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ELECTRONICS ENGINEERING

Objective Practice Sets

Communication Systems

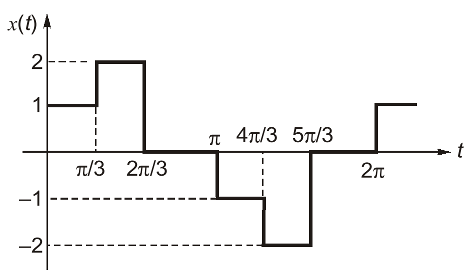
Communication Systems

1 CHAPTER Fourier Analysis of Signal Energy and Power Signals

MCQ and NAT Questions

Q.1 If $G(f)$ represents the Fourier transform of a signal $g(t)$ which is real and odd symmetric in time then
(a) $G(f)$ is complex
(b) $G(f)$ is imaginary
(c) $G(f)$ is real
(d) $G(f)$ is real and non-negative

Q.2 Compute the amplitude of the fundamental component of the waveform given in figure.



(a) 0 (b) 1.00
(c) 1.603 (d) 1.712

Q.3 Let $x(t)$ be a signal with its Fourier transform $X(j\omega)$ suppose we are given the following facts.
1. $x(t)$ is real.
2. $x(t) = 0$ for $t \le 0$.
3. $\frac{1}{2\pi}\int_{-\infty}^{\infty} \text{Re}\{X(j\omega)\}e^{j\omega t}d\omega = 2|t|e^{-|t|}$.

then a closed form expression for $x(t)$ is
(a) $2e^{-t}u(t)$ (b) $e^{-|t|}$
(c) $te^{-2t}u(t)$ (d) $2te^{-t}u(t)$

Q.4 **Assertion (A):** If two signals are orthogonal they will also be orthonormal.
Reason (R): If two signals are orthonormal they also will be orthogonal.
(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not a correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

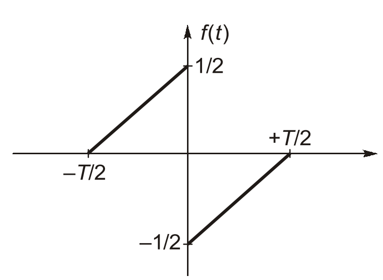
Q.5 Consider the following statements:
The normalized power, $S \equiv v^2(t)$ can be defined as the
1. instantaneous power divided by the maximum power in the circuit.
2. time average power that appears in a one ohm resistor.
3. Total power consumed by the circuit divided by the average power consumed in that circuit.
4. the mean square value of $v(t)$.

Which of the above statements is/are correct?
(a) 2 only (b) 1 and 2
(c) 2 and 3 (d) 2 and 4

Q.6 The auto correlation function of a rectangular pulse of duration T is
(a) A rectangular pulse of duration T
(b) A rectangular pulse of duration $2T$
(c) A triangular pulse of duration T
(d) A triangular pulse of duration $2T$

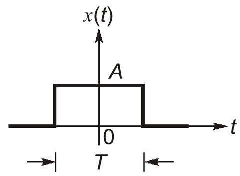
Q.7 The amplitude spectrum of Gaussian pulse is
(a) uniform (b) a sine function
(c) gaussian (d) an impulse function

Q.8 A function $f(t)$ is shown in figure.



The Fourier transform $F(\omega)$ of $f(t)$ is
(a) real and even function of ω
(b) real and odd function of ω
(c) imaginary and odd function of ω
(d) imaginary and even function of ω

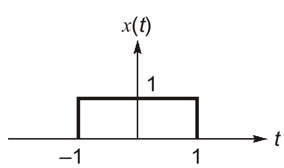
Q.9 What is the total energy of the rectangular pulse shown in the figure below?



(a) AT (b) A^2T
(c) A^2T^2 (d) AT^2

Q.10 Which one of the following is not a property of auto correlation function ($R(\tau)$)?
(a) $R(0) \le R(\tau)$
(B) $R(\tau) = R(-\tau)$
(c) $R(0) = s$ = average power of the waveform
(d) Power spectral density is Fourier transform of auto correlation function for a periodic waveform

Q.11 $x(t)$ is a positive rectangular pulse from $t = -1$ to $t = +1$ with unit height as shown in figure. The value of $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ (where $X(\omega)$ is Fourier transform of $x(t)$) is



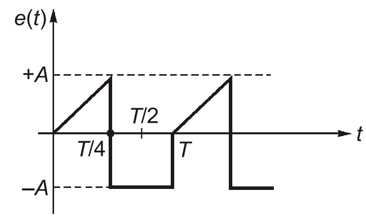
(a) 2 (b) 2π
(c) 4π (d) 4

Q.12 The Fourier transform of $x(t) = \dfrac{2a}{a^2 + t^2}$ is

(a) $2\pi e^{-a|\omega|}$ (b) $\pi e^{-2a|\omega|}$
(c) $\pi e^{-a\omega}$ (d) $\pi e^{-2a\omega}$

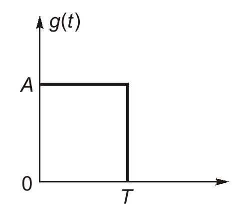
Q.13 Out of the four signal waveforms-sinusoid, rectangular, triangular and saw-tooth, all of them having the same periodicity, the minimum bandwidth corresponds to which one of the following?
(a) Sinusoidal (b) Rectangular
(c) Triangular (d) Saw-tooth

Q.14 The rms value of the periodic waveform $e(t)$ shown in figure is



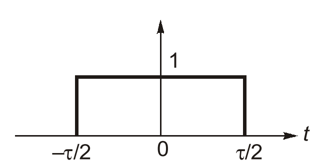
(a) $\sqrt{\dfrac{3}{2}}A$ (b) $\sqrt{\dfrac{2}{3}}A$
(c) $\sqrt{\dfrac{1}{3}}A$ (d) $\sqrt{\dfrac{5}{6}}A$

Q.15 The energy density spectrum $|G(f)|^2$ of a rectangular pulse shown in the given figure is



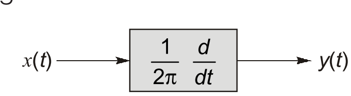
(a) $AT\left(\dfrac{\sin\pi fT}{\pi fT}\right)$ (b) $(AT)^2\left(\dfrac{\sin \pi fT}{\pi fT}\right)^2$
(c) $(AT)^2\left(\dfrac{\sin \pi fT}{\pi fT}\right)$ (d) $A^2\left(\dfrac{\sin\pi fT}{\pi fT}\right)$

Q.16 Fourier transform of the gate function as shown below is
$f(t) = 1$ for $-\tau/2 \le t < \tau/2$
$= 0$ otherwise
(where τ is the width of the gate function).
The value of $F(\omega)$ is



(a) $\dfrac{\tau\sin(\omega\tau)}{\omega\tau}$ (b) $\dfrac{\tau\sin(2\omega\tau)}{2\omega\tau}$
(c) $\dfrac{\tau\sin(\omega\tau/2)}{(\omega\tau/2)}$ (d) $\dfrac{\tau}{2}\cdot\dfrac{\sin(\omega\tau/2)}{(\omega\tau/2)}$

Q.17 A deterministic signal $x(t) = \cos 2\pi t$ is passed through a differentiator as shown in figure.



what is its power spectral density $S_{xx}(f)$?

(a) $\frac{1}{4}[\delta(f-1)+\delta(f+1)]$ (b) $\frac{1}{2}[\delta(f-1)+\delta(f+1)]$

(c) $\frac{1}{4}[\delta(f)+\delta(f+1)]$ (d) None of the above

Q.18 A signal is represented by

$$x(t) = \begin{cases} 1 & |t| < 1 \\ 0 & |t| > 1 \end{cases}$$

The Fourier transform of the convolved signal $y(t) = x(2t) * x(t/2)$.

(a) $\frac{4}{\omega^2}\sin\left(\frac{\omega}{2}\right)$ (b) $\frac{4}{\omega^2}\sin\left(\frac{\omega}{2}\right)\sin(2\omega)$

(c) $\frac{4}{\omega^2}\sin 2\omega$ (d) $\frac{4}{\omega^2}\sin^2\omega$

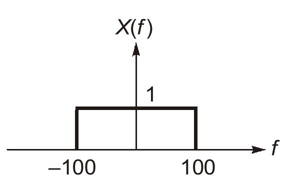
Q.19 A signal has Fourier series coefficients $C_n \Rightarrow C_{-1} = C_1 = 8,\ C_0 = 0,\ C_2 = C_{-2} = 2$ its power is

(a) 0 (b) 136

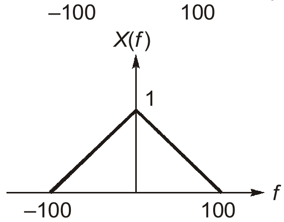
(c) 20 (d) 120

Q.20 Frequency spectrum of signal $\frac{100}{\pi^2}\text{sin}c^2(100t)$ is

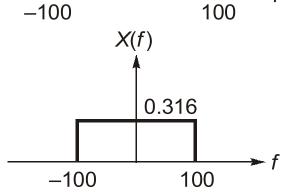
(a)



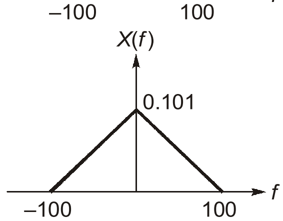
(b)



(c)



(d)



Q.21 Consider the signal defined by

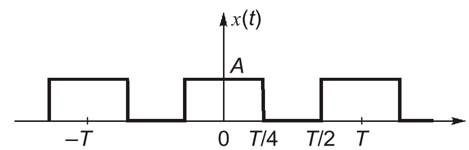
$$x(t)\begin{cases} e^{j10t} & \text{for } |t| \le 1 \\ 0 & \text{for } |t| > 1 \end{cases}$$

its fourier transform is

(a) $\frac{2\sin(\omega-10)}{\omega-10}$ (b) $2e^{j10}\frac{\sin(\omega-10)}{\omega-10}$

(c) $\frac{2\sin\omega}{\omega-10}$ (d) $e^{j10\omega}\frac{2\sin\omega}{\omega}$

Q.22 Determine the Fourier series coefficients for given periodic signal $x(t)$ is



(a) $\frac{A}{j2\pi k}\sin\left(\frac{\pi}{2}k\right)$ (b) $\frac{A}{\pi jk}\cos\left(\frac{\pi}{2}k\right)$

(c) $\frac{2A}{\pi k}\sin\left(\frac{\pi}{2}k\right)$ (d) $\frac{2A}{\pi k}\cos\left(\frac{\pi}{2}k\right)$

Q.23 Suppose we have given following information about a signal $x(t)$

1. $x(t)$ is real odd
2. $x(t)$ is periodic with $T = 2$
3. Fourier coefficients $C_n = 0,\ |n| > 1$
4. $\frac{1}{2}\int_0^2 |x(t)^2|\,dt = 1$

The signal that satisfy these conditions

(a) $\sqrt{2}\sin\pi t$ and unique

(b) $\sqrt{2}\sin\pi t$ but not unique

(c) $2\sin\pi t$ and unique

(d) $2\sin\pi t$ but not unique

Q.24 The Fourier series coefficients, of a periodic signal $x(t)$ expressed as $\sum_{k=-\infty}^{k=+\infty} a_k e^{j2\pi kt/T}$ are given by

$a_{-2} = 2 - j1;\ a_{-1} = 0.5 + j0.2;\ a_0 = j2;\ a_1 = 0.5 - j0.2$

$a_2 = 2 + j1$; and ak = 0; for $|k| > 2$

which of the following is true.

(a) $x(t)$ has finite energy because only finitely many coefficients are non-zero

(b) $x(t)$ has zero average value because it is periodic

(c) the imaginary part of $x(t)$ is constant

(d) the real part of $x(t)$ is even

Q.25 If the energy of a signal $X(t)$ is 9 unit, then the energy of signal $X(2t)$ will be ______ unit.

Q.26 If $x(t) = \frac{1}{t}$, then Hilbert transform of $x(t)$ will be $-K\delta(t)$. Then the value of K will be ______.

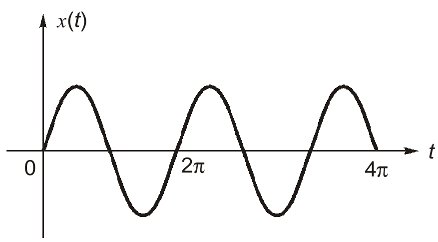
Q.27 What will be the value of following integral _____?

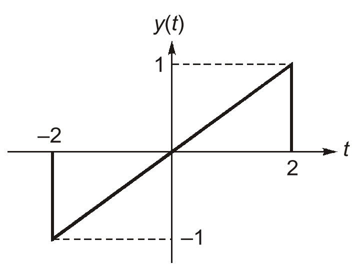
$$\int_{-\infty}^{\infty} S_a^2(2t)dt$$

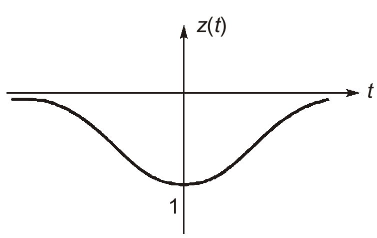
where $S_a(t)$ = Sampling function $S_a(t) = \frac{\sin t}{t}$

Multiple Select Questions (MSQs)

Q.28 Consider the real signals shown below:

(i)


(ii)


(iii)


Which of the below statements are correct?

(a) The Fourier transform of $y(t)$ and $z(t)$ is real-valued.

(b) The Fourier transform of $x(t)$ is conjugate symmetric.

(c) $\int_{-\infty}^{\infty} X(j\omega)\cdot d\omega = 0$

(d) $\int_{-\infty}^{\infty} Z(j\omega)\cdot d\omega = 0$

Q.29 Consider a continuous-time ideal low pass filter having the frequency response

$$H(j\omega) = \begin{cases} 1, & |\omega| \le 80 \\ 0, & |\omega| > 80 \end{cases}$$

The input to this filter is a signal $x(t)$ with fundamental frequency ω_0 = 10 rad/sec and Fourier series coefficients $X[k]$. If $y(t)$ represents the output of the filter and it is given that $Y[k] = X[k]$, then the values of k for which $X[k]$ is non-zero are:

(a) 3
(b) 7
(c) 10
(d) 12

Q.30 For a periodic signal $x(t)$, the Fourier series coefficients are given as below:

$$X[k] = \begin{cases} 5, & k = 0 \\ j\left(\frac{1}{2}\right)^{|k|}, & \text{otherwise} \end{cases}$$

Which of the below statements are correct?

(a) $x(t)$ is real signal.

(b) $x(t)$ is an even signal.

(c) $\frac{dx(t)}{dt}$ is an odd signal.

(d) $x(t)$ is an energy signal.

■■■■

Answers Fourier Analysis of Signal Energy and Power Signals

1. (b)	2. (a)	3. (d)	4. (d)	5. (d)	6. (d)	7. (c)
8. (c)	9. (b)	10. (a)	11. (c)	12. (a)	13. (a)	14. (d)
15. (b)	16. (c)	17. (b)	18. (b)	19. (b)	20. (d)	21. (a)
22. (c)	23. (b)	24. (a)	25. (4.5)	26. (3.14)	27. (1.57)	28. (b, c)
29. (a, b)	30. (b, c)					

Explanations Fourier Analysis of Signal Energy and Power Signals

1. (b)

Function, $g(t)$	Fourier Transform, $G(f)$
Real and odd	Imaginary and odd
Real and even	Real and even
Imaginary and odd	Real and odd
Imaginary and even	Imaginary and even

2. (a)

$$a_0 = \frac{1}{T}\cdot\int_0^T f(t)\,dt$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)\,dt = \frac{1}{2\pi}\times\left[\int_0^{2\pi/3} f(t)\,dt + \int_\pi^{5\pi/3} f(t)\,dt\right]$$

$$= \frac{2}{2\pi}\left[1\cdot\frac{\pi}{3}+2\cdot\frac{\pi}{3}-1\cdot\frac{\pi}{3}-2\cdot\frac{\pi}{3}\right]=0$$

3. (d)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

$\Rightarrow$ Real $(x(t)) = 2|t|e^{|t|}$

Since, $x(t) = 0, \quad t\le 0$

$\Rightarrow x(t) = 2te^{-t} \quad t>0$

$\Rightarrow x(t) = 2te^{-t}u(t)$

4. (d)

Orthogonal: Two vector are perpendicular i.e. their dot product is zero.

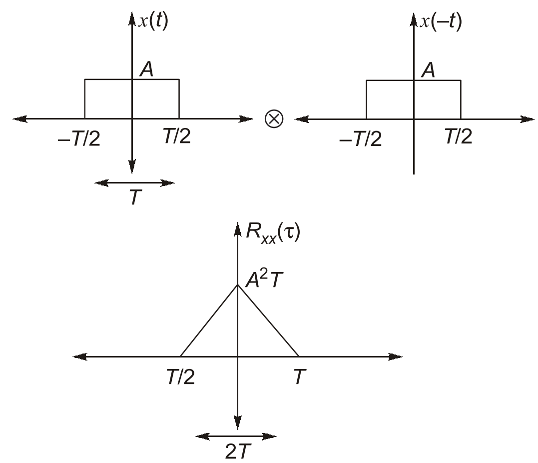
Orthonormal: Two vectors are perpendicular and are of unit length.

5. (d)

- Normalized power $\rightarrow$ Power in 1 Ω resistor
- $P_N = V^2(t)$, normalized power is average and mean of voltage required.
- $V_{rms} = \sqrt{P_N}$

6. (d)

ACF or Auto correlation function is nothing but convolution of $x(t)$ with time reversed form of $x(t)$, i.e. $x(t)$



ACF is a triangular pulse of duration 2T.

7. (c)

Amplitude spectrum of Gaussian pulse is Gaussian.

8. (c)

Signal is odd,

$$x(t) = -x(-t)$$

Signal is half symmetric

$$x(t) = x\left(t+\frac{T_0}{2}\right)$$

∴ contains odd harmonic.

Signal $f(t)$ is real and odd,

∴ $F(\omega)$ is imaginary and odd.

9. (b)

$$x(t) = A; \qquad -\frac{T}{2}\le t\le\frac{T}{2}$$

$$= 0; \qquad \text{for all other } t$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$= \int_{-T/2}^{T/2} A^2 dt = \left[A^2 t\right]_{-T/2}^{T/2} = A^2 T$$

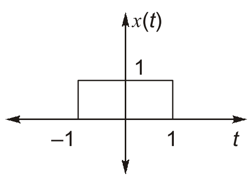
10. (a)

For autocorrelation function

$$R(0) \geq R(\tau)$$

$R(0) \rightarrow$ Maximum value of autocorrelation function.

11. (c)



From Parseval's theorem,

$$\int_{-\infty}^{\infty} |x(t)| dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |x(\omega)|^2\, d\omega$$

$$\Rightarrow 2\pi \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$$

$$= 2\pi \int_{-1}^{1} \cdot 1^2 dt = 2\pi \times 2 = 4\pi$$

$$\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega = 4\pi$$

12. (a)

$$e^{-a|t|} \longleftrightarrow \frac{2a}{a^2+\omega^2}$$

$$\frac{2a}{a^2+t^2} \longleftrightarrow 2\pi e^{-a|-\omega|} = 2\pi e^{-a|\omega|}$$

As per duality property.

13. (a)

Fourier transform of a sinc wave is an impulse. So, it has infinitesimally narrow bandwidth and out of these, sinusoid have minimum BW.

14. (d)

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} (f(t))^2 dt}$$

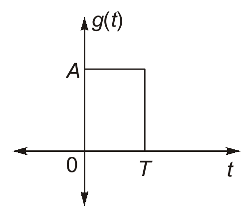
$$V_{\text{rms}}^2 = \frac{1}{T}\cdot\int_0^{T/4}\left(\frac{4A}{T}\cdot t\right)^2 dt + \int_{+T/4}^{T} (-A)^2 dt$$

$$V_{\text{rms}}^2 = \frac{1}{T}\cdot\left[\frac{3A^2T}{4} + \frac{16A^2}{T^2}\cdot\frac{1}{3}\frac{T^3}{16\times 4}\right]$$

$$V_{\text{rms}}^2 = A^2\left[\frac{3}{4}+\frac{1}{12}\right] = A^2\cdot\frac{10}{12}$$

$$V_{\text{rms}} = A\sqrt{\frac{5}{6}}$$

15. (b)



$\longleftrightarrow G(f)$

$$|G(f)| = \text{AT sin c } (f \cdot T)$$

$$|G(f)|^2 = A^2T^2 \sin^2 c(fT)$$

$$= A^2T^2\left(\frac{\sin(\pi fT)}{\pi fT}\right)^2 \qquad \left(\text{sinc} fT = \frac{\sin \pi fT}{\pi fT}\right)$$

16. (c)

$$F(\omega) = \int_{-\tau/2}^{\tau/2} 1\cdot e^{-j\omega t}\, dt$$

$$= \frac{1}{-j\omega}\left[e^{-j\omega\tau/2} - e^{+j\omega\tau/2}\right]$$

$$= \frac{2}{\omega}\left[\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{2j}\right]$$

$$= \frac{\sin\frac{\omega\tau}{2}}{\frac{\omega}{2}} = \frac{\tau\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}$$

17. (b)

$$S_0 = S_i(\omega)\cdot|H(\omega)|^2$$

$$H(\omega) = \frac{j\omega}{2\pi} = \frac{j2\pi f}{2\pi} = jf$$

$$|H(\omega)|^2 = \frac{\omega^2}{4\pi^2} = f^2$$

$$S_i(f) = \frac{1}{2}\cdot[\delta(f-1)+\delta(f+1)]$$

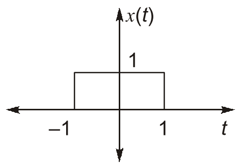
$$S_0(f) = \frac{f^2}{2}\cdot[\delta(f-1)+\delta(f+1)]$$

We know,

$$f^2 \cdot \delta(f-1) = (+1)^2 \cdot \delta(f-1)$$
$$f^2 \cdot \delta(f+1) = (-1)^2 \cdot \delta(f+1)$$
$$S_0(f) = \frac{1}{2}(\delta(f-1)+\delta(f+1))$$

18. (b)

$$X(t) \longleftrightarrow X(\omega)$$



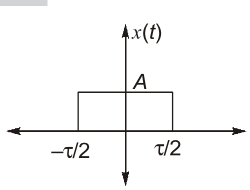
$$\longleftrightarrow X(\omega) = \frac{2\sin\omega}{\omega}$$

$$x(t) \longleftrightarrow X(\omega)$$
$$x(2t) \longleftrightarrow \frac{1}{2}X\left(\frac{\omega}{2}\right)$$
$$x\left(\frac{t}{2}\right) \longleftrightarrow 2\,X(2\,\omega)$$
$$y(t) = x(2t) \otimes x\left(\frac{t}{2}\right) \longleftrightarrow \frac{1}{2}X\left(\frac{\omega}{2}\right)\cdot 2X(2\omega)$$
$$Y(\omega) = X\left(\frac{\omega}{2}\right)\cdot X(2\omega)$$
$$= 4\cdot\frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}}\cdot\frac{\sin 2\omega}{2\omega}$$
$$= \frac{4}{\omega^2}\sin\left(\frac{\omega}{2}\right)\cdot\sin(2\omega)$$

19. (b)

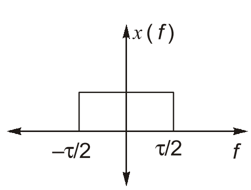
$$\text{Power} = \sum_{n=-\infty}^{\infty}|C_n|^2 = \frac{1}{T}\int_{-T/2}^{T/2}|f(t)|^2\,dt$$
$$= |C_{-2}|^2+|C_2|^2+|C_{-1}|^2+|C_{-1}|^2+|C_0|^2$$
$$= 2^2+2^2+8^2+8^2+0^2 = 136$$

20. (d)



$$\longleftrightarrow A\tau\,\text{sinc}(f\tau)$$

$$A\tau\,\text{sinc}(t\tau) \longleftrightarrow$$



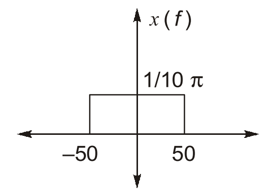
Also:

$$x(t)\otimes h(t) \longleftrightarrow X(f)\cdot H(f)$$
$$x(t)\cdot h(t) \longleftrightarrow X(f)\otimes H(f)$$

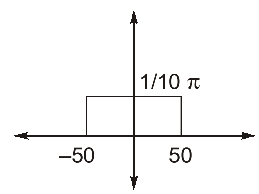
Hence,

$$x(t) = \frac{100}{\pi^2}\sin^2 c(100t)$$
$$x(t) = \underbrace{\frac{10}{\pi}\text{sin}c(100t)}_{x_1(t)}\cdot\underbrace{\frac{10}{\pi}\text{sin}c(100t)}_{x(f)}\,x_2(t)$$

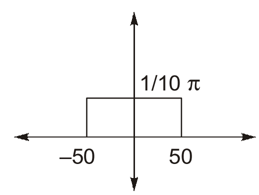
$$\frac{10}{\pi}\text{sin}c(100t) \longleftrightarrow$$



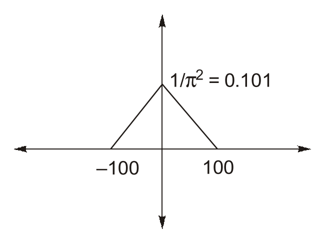
$$X_1(f) \longleftrightarrow$$



$$X_2(f) \longleftrightarrow$$



$$X_1(f)\otimes X_2(f) \longleftrightarrow$$



21. (a)

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$
$$x(t) = \begin{cases} e^{j10t} & |t|<1 \\ 0 & |t|>1 \end{cases}$$
$$\Rightarrow \quad X(\omega) = \int_{-1}^{1} e^{j10t}\cdot e^{-j\omega t}\,dt$$
$$X(\omega) = \frac{e^{-j(\omega-10)}-e^{j(\omega-10)}}{-j(\omega-10)}$$
$$X(\omega) = \frac{2\,[e^{j(\omega-10)}-e^{-j(\omega-10)}]}{2j(\omega-10)}$$
$$X(\omega) = \frac{2\sin(\omega-10)}{\omega-10}$$