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CHAPTER

Propositional Logic

Multiple Choice Questions

Q.1 Argument: $([P \rightarrow (q \vee r)] \wedge \bar{q} \wedge \bar{r}) \rightarrow \bar{P}$ is

(a) Valid argument (b) Invalid argument
(c) Unknown (d) None of these

Q.2 $\neg\forall_x \forall_y[(x < y) \rightarrow (x^2 < y^2)]$ is equivalent to

(a) $\exists_x \exists_y[(x < y) \wedge (x^2 \geq y^2)]$

(b) $\exists_x \exists_y[\neg(x < y) \wedge \neg(x^2 < y^2)]$

(c) $\exists_x \exists_y[\neg(x < y) \vee \neg(x^2 < y^2)]$

(d) $\exists_x \exists_y[(x < y) \vee (x^2 \geq y^2)]$

Q.3 What is the logical translation of the following statement?
"None of my friends are perfect".

(a) $\exists x(F(x) \wedge \neg P(x))$ (b) $\exists x(\neg F(x) \wedge P(x))$
(c) $\exists x(\neg F(x) \wedge \neg P(x))$ (d) $\exists x(\neg F(x) \vee \neg P(x))$

Q.4 **Statement:** Last year, the only Book I read were adventure stories.
Logical representation of the above statement is?

(a) Adventure story $\rightarrow$ book that I read last year
(b) Book that I read last year $\rightarrow$ adventure story
(c) Not a book I read last year $\rightarrow$ not adventure story
(d) None of these

Q.5 Consider the following statements:

(i) Those who like painting like flowers.
(ii) Those who like running like music.
(iii) Those who do not like music do not like flowers.

If all the above statements are true, then consider the following statements.

1. Those who like running do not like paintings.
2. Those who like painting like flowers.
3. Those who like running like flower.
4. Those who like painting like music.

Which is following is true?

(a) 2 only (b) 1, 4 only
(c) 2, 3 only (d) 4 only

Q.6 Which of the following formulas is a formalization of the sentence:
"There is a computer which is not used by any student".

(a) $\exists x(\text{computer}(x) \wedge \forall y(\neg \text{ student } (y) \rightarrow \text{uses } (y, x)))$
(b) $\exists x(\text{computer}(x) \rightarrow \forall y(\text{student } (y) \rightarrow \neg \text{ uses } (y, x)))$
(c) $\exists x(\text{computer}(x) \wedge \forall y(\text{student } (y) \rightarrow \neg \text{ uses } (y, x)))$
(d) $\exists x(\text{computer}(x) \vee \forall y(\text{student } (y) \rightarrow \neg \text{ uses } (y, x)))$

Q.7 $P(x, y) : x + y = x - y$
If the universe is the set of integers which of the following are true

(i) $P(1, 1)$ **(ii)** $P(3, 0)$
(iii) $\exists x P(x, 2)$ **(iv)** $\exists x \forall y P(x, y)$
(v) $\exists y \forall x P(x, y)$ **(vi)** $\forall x \exists x P(x, y)$

(a) (ii) and (v) only (b) (ii), (v) & (iv) only
(c) (ii) only (d) (v) and (vi) only

Q.8 Which of the following is true?

(i) $\exists x\{P(x) \wedge Q(x)\} \equiv \exists x P(x) \wedge \exists x Q(x)$
(ii) $\exists x\{P(x) \wedge Q(x)\} \Rightarrow \exists x P(x) \wedge \exists x Q(x)$
(iii) $\exists x P(x) \wedge \exists x Q(x) \equiv \exists x P(x) \wedge \exists y Q(y)$

(a) (i) only (b) (ii) and (iii) only
(c) (ii) only (d) None of these

Q.9 Which of the following is principle conjunction normal form for $[(p \vee q] \wedge \neg p \neg q]$?

(a) $p \vee \neg q$ (b) $p \vee q$
(c) $\neg p \vee q$ (d) $\neg p \vee \neg q$

Q.64 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I

A. Everybody loves Modi
B. Everybody loves somebody
C. There is somebody whom everybody loves
D. There is somebody whom no one loves

List-II

1. $\forall x$ Loves (x, modi)
2. $\forall x\, \exists y$ Loves (x, y)
3. $\exists y\, \forall x$ Loves (x, y)
4. $\exists y\, \forall x \neg$ Loves (x, y)

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	1	3	2	4
(c)	1	4	3	2
(d)	1	2	4	3

Q.65 What is correct translation of the following statement into mathematical logic?

"Some real number are rational"

(a) $\exists x$ (real (x) $\vee$ rational (x))
(b) $\forall x$ (real (x) $\rightarrow$ rational (x))
(c) $\exists x$ (real (x) $\wedge$ rational (x))
(d) $\exists x$ (rational (x) $\rightarrow$ real (x))

Q.66 Which of the following is negation of $\forall x \forall y[((x > 0) \wedge (y > 0)) \rightarrow (x + y > 0)]$?

(a) $\exists x\, \exists y\, [(x > 0) \wedge (y > 0) \wedge (x + y \leq 0)]$
(b) $\exists x\, \exists y\, [((x > 0) \wedge (y > 0)) \wedge (x + y \leq 0)]$
(c) $\exists x\, \exists y\, [\neg((x > 0) \wedge (y > 0)) \vee (x + y \leq 0)]$
(d) None of the above

Q.67 What is the predicate logic for the following statement?

There are atmost two males in the class.

(a) $\forall x \forall$y((Male (x) $\wedge$ Male (y)) $\rightarrow$ (x = y $\vee$ y = x))
(b) $\exists x \exists$y (Male (x) $\wedge$ Male (y) $\wedge$ $x \neq$ y $\wedge$ $\forall$z(Male (z) $\rightarrow$ (z = x $\vee$ z = y)))
(c) $\forall x \forall$y$\forall$z ((Male (x) $\wedge$ Male (y) $\wedge$ Male (z)) $\rightarrow$ (x = y $\vee$ x = z $\vee$ y = z))
(d) None of these

Q.68 Which one of following is most appropriate logical formula to represent the statement?

"Gold and silver ornaments are precious" the following notation are used:

$G(x)$: x is a gold ornament.
$S(x)$: x is a silver ornament.
$P(x)$: x is precious.

(a) $\forall x(P(x) \rightarrow (G(x) \wedge S(x))$
(b) $\forall x((G(x) \wedge S(x)) \rightarrow P(x))$
(c) $\exists x((G(x) \wedge S(x)) \rightarrow P(x)$
(d) $\forall x((G(x) \vee S(x)) \rightarrow P(x)$

Q.69 Which of the above two are equivalent?

1. $\neg\forall x(P(x))$
2. $\neg\exists x(P(x))$
3. $\neg\exists x(\neg P(x))$
4. $\exists x(\neg P(x))$

Select the correct option:

(a) 1 and 3
(b) 1 and 4
(c) 2 and 3
(d) 2 and 4

Multiple Select Questions (MSQ)

Q.70 Which of the following is/are true?

(a) $\forall x\, (P(x) \vee y)$ is equivalent to $(\forall x\, P(x) \vee y)$
(b) $(\forall x\, (P(x) \vee \exists y\, P(y)))$ is equivalent to $\exists x\, P(x)$
(c) $\forall x\, (P(x) \vee y)$ is not equivalent to $(\forall x\, P(x) \wedge$ y)
(d) $\forall x\, (P(x) \wedge y)$ is not equivalent to $(\forall x\, P(x) \wedge$ y)

Q.71 Which of the following are valid?

(a) $\exists x\, \exists y\, P(x, y) \rightarrow \exists y\, \exists x\, P(x, y)$
(b) $\forall x\, \exists y\, Q(x, y) \rightarrow \exists y\, \forall x\, Q(x, y)$
(c) $\exists x\, \forall y\, P(x, y) \rightarrow \forall y\, \exists x\, R(x, y)$
(d) Not $[\exists x\, S(x)]$ IFF $\forall x$ NOT $[S(x)]$

Q.72 Suppose $P(x, y)$ is some binary predicate defined on a very small domain of discourse: Just the integers 1, 2, 3 and 4. For each of the 16 pairs of these numbers, $P(x, y)$ is either true or false, according to the following table (x values are rows, y values are columns).

	1	2	3	4
1	T	F	F	F
2	F	T	T	F
3	T	T	T	T
4	F	F	F	F

Then which if the following statement(s) is/are true?

(a) $\forall x\, \exists y\, P(x, y)$
(b) $\forall y\, \exists x\, P(x, y)$
(c) $\exists x\, \forall y\, P(x, y)$
(d) $\exists y\, \forall x\, P(x, y)$

Explanations **Propositional Logic**

1. (a)

$([P \rightarrow (q \vee r)] \wedge \bar{q} \wedge \bar{r}) \rightarrow \bar{P}$ is tautology hence it is valid argument.

2. (a)

$\neg \forall x \forall y [(x < y) \rightarrow (x^2 < y^2)]$

$\cong \exists x \exists y \neg [(x < y) \rightarrow (x^2 < y^2)]$

$\cong \exists x \exists y \neg [\neg (x < y) \vee (x^2 < y^2)]$

$\cong \exists x \exists y [(x < y) \wedge \neg (x^2 < y^2)]$

$\cong \exists x \exists y [(x < y) \wedge (x^2 \geq y^2)]$

3. (d)

$F(x) \Rightarrow x$ is my friend.
$P(x) \Rightarrow x$ is perfect.
(d) is correct answer.
(a) There exist some friend which are not perfect.
(b) There are some people who are not my friend and are perfect.
(c) There exist some people who are not my friend and are not perfect.
(d) There does not exist any person who is my friend and perfect.

So, option (d) is correct.

4. (b)

This states the original accurately. If something is a book that I read last year, then it is guaranteed to be adventure story. The equivalent rule is that if a book is not an adventure story, then i definitely did not read it last year.

5. (d)

Let $P(x) = x$ likes paintings, $F(x) = x$ likes flowers, $R(x) = x$ likes running, $M(x) = x$ likes music
Statement (i) implies $P(x) \rightarrow F(x)$
Statement (ii) implies $R(x) \rightarrow M(x)$
Statement (iii) implies $\sim M(x) \rightarrow \sim P(x)$ can be written as $F(x) \rightarrow M(x)$
From statement 2 and 3, we can get $P(x) \rightarrow M(x)$
Only statement (4) is correct.
So, option (d) is correct.

6. (c)

(c) is the correct option.

7. (b)

$P(1, 1) : 1 + 1 = 1 - 1$ is false
$P(3, 0) : 3 + 0 = 3 - 0$ is true
$\exists x P(x, 2) : \exists x (x + 2 = x - 2)$ is false as there is no solution
$\exists x \forall y P(x, y) : \exists x \forall y (x + y = x - y)$ is false since this can be made true only if y = 0
$\exists y \forall x P(x, y) : \exists y \forall x (x + y = x - y)$ is true since for y = 0 the equation is true for all x
$\forall x \exists y P(x, y) : \forall x \exists y (x + y = x - y)$ is true since for all x, $y = 0$ will satisfy the equation.

8. (b)

III is true since once a variable is bound to a qualifier it's name does not matter.
So $\exists x Q(x)$ is same $\exists y Q(y)$ and so II is true since LHS and RHS is same.

I is false, since
LHS: some value of x satisfies both P and Q
RHS: some values satisfies P and some value satisfies Q, but these 2 values need not be same.

II is true, since
If the same value satisfies both P and Q surely some value satisfies P and some values satisfies Q.
In other words LHS of implication is stronger than RHS and hence implication will be true.

9. (a)

Given $[(p \vee q] \wedge \neg p \neg q]$

The precedence of the used operators are: $\wedge > \vee > \rightarrow$

Therefore, $[(p \vee q] \wedge \neg p \rightarrow \neg q]$

$\Rightarrow \quad [((p \vee q) \wedge \neg p) \rightarrow \neg q]$

$\Rightarrow \quad [((p \wedge \neg p) \vee (q \wedge \neg p)) \rightarrow \neg q]$

(using distributing law)

$\Rightarrow \quad [(0 \vee (q \wedge \neg p)) \rightarrow \neg q]$

$\Rightarrow \quad [(q \wedge \neg p) \rightarrow \neg q]$

$\Rightarrow \quad [\neg(q \wedge \neg p) \vee \neg q)]$ (using Demorgan's law)

$\Rightarrow \quad [\neg q \vee p \vee \neg q]$

$\Rightarrow \quad [p \vee \neg q]$

$\therefore$ Option (a) is correct.

10. (b)

I. $\forall x \exists y\, R(x,y) \to \exists y(\exists x\, R(x,y))$ is true, since $\exists y\, (\exists x\, R(x,y)) \equiv \exists x\, (\exists y\, R(x,y))$

II. $\forall x \exists y\, R(x,y) \to \exists y(\forall x\, R(x,y))$ is false

Since $\exists y$ when it is outside is stronger then when it is inside.

III. $\forall x \exists y\, R(x,y) \to \forall y \exists x\, R(x,y)$ is false

Since $R(x, y)$ may not be symmetric in x and y.

IV. $\forall x \exists y\, R(x,y) \to \neg(\exists x \forall y\, \neg R(x,y))$ is true

Since $\neg(\exists x \forall y \neg\, R(x,y)) \equiv \forall x \exists y\, R(x,y)$

So, IV will reduce to

$\forall x \exists y\, R(x,y) \to \forall x \exists y\, R(x,y)$ which is trivially true.

So correct answer is I and IV only which is option (b).

11. (b)

(a) $(a \vee b) \to (b \wedge c)$

$\equiv (a + b)' + bc$

$\equiv a'\, b' + bc$

Therefore, $((a \vee b) \to (b \wedge c))$ is contingency and not tautology.

(b) $(a \wedge b) \to (b \vee c)$

$\equiv ab \to b + c$

$\equiv (ab)' + b + c$

$\equiv a' + b' + b + c$

$\equiv a' + 1 + c \equiv 1$

So $((a \wedge b) \to (b \vee c))$ is tautology.

(c) $(a \vee b) \to (b \to c)$

$\equiv (a + b) \to (b' + c)$

$\equiv (a + b)' + b' + c$

$\equiv a'\, b' + b' + c$

$\equiv b' + c$

So $((a \vee b) \to (b \to c))$ is contingency but not tautology.

(d) $(a \to b) \to (b \to c)$

$\equiv (a' + b) \to (b' + c)$

$\equiv (a' + b)' + b' + c$

$\equiv ab' + b' + c$

$\equiv b' + c$

Therefore, $((a \to b) \to (b \to c))$ is contingency but not tautology.

12. (a)

$P(x) : x <= 4$

(a) $P(0)$: $0 <= 4$ True

(b) $P(6)$: $6 <= 4$ False

(c) $P(8)$: $8 <= 4$ False

(d) $P(9)$: $9 <= 4$ False

So, option (a) is correct.

13. (d)

All statement S(I), S(II) and S(III) is correct.

14. (a)

$F_1 : P \to \sim P \equiv p \to p' \equiv p' + p' \equiv p'$

So F_1 is contingency. Hence, F_1 is satisfiable but not valid.

$F_2 : (P \to \sim P) \vee (\sim P \to P)$

$\equiv (p \to p') + (p' \to p)$

$\equiv (p' + p') + (p + p)$

$\equiv p' + p \equiv 1$

So F_2 is tautology and therefore valid.

15. (b)

$(a \leftrightarrow c) \to (\sim b \to (a \wedge c))$

$(a'c' + ac) \to (b + (ac))$

$(a'c + ac') + (b + ac)$

$a(c + c') + a'c + b$

$a + a'c + b$

$a + c + b$

So not tautology but contingency.

16. (c)

X = {integer}

$X = \{-2, -1, 0, 1, 2 \ldots\}$

$P(X) : X = X^2$

(a) $P(2)$: False, since $2 \neq 2 \wedge 2$

(b) $P(-1)$: False, since $-1 \neq (-1) \wedge 2$

(c) $\exists x P x$: True; let $x = 0$, since $0 = 0^2$

(d) $\forall x P(x)$; False; let $x = 2$, since 2 ± 2^2

Hence, option (c) is correct.

2 CHAPTER

Combinatorics

Multiple Choice Questions & NAT Questions

Q.1 The number of distinguishable permutations of the letters in the word ELEPHANT are
(a) 40320 (b) 20160
(c) 20610 (d) 40230

Q.2 How many words can be formed out of the letters of the word "TECHNOLOGY" starting with T and ending with Y
(a) 40320 (b) 20160
(c) 10080 (d) 1814400

Q.3 A general electron is to be scheduled on 5 days in May such that it is not scheduled on two consecutive days. In how many ways can the 5 days be chosen to hold the electron?
(a) $\binom{26}{5}$ (b) $\binom{27}{5}$
(c) $\binom{30}{5}$ (d) $\binom{31}{5}$

Q.4 It is required to divide the $2n$ members of a club into n disjoint teams of 2 members each. The teams are not labelled. The number of ways in which this can be done is:
(a) $\frac{(2n!)}{(2^n)}$ (b) $\frac{(2n!)}{n!}$
(c) $\frac{(2n!)}{(2^n \cdot n!)}$ (d) $\frac{(n!)}{2}$

Q.5 How many ordered pairs of non-negative integers (M, N) are there which satisfy the inequality $M + N \leq 25$?

Q.6 How many ways are there to travel in xyz space from the origin (0, 0, 0) to the point (4, 3, 5) by taking steps one unit in positive x direction, one unit in the positive y direction, or one unit in the positive z direction? (Moving in the negative x, y and z direction is prohibited, so that no back tracking is allowed)?

Q.7 Suppose that the name of a file in computer directory consists of three digits followed by two lowercase letters and each digit is 0, 1 or 2, and each letter is either *a* or *b*. What is total number of names of these files in lexicographic order, where we order letter using the usual alphabetic order of letters?

Q.8 Let A be a sequence of 8 distinct integers sorted in ascending order. How many distinct pairs of sequences, B and C are there such that (i) each is sorted in ascending order, (ii) B has 5 and C has 3 elements, and (iii) the result of merging B and C gives A?
(a) 2 (b) 30
(c) 56 (d) 256

Q.9 Mala has a colouring book in which each English letter is drawn two times. She wants to paint each of these 52 prints with one of k colours, such that the colour-pairs used to colour any two letters are different. Both prints of a letter can also be coloured with the same colour. What is the minimum value of *k* that satisfies this requirement?
(a) 9 (b) 8
(c) 7 (d) 6

Q.10 Out of 7 consonants and 4 vowels, how many words of 8 consonants and 2 vowels can be formed?
(a) 25200 (b) 21300
(c) 24400 (d) 210

Q.11 In a group of 6 boys and 4 girls, four children to be selected. In how many different ways can they cane be selected such that atleast one boy be there?
(a) 212 (b) 209
(c) 159 (d) 201

Q.12 In how many ways can 11 men and 8 women sit in a row if all the men sit together and all the women sit together.

(a) 11! 8! (b) $\frac{11!\,8!}{2}$

(c) 2.11! 8! (d) None of these

Q.13 Suppose that a state's license plates consist of 3 letters followed by 3 digit then total number of different places can be formed (no repetitions allowed) ________?

Q.14 n couples are invited to a party with the condition that every husband should be accompanied by his wife. However, a wife need not be accompanied by her husband. The number of different gatherings possible at the party is

(a) $\binom{2n}{n} * 2^n$ (b) 3^n

(c) $\frac{(2n)!}{2^n}$ (d) $\binom{2n}{n}$

Q.15 In how many different ways can be the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

Q.16 Ten different letters of alphabet are given. Words with six letters formed from these given letters. Find the number of words which can have atleast one letter repeated?

(a) $10C_6$ (b) 10^6

(c) $10^6 - 10P_6$ (d) $10P_6$

Q.17 How many factors of $2^5 \times 3^6 \times 5^2$ are perfect squares?

(a) 12 (b) 22

(c) 24 (d) 16

Q.18 In how many ways can 12 people be divided into 3 groups where 4 persons must be there in each group?

(a) Insufficient data (b) $\frac{12!}{(4!)^3}$

(c) $\frac{12!}{(4!)^3 \times 3!}$ (d) None of these

Q.19 Find the number of arrangement that can be made from the letters of the word MADEEASY, without changing the place of vowels in the word?

(a) 6 (b) 24

(c) 120 (d) None of these

Q.20 Find the number of possible 5 character passwords, if all characters must be lower case letters and distinct. Assume characters are only from *a* to *z*.

(a) $(26)^5$ (b) $\frac{26!}{16!}$

(c) $\frac{26!}{21!}$ (d) ${}^{26}C_5$

Q.21 In a chess competition involving some men and women, every player needs to play exactly one game with every other player. It was found that in 45 games both the players were women and in 190 games, both players were men. What is the number of games in which one person was a man and other person was a women?

(a) 200 (b) 180

(c) 120 (d) 40

Q.22 If the ordinary generating function of a sequence $\{a_n\}_{n=0}^{\infty}$ is $\frac{1+z}{(1-z)^3}$, then $a_3 - a_0$ is equal to _________.

Q.23 There are 12 intermediate stations between two places A and B. Find the number of ways in which a train can be made to stop at 4 of these intermediate stations so that no two stopping stations are consecutive?

Q.24 In how many ways can 10 engineers and 4 doctors be seated at a round table without any restrictions?

(a) $14C_{10}$ (b) 14!

(c) 13! (d) 16!

Q.25 In how many ways can 4 girls and 5 boys be arranged in a row so that all the 4 girls are together?

(a) 1440 (b) 4320

(c) 17280 (d) 86400

Q.26. How many numbers [Each with different digits] between 1000 and 10000 can be formed with 7, 6, 5, 4, 3, 2?

(a) 60 (b) 360

(c) 720 (d) None of these

Q.27 In how many ways can 30 identical apples be divided among 10 boys.

Q.55 Which of the following statement(s) is/are true?

(a) Number of cards that must be chosen from a standard deck of 52 cards to guarantee that at least two of the four aces are chosen are 50.

(b) Number of cards that cards must be chosen from a standard deck of 52 cards to guarantee that at least two of the four aces and at least two of the 13 kinds are chosen are 50.

(c) Number of cards that must be chosen from a standard deck of 52 cards to guarantee that at least two of the four aces are chosen are 48.

(d) Number of cards that cards must be chosen from a standard deck of 52 cards to guarantee that at least two of the four aces and at least two of the 13 kinds are chosen are 48.

Q.56 Which of the following statement(s) is/are true?

(a) Number of cards that must be chosen from a standard deck of 52 cards to guarantee that there are at least two cards of the same kind are 17.

(b) Number of cards that must be chosen from a standard deck of 52 cards to guarantee that there are at least two cards of each of two different kinds are 14.

(c) Number of cards that must be chosen from a standard deck of 52 cards to guarantee that there are at least two cards of the same kind are 14.

(d) Number of cards that must be chosen from a standard deck of 52 cards to guarantee that there are at least two cards of each of two different kinds are 17.

■■■■

Answers Combinatorics

1.	(b)	**2.**	(b)	**3.**	(b)	**4.**	(c)	**5.**	(351)	**6.**	(60)	**7.**	(12)	**8.**	(c)	**9.**	(d)		
10.	(a)	**11.**	(b)	**12.**	(c)	**13.**	(11232000)			**14.**	(b)	**15.**	(720)	**16.**	(c)	**17.**	(c)		
18.	(c)	**19.**	(b)	**20.**	(c)	**21.**	(a)	**22.**	(15)	**23.**	(126)	**24.**	(c)	**25.**	(c)	**26.**	(b)		
27.	(c)	**28.**	(60)	**29.**	(d)	**30.**	(61)	**31.**	(c)	**32.**	(44)	**33.**	(b)	**34.**	(b)	**35.**	(a)		
36.	(24)	**37.**	(a)	**38.**	(c)	**39.**	(21)	**40.**	(a)	**41.**	(d)	**42.**	(c)	**43.**	(c)	**44.**	(c)		
45.	(d)	**46.**	(d)	**47.**	(d)	**48.**	(c)	**49.**	(a)	**50.**	(84)	**51.**	(c)	**52.**	(49)	**53.**	(a, d)		
54.	(b, c)	**55.**	(a, b)	**56.**	(c, d)														

Explanations Combinatorics

1. (b)

Total number of distinguishable permutation of word ELEPHANT

$$= \frac{\underline{|8}}{\underline{|2}} = 8\times7\times6\times5\times4\times3 = 20160$$

2. (b)

TECHNOLOGY

Excluding T and Y there are 8 letters left. Among these 8 letters two O's present.

So total permutation

$$= \frac{8!}{2!} = 8\times7\times5\times6\times5\times4\times3 = 20160$$

3. (b)

There are 31 days in May.

Number of election days = 5

Number of non-electron days = 31 – 5 = 26

Let the days on which electrons is to be held is denoted by d. So, 31 days will look like $X_1 d\, X_2 d\, X_3 d\, X_4 d\, X_5 d\, X_6$, where X_i are the number of days between the electron days. They will satisfy the following constraints.

$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 26$

where $X_1 \geq 0$, $X_2 \geq 1$, $X_3 \geq 1$, $X_4 \geq 1$, $X_5 \geq 1$, $X_6 \geq 0$

Now, Add 2 on both sides of the equation and substitute

$Y_1 = 1 + X_1$ and $Y_6 = 1 + X_6$
We will get $Y_1 + X_2 + X_3 + X_4 + X_5 + Y_6 = 28$
where $Y_1 \geq 1, X_2 \geq 1, X_3 \geq 1, X_4 \geq 1, X_5 \geq 1, X_6 \geq 1$
By using generating functions or otherwise the answer is $\binom{n-1}{r-1} = \binom{27}{5}$
So, (b) is correct.

4. (c)

In member to be n teams with 2 member each and teams are unordered so we can exchange n team member among them.
$(2n!)/(2^* 2^* \ldots.n \text{ times } ^* n!)$
$= (2n!)/(2^{n*} n!)$
So, option (c) is correct.

5. (351)

Let, $A = 25 - M - N,$
Then A, M and N are non-negative integers that sum to 25.
$A + M + N = 25$
Number of non-negative integer solutions = C(25 + 3 – 1, 3 – 1) = C(27, 2) = 351
So, answer is 351.

6. (60)

We can choose x direction movement by 4 ways as the movement unit step wise. Similarly, y direction movement can be done in 3 ways and z direction movement can be done in 5 ways.
Hence, total number of ways to travel = 4 * 3 * 5 = 60 ways.

7. (12)

Digit = {0, 1, 2}, Letter = {a, b}
Lexicographic order three digits followed by two lowercase letter.
012*ab*
012*ba*
021*ab*
021*ba*
102*ab*
102*ba*
120*ab*
120*ba*
201*ab*
201*ba*
210*ab*
210*ba*
So, Total = 12

8. (c)

This corresponds to an ordered partition of 8 elements into two groups, the first with 5 elements and second with 3 elements. The number of ways of doing this is

$$P(8; 5, 3) = \frac{8!}{5!\,3!} = 56$$

9. (d)

The problem reduces to finding how many distinct ordered colour pairs (C_1, C_2) are possible with k colors.
Since the first color C_1 can be any one of the k colours and the second color C_2 also can be any one of the k colors (both prints of a letter can be colored with same color), the total no. of such order color pairs is equal to $k \times k = k^2$.
Since each pair of letters must be colored with different color pairs, at least 26 color pairs are required to do this.
Therefore the requirement is $k^2 \geq 26$.
The minimum value of k that satisfies this equation is $k = 6$.

10. (a)

Number of ways of selecting 3 consonants from 7 = $7C_3$
Number of ways of selecting 2 vowels from 4 = $4C_2$
Number of ways of selecting 3 consonants from 7 and 2 vowels from 4 = $7C_3 \times 4C_2$

$$= \left(\frac{7\times6\times5}{3\times2\times1}\right)\times\left(\frac{4\times3}{2\times1}\right) = 210$$

It means, we can have 210 groups where each group contains total 5 letters (3 consonants and 2 vowels)
Number of ways arranging 5 letters among themselves = 5! = 5 × 4 × 3 × 2 = 1 = 120
Hence, Required number of ways = 210 × 120
= 25200
Hence, option (a) is correct.

11. (b)

In a group of 6 boys and 4 girls, four children are to be selected such that atleast one boy should be there.
Hence we have 4 options as given below:
We can select 4 boys .... (option a)
Number of ways to this = $6C_4$

3 CHAPTER

Set Theory and Algebra

Multiple Choice Questions & NAT Questions

Q.1 Let $|A| = 56$, $|A| = 57$, $|C| = 59$, $|A \cap C| = 46$, $|B \cap C| = 44$, $|A \cup B \cup C| = 80$ and $|A \cap B \cap C| = 41$. Then find $|A \cup B|$?

(a) 50 (b) 60
(c) 70 (d) None of these

Q.2 Suppose X and Y are sets and $|X|$ and $|Y|$ are their respective cardinalities. It is given that there are exactly 97 functions from X to Y. From this one can conclude that

(a) $|X| = 1$, $|Y| = 97$
(b) $|X| = 97$, $|Y| = 1$
(c) $|X| = 97$, $|Y| = 97$
(d) None of the above

Q.3 Let A be a finite set of size n. The number of elements in the power set of $A \times A$ is

(a) 2^{2^n} (b) 2^{n^2}
(c) $2n$ (d) 2^n

Q.4 Let Y be a set with n elements. How many subsets of Y have odd cardinality?

(a) n (b) 2^n
(c) $2^{n/2}$ (d) 2^{n-1}

Q.5 Let $R_1 = \{(a, b), (a, c), (c, a), (a, a)\}$ and

$R_2 = \{(c, c), (c, b), (a, d), (d, b), (c, d)\}$.

Where R_1 and R_2 are relations on set $A = \{a, b, c, d\}$. Then $R_1 \cup R_2$ is

(a) Reflexive (b) Symmetric
(c) Transitive (d) Asymmetric

Q.6 The number of surjective junctions defined from A to B where $|A| = 5$, $|B| = 4$ is ______.

Q.7 Which of the following relation is/are partial order?

R_1 for $a, b \in R$, $(a^R b) \Leftrightarrow (a \geq b)$
R_2 for $a, b \in R$, $(a^R b) \Leftrightarrow (a < b)$
R_3 for $a, b \in R$, $a^R b \Leftrightarrow a^2 \geq b^2$

(a) R_1 and R_2 (b) R_2 and R_3
(c) R_1 and R_3 (d) None of the above

Q.8 Consider $(S, *)$ is a semigroup where $S = \{a, b, c, d\}$. Assume $a * b = c$, $b * b = a$ and $d * a = b$. Then $d * c =$

(a) a (b) b
(c) c (d) None of these

Q.9 Some group $(G, \circ)$ is known to be abelian. Then, which one of the following is true for G?

(a) $g = g^{-1}$ for every $g \in G$
(b) $g = g^2$ for every $g \in G$
(c) $(g \circ h)^2 = g^2 \circ h^2$ for every $g, h \in G$
(d) G is of finite order

Q.10 Consider of the following statements:

1. Union of two subgroup is a subgroup.
2. Intersection of two subgroups is not always a subgroup.
3. If G is a cyclic group, then every subgroup of G is also cyclic.

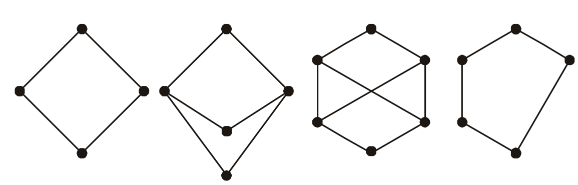
Which of the following is not true?

(a) 1 and 3 are true (b) 1 and 2 are false
(c) 3 is true (d) None of these

Q.11 Let G be group with identity e and xy be two elements of G satisfying $x^5 y^3 = x^8 y^5 = e$. Which of the following is true?

(a) $x = e$, $y = e$ (b) $x = e$, $y \neq e$
(c) $x \neq e$, $y = e$ (d) $x \neq e$, $y \neq e$

Q.12 Consider the following Hasse diagrams:



(i) (ii) (iii) (iv)

Which all of the above represent a lattice?

(a) (i) and (iv) only (b) (ii) and (iii) only
(c) (iii) only (d) (i), (ii) and (iv) only

Q.13 The relation {(1, 2), (1, 3), (3, 1), (2, 1), (2, 2), (3, 2), (3, 1), (1, 4), (4, 2), (3, 3)}
(a) Reflexive (b) Symmetric
(c) Transitive (d) Asymmetric

Q.14 In a class of 200 students, 125 students have taken Programming Language course, 85 students have taken Data Structures course, 65 students have taken Computer Organization course; 50 students have taken both Programming Language and Data Structures, 35 students have taken both Data Structures and Computer Organization; 30 students have taken both Data Structures and Computer Organization, 15 students have taken all the three courses. How many students have not taken any of the three courses?
(a) 15 (b) 20
(c) 25 (d) 30

Q.15 Consider the following statements:
1. If R_1 and R_2 are transitive relations on a set A then $R_1 \cup R_2$ is also transitive.
2. Number of relations on a set of n elements that are both symmetric and antisymmetric is $3^{\binom{n}{2}}$ which of following is true?

(a) 1 is true (b) 2 is true
(c) Both are true (d) Both are false

Q.16 Consider a cyclic group G or order n, and $a \in G$. Then which of the following can be generation of G?
(a) a^{n+1} (b) a^{n^2}
(c) a^{2n} (d) None of these

Q.17 Let f is a function from set of integers to the set of integers. Find which of the following function is neither one to one nor onto function.
(a) $f(x) = x + 5$ (b) $f(x) = 3x - 100$
(c) $f(x) = x^3 + 1$ (d) $f(x) = x^2 + 1$

Q.18 The function $f : Z \times Z \to Z$ defined as $f(m, n) = m - n$ is
(a) one-to-one
(b) onto
(c) bijective
(d) neither one-to-one nor onto

Q.19 Consider the following statements:
1. A relation can be both symmetric and antisymmetric.
2. There exists a relation which is both reflexive and irreflexive.

Which of the following is true?
(a) 1 is true (b) 2 is true
(c) Both are true (d) Both are false

Q.20 Let N be the set of natural numbers and G is a set defined as:

$G = \{x | x \in N \text{ and } \forall_y \in N, x = y \text{ modulo } 5\}$

Relation $R \subseteq G \times G$ which is defined as:

$R = \{(a, b) | \, |a - b| = 1\}$.

Consider the following relations:
(i) Reflexive
(ii) Symmetric and
(iii) Transitive

Find the relation R?
(a) (i) only (b) (ii) only
(c) (i), (ii) and (iii) (d) None of these

Q.21 Let A be a totally ordered set of size 'n' which is boolean algebra then maximum value of n is _________?

Q.22 Let G be group with order 20 and H is a non-abelian subgroup of G. Assuming HG, what is the order of H?

Q.23 Let H_1 and H_2 be two distinct subgroups of a finite group G, each of order 2. Let H be the smallest subgroup containing H_1 and H_2. Then the order of H is
(a) Always 2 (b) Always 4
(c) Always 8 (d) None of the above

Q.24 Number of edges in the Hasse diagram of D_{5423} is _______?

Q.25 Let # be a binary operator defined as $X \# Y = X' + Y'$ where X and Y are Boolean variables. Consider the following two statements:

S_1: $(P \# Q) \# R = P \# (Q \# R)$
S_2: $Q \# R = R \# Q$

Which of the following is/are true for the Boolean variables P, Q and R?
(a) Only S_1 is True
(b) Only S_2 is True
(c) Both S_1 and S_2 are True
(d) Neither S_1 nor S_2 are True

Answers — Set Theory and Algebra

1.	(c)	2.	(a)	3.	(b)	4.	(d)	5.	(c)	6.	(240)	7.	(d)	8.	(a)	9.	(c)
10.	(a)	11.	(a)	12.	(a)	13.	(c)	14.	(c)	15.	(d)	16.	(a)	17.	(d)	18.	(b)
19.	(a)	20.	(b)	21.	(2)	22.	(10)	23.	(b)	24.	(12)	25.	(b)	26.	(b)	27.	(1)
28.	(b)	29.	(b)	30.	(d)	31.	(306)	32.	(c)	33.	(a)	34.	(d)	35.	(a)	36.	(b)
37.	(2)	38.	(c)	39.	(b)	40.	(b)	41.	(c)	42.	(d)	43.	(1)	44.	(b)	45.	(a)
46.	(d)	47.	(d)	48.	(b)	49.	(c)	50.	(c)	51.	(c)	52.	(a)	53.	(Sol.)	54.	(b)
55.	(38)	56.	(0)	57.	(d)	58.	(d)	59.	(d)	60.	(d)	61.	(d)	62.	(d)	63.	(a)
64.	(96)	65.	(c)	66.	(1)	67.	(c)	68.	(c)	69.	(b)	70.	(c)	71.	(b)	72.	(39)
73.	(28)	74.	(a, b, c)	75.	(a, c, d)	76.	(a, b)	77.	(b, d)	78.	(a, b, c)	79.	(b, c)				
80.	(a, b, c, d)	81.	(a, b, c, d)	82.	(a, b, c, d)	83.	(b, c, d)	84.	(b, c, d)	85.	(c)						
86.	(a, b, c)	87.	(a)	88.	(b, c, d)												

Explanations — Set Theory and Algebra

1. (c)

$$\boxed{|A\cup B| = |A| + |B| - |A\cap B|} \rightarrow (1)$$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cup B|$
$- |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$80 = 56 + 57 + 59 - |A \cap B| - 46 - 44 + 41$

$|A \cap B| = 43$

$|A \cup B| = |A| + |B| - |A \cap B|$
$= 56 + 57 - 43 = 70$

2. (a)

X and Y are sets. The cardinalities of X and Y are $|X|$ and $|Y|$ respectively.

The number of functions from X to $Y = (|Y|)^{|X|}$

Given that number of functions from X to $Y = 97$

$\therefore \quad 97 = (|Y|)^{|X|}$

So above implies that $|X| = 1$ and $Y| = 97$

3. (b)

Number of elements in $A \times A = n^2$

$\therefore$ Number of elements in the power set of

$A \times A = 2^{n^2}$

4. (d)

Y is a set of n elements then there are total 2^n subsets out of which 2^{n-1} have odd cardinality and 2^{n-1} have even cardinality.

So, option (d) is correct.

5. (c)

$X = R_1 \cup R_2$
$= \{(a, b), (a, c), (c, a), (a, a), (c, c), (c, b), (a, d), (d, b)\}$

X is not reflexive, $(b, b) \notin X$

X is not symmetric, $(a, b) \in X$ but $(b, a) \notin X$.

X is not asymmetric, $(a, c) \in X$ but $(c, a) \in X$

X is transitive.

6. (240)

We have $S(r + 1, n) = n \times S(r, n) + S(r, n - 1)$

1
1 1
1 3 1
1 76 1
1 15 25 1 0 1

So, $S(5, 4) = 10$ and $4! = 24$,

Number of surjective junctions $= 24 \times 10 = 240$.

7. (d)

R_1: is definitely partial order set (> = is a classic example of POSET)

R_2: is clearly not reflexive therefore not partial order set.

R_3: We have to check whether it is antisymmetric or not: i.e. (aRb and bRa) implies $a = b$

Suppose we take +3 and –3, now $(3)^2 \le (-3)^2$ and $(-3)^2 < = (3)^2$ implies that $3 = -3$ which is false therefore it not antisymmetric in nature following not a partial order.

Hence, option (d) is correct.