 \text{ M}\Omega$, $C = 1 \text{ }\mu\text{F}$. Calculate the initial value of the charging current.

(b) A series circuit consisting of resistance R and an inductance of L is connected to a d.c. supply voltage of E . Derive an expression for the steady-state value of the current flowing in the circuit using final value theorem.

Solution:

(a) Since, $E = 100 \text{ v}(t)$
Taking Laplace Transform, $E = 100 (t) \text{ volts,}$

$$\therefore E(s) = \frac{100}{s}$$

Substituting the given values,

$$I(s) = \frac{1 \times 10^{-6} s}{(2 \times 10^6 \times 1 \times 10^{-6} s + 1)} \cdot \frac{100}{s} = \frac{10^{-6} s}{2s + 1} \cdot \frac{100}{s}$$

Applying the initial value theorem,

$$i(0^+) = \lim_{t \rightarrow 0} i(t) = \lim_{s \rightarrow \infty} s I(s)$$

$$i(0^+) = \lim_{s \rightarrow \infty} s \cdot \frac{10^{-4}}{1 + 2s} = \lim_{s \rightarrow \infty} \frac{10^{-4}}{\frac{1}{s} + 2} = 50 \mu\text{A}$$

(b) The differential equation relating the current $i(t)$ flowing in the circuit and the input voltage E is given by

$$E = R i(t) + L \frac{di(t)}{dt}$$

Taking Laplace transform of the equation yields,

$$E(s) = R I(s) + L [sI(s) - i(0^+)]$$

Assume, $i(0^+) = 0$

$$\therefore E(s) = R I(s) + Ls I(s)$$

$\therefore E$ is constant (d.c. voltage)

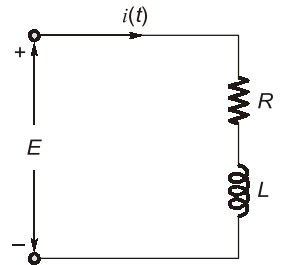
$$E(s) = \frac{E}{s} = R I(s) + Ls I(s)$$

$$I(s) = \frac{E}{s(R + sL)}$$

Applying the final value theorem,

$$i_{ss} = \lim_{t \rightarrow \infty} i(t) = \lim_{s \rightarrow 0} s I(s) = \lim_{s \rightarrow 0} \frac{sE}{s(R + sL)}$$

$$i_{ss} = \frac{E}{R}$$



Q3 The impulse response of a system S_1 is given by $y_1(t) = 4e^{-2t}$. The step response of a system S_2 is given by $y_2(t) = 2(1 - e^{-3t})$. The two systems are cascaded together without any interaction. Find response of the cascaded system for unit ramp input.

Solution:

(a) Taking the Laplace transform of the response of S_1 , we get

$$Y_1(s) = \frac{4}{s + 2}$$

$$X_1(s) = 1 \dots (x(t) = \delta(t))$$

Therefore, $G_1(s) = \frac{Y_1(s)}{X_1(s)} = \frac{4}{s + 2}$

[$\therefore Y_1(s) = 1$]

Taking the Laplace transform of the response of S_2 , we get

$$Y_2(s) = 2 \left(\frac{1}{s} - \frac{1}{s + 3} \right) = \frac{6}{s(s + 3)}$$

$$Y_2(s) = \frac{1}{s} \dots (x_2(t) = u(t))$$

Thus,
$$G_2(s) = \frac{Y_2(s)}{X_2(s)} = \frac{6}{s(s+3)} \cdot s = \frac{6}{s+3}$$

(b) The transfer function of the cascaded system is

$$G(s) = G_1(s)G_2(s) = \frac{24}{(s+2)(s+3)}$$

The Laplace transform of unit ramp is $R(s) = \frac{1}{s^2}$. Therefore,

$$G(s) = \frac{C(s)}{R(s)}$$

$$\begin{aligned} C(s) &= \frac{24}{(s+2)(s+3)} \cdot \frac{1}{s^2} \\ &\equiv \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+2} + \frac{D}{s+3} \end{aligned}$$

$$A = \left. \frac{24}{(s+2)(s+3)} \right|_{s=0} = 4$$

$$\begin{aligned} B &= \left. \frac{d}{ds} [s^2 C(s)] \right|_{s=0} \\ &= \left. \frac{d}{ds} \left[\frac{24}{(s+2)(s+3)} \right] \right|_{s=0} = - \left. \frac{24(2s+5)}{(s+2)^2(s+3)^2} \right|_{s=0} \\ &= -\frac{10}{3} \end{aligned}$$

$$C = \left. \frac{24}{s^2(s+3)} \right|_{s=-2} = 6$$

$$D = \left. \frac{24}{s^2(s+2)} \right|_{s=-3} = -\frac{8}{3}$$

$$C(s) = \frac{4}{s^2} - \frac{10}{3}s + \frac{6}{s+2} - \frac{8}{3}e^{-3t}$$

Taking inverse Laplace transform.

Therefore,
$$c(t) = 4t - \frac{10}{3}u(t) + 6e^{-3t} - \frac{8}{3}e^{-3t}$$

