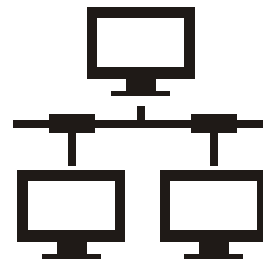


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Propositional Logic

Multiple Choice Questions

Q.1 Argument: $([P \rightarrow (q \vee r)] \wedge \bar{q} \wedge \bar{r}) \rightarrow \bar{P}$ is

- (a) Valid argument (b) Invalid argument
(c) Unknown (d) None of these

Q.2 $\neg \forall_x \forall_y [(x < y) \rightarrow (x^2 < y^2)]$ is equivalent to

- (a) $\exists_x \exists_y [(x < y) \wedge (x^2 \geq y^2)]$
(b) $\exists_x \exists_y [\neg(x < y) \wedge \neg(x^2 < y^2)]$
(c) $\exists_x \exists_y [\neg(x < y) \vee \neg(x^2 < y^2)]$
(d) $\exists_x \exists_y [(x < y) \vee (x^2 \geq y^2)]$

Q.3 What is the logical translation of the following statement?

"None of my friends are perfect".

- (a) $\exists x(F(x) \wedge \neg P(x))$ (b) $\exists x(\neg F(x) \wedge P(x))$
(c) $\exists x(\neg F(x) \wedge \neg P(x))$ (d) $\exists x(\neg F(x) \vee \neg P(x))$

Q.4 **Statement:** Last year, the only Book I read were adventure stories.

Logical representation of the above statement is?

- (a) Adventure story \rightarrow book that I read last year
(b) Book that I read last year \rightarrow adventure story
(c) Not a book I read last year \rightarrow not adventure story
(d) None of these

Q.5 Consider the following statements:

- (i) Those who like painting like flowers.
(ii) Those who like running like music.
(iii) Those who do not like music do not like flowers.

If all the above statements are true, then consider the following statements.

1. Those who like running do not like paintings.
2. Those who like painting like flowers.
3. Those who like running like flower.

4. Those who like painting like music.

Which is following is true?

- (a) 2 only (b) 1, 4 only
(c) 2, 3 only (d) 4 only

Q.6 Which of the following formulas is a formalization of the sentence:

"There is a computer which is not used by any student".

- (a) $\exists x(\text{computer}(x) \wedge \forall y(\neg \text{student}(y) \rightarrow \text{uses}(y, x)))$
(b) $\exists x(\text{computer}(x) \rightarrow \forall y(\text{student}(y) \rightarrow \neg \text{uses}(y, x)))$
(c) $\exists x(\text{computer}(x) \wedge \forall y(\text{student}(y) \rightarrow \neg \text{uses}(y, x)))$
(d) $\exists x(\text{computer}(x) \vee \forall y(\text{student}(y) \rightarrow \neg \text{uses}(y, x)))$

Q.7 $P(x, y) : x + y = x - y$

If the universe is the set of integers which of the following are true

- (i) $P(1, 1)$ (ii) $P(3, 0)$
(iii) $\exists x P(x, 2)$ (iv) $\exists x \forall y P(x, y)$
(v) $\exists y \forall x P(x, y)$ (vi) $\forall x \exists x P(x, y)$
(a) (ii) and (v) only (b) (ii), (v) & (iv) only
(c) (ii) only (d) (v) and (vi) only

Q.8 Which of the following is true?

- (i) $\exists x \{P(x) \wedge Q(x)\} \equiv \exists x P(x) \wedge \exists x Q(x)$
(ii) $\exists x \{P(x) \wedge Q(x)\} \Rightarrow \exists x P(x) \wedge \exists x Q(x)$
(iii) $\exists x P(x) \wedge \exists x Q(x) \equiv \exists x P(x) \wedge \exists y Q(y)$
(a) (i) only (b) (ii) and (iii) only
(c) (ii) only (d) None of these

Q.9 Which of the following is principle conjunction normal form for $[(p \vee q) \wedge \neg p \neg q]$?

- (a) $p \vee \neg q$ (b) $p \vee q$
(c) $\neg p \vee q$ (d) $\neg p \vee \neg q$

Q.10 Consider the first-order logic sentence $F : \forall x(\exists y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are implied by F ?

- I. $\exists y(\exists x R(x, y))$ II. $\exists y(\forall x R(x, y))$
 III. $\forall y(\exists x R(x, y))$ IV. $\neg \exists x(\forall y \neg R(x, y))$
 (a) IV only (b) I and IV only
 (c) II only (d) II and III only

Q.11 Which of the following is/are tautology:

- (a) $(a \vee b) \rightarrow (b \wedge c)$ (b) $(a \wedge b) \rightarrow (b \vee c)$
 (c) $(a \vee b) \rightarrow (b \rightarrow c)$ (d) $(a \rightarrow b) \rightarrow (b \rightarrow c)$

Q.12 Let $P(x)$ denote the statement " $x \leq 4$ ". What is truth value?

- (a) $P(0)$ (b) $P(6)$
 (c) $P(8)$ (d) $P(9)$

Q.13 Let $N(x)$ be the statements " x " has visited North Dakota" where domain consists of the students in your school. The qualifications are as follows with English expression.

S(I) $\exists x \sim N(x)$: Some student has not visited North Dakota.

S(II) $\sim \forall x N(x)$: Not all student has not visited North Dakota.

S(III) $\forall x \sim N(x)$: All students have not visited North Dakota.

Choose the correct option from above statements:

- (a) S(I) true; S(II) is true
 (b) Only S(III) is true
 (c) S(II) is true and S(II) is true
 (d) All statement S(I) S(II) and S(III) is correct.

Q.14 Consider two well-formed formulas in propositional logic:

$$F_1 : P \Rightarrow \neg P$$

$$F_2 : (P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$$

Which of the following statements is correct?

- (a) F_1 is satisfiable, F_2 is valid
 (b) F_1 is unsatisfiable, F_2 is satisfiable
 (c) F_1 is unsatisfiable, F_2 is valid
 (d) F_1 and F_2 are both satisfiable

Q.15 Which one of the following Boolean expresses is NOT a tautology?

- (a) $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$
 (b) $(a \leftrightarrow b) \rightarrow (\sim b \rightarrow (a \wedge c))$

$$(c) (a \wedge b \wedge c) \rightarrow (c \vee a)$$

$$(d) a \rightarrow (b \rightarrow a)$$

Q.16 Let $P(x)$ be the statement " $x = x^2$ ". If the domain consists of the integers, then which is true value?

- (a) $P(2)$ (b) $P(-1)$
 (c) $\exists x P(x)$ (d) $\forall x P(x)$

Q.17 Either everything is material or there are some things that are not material.

- (a) $(\forall x)M(x) \vee (\exists x) \sim M(x)$
 (b) $\forall x(M(x) \vee \sim M(x))$
 (c) $(\forall x)M(x) \wedge (\exists x) \sim M(x)$
 (d) $(\forall x)M(x) \vee (\exists y) \sim M(y)$

Q.18 Select which quantifier is Tautology.

- (a) $\sim (P \rightarrow q) \rightarrow P$
 (b) $(\sim P \wedge (P \rightarrow q)) \rightarrow P$
 (c) $(\sim P \wedge (P \rightarrow q)) \rightarrow \sim q$
 (d) $(\sim P \wedge (P \rightarrow q)) \rightarrow q$

Q.19 Consider the following predicates:

$S(x)$: " x is student"

$GATE(x, y)$: " x has written gate in every stream".
 Find the equivalent predicate logic for the following statement:

"There does not exist a student who has written a GATE in every stream."

- (a) $\exists y \exists x [S(x) \wedge \sim GATE(x, y)]$
 (b) $\forall y \exists x [\sim S(x) \vee \sim GATE(x, y)]$
 (c) $\exists y \forall x [\sim S(x) \vee \sim GATE(x, y)]$
 (d) $\exists y \exists x [\sim S(x) \wedge \sim GATE(x, y)]$

Q.20 Consider the following compound proposition:

$$[(A \vee B) \wedge (A \rightarrow C) \wedge (B \rightarrow D)] \rightarrow (D \vee C)$$

Which of the following is true for the above proposition?

- (a) Satisfiable (b) Contradiction
 (c) Tautology (d) None of these

Q.21 Consider the following well-formed formulae:

$$\text{I. } \neg \forall x (P(x))$$

$$\text{II. } \neg \exists x (P(x))$$

$$\text{III. } \neg \exists x (\neg P(x))$$

$$\text{IV. } \exists x (\neg P(x))$$

Which of the above are equivalent?

- (a) I and III (b) I and IV
(c) II and III (d) II and IV

Q.22 Which one of the following is the most appropriate logical formula to represent the statement:

"Gold and silver ornaments are precious"

The following notations are used:

$G(x)$: x is a gold ornament

$S(x)$: x is a silver ornament

$P(x)$: x is precious

- (a) $\forall x(P(x) \rightarrow (G(x) \wedge S(x)))$
(b) $\forall x(G(x) \wedge (S(x) \rightarrow P(x)))$
(c) $\exists x((G(x) \wedge S(x)) \rightarrow P(x))$
(d) $\forall x((G(x) \vee S(x)) \rightarrow P(x))$

Q.23 Match List-I with List-II:

- | List-I | List-II |
|----------------------------|---------------------------------|
| A. $P \rightarrow q$ | 1. $\neg(q \rightarrow \neg P)$ |
| B. $P \vee q$ | 2. $P \wedge \neg q$ |
| C. $P \wedge q$ | 3. $\neg P \rightarrow q$ |
| D. $\neg(P \rightarrow q)$ | 4. $\neg P \vee q$ |

Choose the correct option from those given below:

- (a) A-2; B-3; C-1; D-4 (b) A-2; B-1; C-3; D-4
(c) A-4; B-1; C-3; D-2 (d) A-4; B-3; C-1; D-2

Q.24 If $x + y \geq 2$, where x and y are real number, then

- (a) $x \geq 1$ or $y \geq 1$ (b) $x \geq 1$ or $y \geq 0$
(c) $x \geq 0$ or $y \geq 0$ (d) $x \neq 1$

Q.25 Consider the following statement:

"Every bird can fly"

The negation of the above statement in simple English.

- (a) No bird cannot fly
(b) There is a bird that can fly
(c) Every bird cannot fly
(d) There is a bird that cannot fly

Q.26 $\neg(X \wedge Y) \rightarrow (\neg X \vee (\neg X \vee Y))$ is logically equivalent to

- (a) $\sim(X \vee Y)$ (b) $\sim X \vee Y$
(c) $\sim X \wedge Y$ (d) $\sim(X \wedge Y)$

Q.27 Which one of the following is NOT logically equivalent to $\neg \exists x(\forall y(\alpha) \wedge \forall z(\beta))$

- (a) $\forall x(\exists z(\neg \beta) \rightarrow \forall y(\alpha))$
(b) $\forall x(\forall z(\beta) \rightarrow \exists y(\neg \alpha))$

(c) $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg \beta))$

(d) $\forall x(\exists y(\neg \alpha) \vee \exists z(\neg \beta))$

Q.28 Given that

$A(x)$ means " x is an alligator",

$H(x)$ means " x is an Human", and

$E(x, y)$ means " x eats y ",

Which of the given choices is the best English translation for the following first order logic statement?

$\forall x(H(x) \rightarrow \forall y[E(y, x) \rightarrow A(y)])$

- (a) All humans eat alligators.
(b) Alligators eat only humans.
(c) Every Alligators eats humans.
(d) Only alligators eat humans.

Q.29 If the binary operation $*$ is defined on a set of ordered pairs of real numbers as $(a, b) * (c, d) = (ad + bc, bd)$ where $b \neq 0$, then the inverse of (a, b) is _____.

- (a) $\left(\frac{a}{b^2}, \frac{1}{b}\right)$ (b) $\left(\frac{a}{b^2}, \frac{-1}{b}\right)$
(c) $\left(\frac{-a}{b^2}, \frac{1}{b}\right)$ (d) None of these

Q.30 Choose the correct statement

- (a) $\sim(p \leftrightarrow q)$ and $((p \wedge \sim q) \vee (q \wedge \sim p))$ is equivalent
(b) $\sim(p \leftrightarrow q)$ and $(p \rightarrow \sim q \wedge q \rightarrow \sim p)$ is equivalent
(c) Both (a) and (b)
(d) None of these

Q.31 $(p \wedge q) \wedge \sim(p \vee q)$ is a negation of

- (a) Tautology (b) Fallacy
(c) Both (a) and (b) (d) None of these

Q.32 Which of the following set of premises is not inconsistent?

- (a) $\{P \rightarrow Q, P \rightarrow R, Q \rightarrow \sim R, P\}$
(b) $\{R \cup M \sim R \cup S, \sim M \cup \sim S\}$
(c) $\{R \cup M \sim R \cup S, \sim M, \sim S\}$
(d) None of these

Q.33 The statement formula

$\{(a \vee b) \wedge (\sim a \vee c) \wedge \sim(b \vee c)\}$ is _____,

- (a) a tautology (b) a contradiction
(c) a contingency (d) None of these

Q.73 Consider the statement, "If you will give me a cow, then I will give you magic beans." Then which of the following statement does not represent the contrapositive statement?

- (a) If you will give me a cow, then I will not give you magic beans.
- (b) If I will not give you magic beans, then you will not give me a cow.
- (c) If I will give you magic beans, then you will give me a cow.
- (d) If you will not give me a cow, then I will not give you magic beans.

Q.74 You have discovered an old paper on graph theory that discusses the viscosity of a graph (which for all you know, is something completely made up by the author). A theorem in the paper claims that "if a graph satisfies condition (V), then the graph is viscous." Which of the following are equivalent ways of stating this claim?

- (a) A graph is viscous only if it satisfies condition (V).
- (b) A graph is viscous if it satisfies condition (V).
- (c) For a graph to be viscous, it is necessary that it satisfies condition (V).
- (d) For a graph to be viscous, it is sufficient for it to satisfy condition (V).

Q.75 You have discovered an old paper on graph theory that discusses the viscosity of a graph (which for all you know, is something completely made up by the author). A theorem in the paper claims that "if a graph satisfies condition (V), then the graph is viscous." Which of the following are equivalent ways of stating the converse of this claim?

- (a) Satisfying condition (V) is a sufficient condition for a graph to be viscous.
- (b) Satisfying condition (V) is a necessary condition for a graph to be viscous.
- (c) Every viscous graph satisfies condition (V).
- (d) Only viscous graphs satisfy condition (V).

Q.76 Consider the statement:

$$\forall x(\forall y(x + y = y) \rightarrow \forall z(x \cdot z = 0))$$

Then which of the following statement(s) is/are true?

- (a) The converse of above statement is $\forall x(\forall z(x \cdot z = 0) \rightarrow \forall y(x + y = y))$
- (b) The contrapositive of above statement is $\forall x(\exists z(x \cdot z \text{ not equals to } 0) \rightarrow \exists y(x + y \text{ not equals to } y))$.
- (c) The negation of above statement is $\exists x(\forall y(x + y = y) \wedge \exists z(x \cdot z \text{ not equals to } 0))$.
- (d) The above statement can also be represented as $\forall x(\forall z(x \cdot z = 0) \rightarrow \forall y(x + y = y))$

Q.77 Consider the following statement:

$$\forall x(x < 1 \rightarrow x^2 < 1)$$

Then which of the following statement(s) is/are True with respect to given statement

- (a) The negation of the statement is $\exists x(x < 1 \wedge x^2 \geq 1)$
- (b) The converse of the statement is $\exists x(x < 1 \wedge x^2 \geq 1)$
- (c) The contrapositive of the statement is $\forall x(x^2 \geq 1 \rightarrow x \geq 1)$
- (d) The converse of the statement is $\forall x(x^2 < 1 \rightarrow x < 1)$



Answers		Propositional Logic									
1.	(a)	2.	(a)	3.	(d)	4.	(b)	5.	(d)	6.	(c)
7.	(b)	8.	(b)	9.	(a)	10.	(b)	11.	(b)	12.	(a)
13.	(d)	14.	(a)	15.	(b)	16.	(c)	17.	(a)	18.	(a)
19.	(c)	20.	(c)	21.	(b)	22.	(d)	23.	(d)	24.	(a)
25.	(d)	26.	(b)	27.	(a)	28.	(d)	29.	(c)	30.	(c)
31.	(a)	32.	(b)	33.	(b)	34.	(b)	35.	(a)	36.	(a)
37.	(d)	38.	(b)	39.	(c)	40.	(a)	41.	(c)	42.	(b)
43.	(b)	44.	(c)	45.	(b)	46.	(b)	47.	(d)	48.	(c)
49.	(c)	50.	(b)	51.	(b)	52.	(a)	53.	(d)	54.	(d)
55.	(c)	56.	(d)	57.	(c)	58.	(a)	59.	(a)	60.	(c)
61.	(c)	62.	(d)	63.	(c)	64.	(a)	65.	(c)	66.	(b)
67.	(c)	68.	(d)	69.	(b)	70.	(a, c)	71.	(a, c, d)	72.	(b, c)
73.	(a, c, d)	74.	(b, d)	75.	(b, c)	76.	(a, b, c)	77.	(a, c, d)		

Explanations Propositional Logic

1. (a)

$([P \rightarrow (q \vee r)] \wedge \bar{q} \wedge \bar{r}) \rightarrow \bar{P}$ is tautology hence it is valid argument.

2. (a)

$$\begin{aligned} & \neg \forall x \forall y [(x < y) \rightarrow (x^2 < y^2)] \\ & \equiv \exists x \exists y \neg [(x < y) \rightarrow (x^2 < y^2)] \\ & \equiv \exists x \exists y \neg [\neg(x < y) \vee (x^2 < y^2)] \\ & \equiv \exists x \exists y [(x < y) \wedge \neg(x^2 < y^2)] \\ & \equiv \exists x \exists y [(x < y) \wedge (x^2 \geq y^2)] \end{aligned}$$

3. (d)

$F(x) \Rightarrow x$ is my friend.

$P(x) \Rightarrow x$ is perfect.

(d) is correct answer.

(a) There exist some friend which are not perfect.

(b) There are some people who are not my friend and are perfect.

(c) There exist some people who are not my friend and are not perfect.

(d) There does not exist any person who is my friend and perfect.

So, option (d) is correct.

4. (b)

This states the original accurately. If something is a book that I read last year, then it is guaranteed to be adventure story. The equivalent rule is that if a book is not an adventure story, then i definitely did not read it last year.

5. (d)

Let $P(x) = x$ likes paintings, $F(x) = x$ likes flowers, $R(x) = x$ likes running, $M(x) = x$ likes music

Statement (i) implies $P(x) \rightarrow F(x)$

Statement (ii) implies $R(x) \rightarrow M(x)$

Statement (iii) implies $\sim M(x) \rightarrow \sim P(x)$ can be written as $F(x) \rightarrow M(x)$

From statement 2 and 3, we can get $P(x) \rightarrow M(x)$

Only statement (4) is correct.

So, option (d) is correct.

6. (c)

(c) is the correct option.

7. (b)

$P(1, 1) : 1 + 1 = 1 - 1$ is false

$P(3, 0) : 3 + 0 = 3 - 0$ is true

$\exists x P(x, 2) : \exists x (x + 2 = x - 2)$ is false as there is no solution

$\exists x \forall y P(x, y) : \exists x \forall y (x + y = x - y)$ is false since this can be made true only if $y = 0$

$\exists y \forall x P(x, y) : \exists y \forall x (x + y = x - y)$ is true since for $y = 0$ the equation is true for all x

$\forall x \exists y P(x, y) : \forall x \exists y (x + y = x - y)$ is true since for all x , $y = 0$ will satisfy the equation.

8. (b)

III is true since once a variable is bound to a qualifier it's name does not matter.

So $\exists x Q(x)$ is same $\exists y Q(y)$ and so II is true since LHS and RHS is same.

I is false, since

LHS: some value of x satisfies both P and Q

RHS: some values satisfies P and some value satisfies Q , but these 2 values need not be same.

II is true, since

If the same value satisfies both P and Q surely some value satisfies P and some values satisfies Q .

In other words LHS of implication is stronger than RHS and hence implication will be true.

9. (a)

Given $[(p \vee q) \wedge \neg p \rightarrow q]$

The precedence of the used operators are:
 $\wedge > \vee > \rightarrow$

Therefore, $[(p \vee q) \wedge \neg p \rightarrow q]$

$$\Rightarrow [((p \vee q) \wedge \neg p) \rightarrow q]$$

$$\Rightarrow [(p \wedge \neg p) \vee (q \wedge \neg p) \rightarrow q]$$

(using distributing law)

$$\Rightarrow [(0 \vee (q \wedge \neg p)) \rightarrow q]$$

$$\Rightarrow [(q \wedge \neg p) \rightarrow q]$$

$$\Rightarrow [\neg(q \wedge \neg p) \vee \neg q] \text{ (using Demorgan's law)}$$

$$\Rightarrow [\neg q \vee p \vee \neg q]$$

$$\Rightarrow [p \vee \neg q]$$

\therefore Option (a) is correct.

$$\equiv (a' + b)' + b' + c$$

$$\equiv ab' + b' + c$$

$$\equiv b' + c$$

Therefore, $((a \rightarrow b) \rightarrow (b \rightarrow c))$ is contingency but not tautology.

10. (b)

I. $\forall x \exists y R(x, y) \rightarrow \exists y (\exists x R(x, y))$ is true, since
 $\exists y (\exists x R(x, y)) \equiv \exists x (\exists y R(x, y))$

II. $\forall x \exists y R(x, y) \rightarrow \exists y (\forall x R(x, y))$ is false
 Since $\exists y$ when it is outside is stronger than when it is inside.

III. $\forall x \exists y R(x, y) \rightarrow \forall y \exists x R(x, y)$ is false
 Since $R(x, y)$ may not be symmetric in x and y .

IV. $\forall x \exists y R(x, y) \rightarrow \neg(\exists x \forall y \neg R(x, y))$ is true
 Since $\neg(\exists x \forall y \neg R(x, y)) \equiv \forall x \exists y R(x, y)$
 So, IV will reduce to

$\forall x \exists y R(x, y) \rightarrow \forall x \exists y R(x, y)$ which is trivially true.
 So correct answer is I and IV only which is option (b).

11. (b)

$$\begin{aligned} \text{(a)} \quad (a \vee b) \rightarrow (b \wedge c) \\ \equiv (a + b)' + bc \\ \equiv a' b' + bc \end{aligned}$$

Therefore, $((a \vee b) \rightarrow (b \wedge c))$ is contingency and not tautology.

$$\begin{aligned} \text{(b)} \quad (a \wedge b) \rightarrow (b \vee c) \\ \equiv ab \rightarrow b + c \\ \equiv (ab)' + b + c \\ \equiv a' + b' + b + c \\ \equiv a' + 1 + c \equiv 1 \end{aligned}$$

So $((a \wedge b) \rightarrow (b \vee c))$ is tautology.

$$\begin{aligned} \text{(c)} \quad (a \vee b) \rightarrow (b \rightarrow c) \\ \equiv (a + b) \rightarrow (b' + c) \\ \equiv (a + b)' + b' + c \\ \equiv a' b' + b' + c \\ \equiv b' + c \end{aligned}$$

So $((a \vee b) \rightarrow (b \rightarrow c))$ is contingency but not tautology.

$$\begin{aligned} \text{(d)} \quad (a \rightarrow b) \rightarrow (b \rightarrow c) \\ \equiv (a' + b) \rightarrow (b' + c) \end{aligned}$$

12. (a)

$$P(x) : x \leq 4$$

- (a) $P(0) : 0 \leq 4$ True
 (b) $P(6) : 6 \leq 4$ False
 (c) $P(8) : 8 \leq 4$ False
 (d) $P(9) : 9 \leq 4$ False
 So, option (a) is correct.

13. (d)

All statement S(I), S(II) and S(III) is correct.

14. (a)

$F_1 : P \rightarrow \sim P \equiv p \rightarrow p' \equiv p' + p' \equiv p'$
 So F_1 is contingency. Hence, F_1 is satisfiable but not valid.

$$\begin{aligned} F_2 : (P \rightarrow \sim P) \vee (\sim P \rightarrow P) \\ \equiv (p \rightarrow p') + (p' \rightarrow p) \\ \equiv (p' + p') + (p + p) \\ \equiv p' + p \equiv 1 \end{aligned}$$

So F_2 is tautology and therefore valid.

15. (b)

$$\begin{aligned} (a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c)) \\ (a'c' + ac) \rightarrow (b + (ac)) \\ (a'c' + ac') + (b + ac) \\ a(c + c') + a'c + b \\ a + a'c + b \\ a + c + b \end{aligned}$$

So not tautology but contingency.

16. (c)

$X = \{\text{integer}\}$
 $X = \{-2, -1, 0, 1, 2, \dots\}$
 $P(X) : X = X^2$
 (a) $P(2)$: False, since $2 \neq 2 \wedge 2$
 (b) $P(-1)$: False, since $-1 \neq (-1) \wedge 2$
 (c) $\exists x P_x$: True; let $x = 0$, since $0 = 0^2$
 (d) $\forall x P(x)$: False; let $x = 2$, since $2 \neq 2^2$
 Hence, option (c) is correct.

17. (a)

"Either everything is material or there are somethings that are not material"

Either-OR means XOR.

We will define P as $\forall x (\text{material}(x))$

q as $\exists x (\neg \text{material}(x))$

Either P or $q \equiv (\neg P \wedge q) \vee (P \wedge \neg q)$

$$\Rightarrow (\neg \forall x (\text{material}(x)) \wedge \exists x (\neg \text{material}(x))) \vee (\forall x (\text{material}(x)) \wedge \neg \exists x (\neg \text{material}(x)))$$

$$\Rightarrow (\exists x (\neg \text{material}(x)) \wedge \exists x (\neg \text{material}(x)))$$

$$(\forall x (\text{material}(x)) \wedge \forall x (\text{material}(x)))$$

$$\Rightarrow (\exists x (\neg \text{material}(x)) \vee (\forall x (\text{material}(x))))$$

Hence, option (a) is correct.

18. (a)

By truth table,

let S											
P	q	P'	q'	P → q	~(P → q)	~(P → q) → P	~P ∧ (P → q)	S → P	S → ~q	S → q	
0	0	1	1	1	0	1	1	0	1	0	
0	1	1	1	1	0	1	1	0	0	1	
1	0	0	0	0	1	1	0	1	1	1	
1	1	0	1	1	0	1	0	1	1	1	
						(a)	(b)	(c)	(d)		

So, from above truth table, (a) option is correct.

19. (c)

$$\sim [\exists x S(x) \wedge \forall y \text{GATE}(x, y)]$$

$$= \sim \forall y \exists x [S(x) \wedge \text{GATE}(x, y)]$$

$$= \exists y \forall x [\sim S(x) \vee \sim \text{GATE}(x, y)]$$

$$\text{II} \quad \neg \exists x P(x) \equiv \forall x \neg P(x)$$

$$\text{and III} \quad \neg \exists x [\neg P(x)] \equiv \forall x P(x)$$

Clearly II and III are not equivalent to each other or to I and IV.

20. (c)

$$[(A \vee B) \wedge (A \rightarrow C) \wedge (B \rightarrow D)] \rightarrow (D \vee C)$$

$$= \neg [(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D)] \vee (D \vee C)$$

$$= \neg (A \vee B) \vee \neg (\neg A \vee C) \vee \neg (\neg B \vee D) \vee D \vee C$$

$$= (\neg A \wedge \neg B) \vee (A \wedge \neg C) \vee (B \wedge \neg D) \vee D \vee C$$

$$= (\neg A \wedge \neg B) \vee (A \wedge \neg C) \vee B \vee D \vee C$$

$$= (\neg A \wedge \neg B) \vee A \vee B \vee D \vee C$$

$$= A \vee A \vee B \vee D \vee C$$

$$= T \vee B \vee D \vee C$$

$$= \text{true}$$

∴ It is tautology

21. (b)

$$\text{I} \quad \neg \forall x P(x) \equiv \exists x \neg P(x)$$

$$\text{and IV} \quad \exists x \neg P(x)$$

Clearly, choices I and IV are equivalent.

22. (d)

The correct translation of "Gold and silver ornaments are precious" is choice (d)

$$\forall x ((G(x) \vee S(x)) \rightarrow P(x))$$

which is read as "if an ornament is gold or silver, then it is precious".

Now since a given ornament cannot be both gold and silver at the same time.

Choice (b) $\forall x ((G(x) \wedge S(x)) \rightarrow P(x))$ is incorrect.

23. (d)

Option (d) is correct

∴ A-4; B-3; C-1; D-2

24. (a)

Let us assume the contrapositive, i.e., both x, y are less than 1.

Now, we are given that fact that $x + y \geq 2$, but from our assumption that $x < 1$ and $y < 1$, we get that $x + y < 2$ which defines the given fact.

Hence, option (a) is correct option.