# COMPUTER SCIENCE & INFORMATION TECHNOLOGY

# Digital Logic

Comprehensive Theory
*with* Solved Examples *and* Practice Questions

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# Digital Logic

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## GOAL OF THE SUBJECT

Digital logic design is a system in electrical and computer engineering that uses simple number values to produce input and output operations. As a digital design engineer, you may assist in developing cell phones, computers, and related personal electronic devices. Digital logic is the representation of signals and sequences of a digital circuit through numbers. It is the basis for digital computing and provides a fundamental understanding on how circuits and hardware communicate within a computer. Digital logic is typically embedded into most electronic devices, including calculators, computers, video games, and watches. This field is utilized by many careers that work with computers and technology, such as engineers and repair technicians.

More specifically, DLD provides following things:

- It dictates how the number can be represented in computers and it's conversion in various bases.
- It includes various gates which are helpful in designing of circuits.
- It also allows us to minimize the functions using Karnaugh map technique which is widely popular in digital world.
- Moreover, DLD also defines flip flop which helps in recording the count of values and can be used to store values in registers which are very fast to access.

## INTRODUCTION

Digital logic design deals with electronics that operate on digital signals. Digital techniques are helpful because it is much easier to get an electronic device to switch into one of a number of unknown states than to accurately reproduce a continuous range of values.

**Chapter 1: Basics of Digital Logic:** In this chapter, we discuss digital number systems, codes, arithmetic operations on signed number representation and its overflow concept.

**Chapter 2: Boolean Algebra and Minimization Techniques:** In this chapter, we discuss about boolean algebra, its laws and postulates, minimization of logic functions using K-map.

**Chapter 3: Logic Gates and Switching Circuits:** In this chapter, we study basic gates and its properties. Moreover, universal gates and its properties have also been discussed.

**Chapter 4: Combinational Logic Circuits:** In combinational circuits, we discuss full adder/half adder, subtractors with different propers. Hazards mainly static 1 has also been introduced.

**Chapter 5: Sequential Logic Circuits** In this chapter we get to know about latches, flip-flops using NAND/NOR gates. All kinds of flip-flops are defined in this very particular chapter.

**Chapter 6: Registers and Counters:** In this chapter we discuss application of flip-flops which includes registers and counters. Some standard counters like ring and Jhonson are also discussed.

■■■■

CHAPTER

1

# Basics of Digital Logic

## 1.1 INTRODUCTION

Electronic systems are of two types:

(*i*) Analog systems

(*ii*) Digital systems

Analog systems are those systems in which voltage and current variations are continuous through the given range and they can take any value within the given specified range, whereas a digital system is one in which the voltage level assumes finite number of distinct values. In all modern digital circuits there are just two discrete voltage level.

Digital circuits are often called switching circuits, because the voltage levels in a digital circuit are assumed to be switched from one value to another instantaneously. Digital circuits are also called logic circuits, because every digital circuit obeys a certain set of logical rules.

Digital systems are extensively used in control systems, communication and measurement, computation and data processing, digital audio and video equipments, etc.

### 1.1.1 Advantages of Digital Systems

Digital systems have number of advantages over analog systems which are summarized below:

**I. Ease of Design**

The digital circuits having two voltage levels, OFF and ON or LOW and HIGH, are easier to design in comparison with analog circuits in which signals have numerical significance ; so their design is more complicated.

**II. Greater Accuracy and Precision**

Digital systems are more accurate and precise than analog systems because they can be easily expanded to handle more digits by adding more switching circuits.

**III. Information Storage is Easy**

There are different types of semiconductor memories having large capacity, which can store digital data.

**IV. Digital Systems are More Versatile**

It is easy to design digital systems whose operation is controlled by a set of stored instructions called program. However in analog systems, the available options for programming is limited.

### V. Digital Systems are Less Affected by Noise

The effect of noise in analog system is more. Since in analog systems the exact values of voltages are important. In digital system noise is not critical because only the range of values is important.

### VI. Digital Systems are More Reliable

As compared to analog systems, digital systems are more reliable.

**Limitations of Digital System**

(*i*) The real world is mainly analog.
(*ii*) Human does not understand the digital data.

## 1.2 RADIX NUMBER SYSTEMS

The numeric system we use daily is the decimal system, but this system is not convenient for machines since the information is handled codified in the shape of on or off bits, this way of codifying takes us to the necessity of knowing the positional calculation which will allow us to express a number is any base where we need it.

A base of a number system or radices defines the range of values that a digit may have.

1. In the binary system or base 2, there can be only two values for each digit of a number, either a "0" or a "1".
2. In the octal system or base 8, there can be eight choices for each digit of a number:
   "0", "1", "2", "3", "4", "5", "6", "7"
3. In the decimal system or base 10, there are ten different values for each digit of a number:
   "0", "1", "2", "3", "4", "5", "6", "7", "8", "9"
4. In the hexadecimal system, we allow 16 values for each digit of a number:
   "0", "1", "2", "3", "4", "5", "6", "7", "8", "9", "A", "B", "C", "D" and "F"
   where "A" stands for 10, "B" for 11 and so on.

In general, a positive number N can be written in positional notation as

$$N = (a_{n-1}\, a_{n-2} \ldots a_1\, a_0 \,.\, a_{-1}\, a_{-2} \ldots a_{-m})$$

where,

$.$ = radix point separating the integer and fractional digits.
$r$ = radix or base of the number system being used
$n$ = number of integer digits to the left of the radix point
$m$ = number of fractional digits to the right of the radix point
$a_i$ = integer digits $i$ when $n-1 \geq i \geq 0$
$a_j$ = fractional digits $j$ when $-1 \geq i \geq -m$
$a_{n-1}$ = most significant digit
$a_{n-2}$ = least significant digit

A number system with base or radix '$r$' will have $r$ number of different digits from $0 \rightarrow (r-1)$ thus, number system is represented by $(N)_b$

where, $N$ = Number ; $b$ = Base or radix

## 1.3 CONVERSION AMONG RADICES

### 1.3.1 Convert from Decimal to Any Base

Let's think about what you do to obtain each digit. As an example, let's start with a decimal number 1234 and convert it to decimal notation. To extract the last digit, you move the decimal point digit, you move the decimal point left by one digit, which means you divide the given number by its base 10.

$$1234/10 = 123 + 4/10$$

To remainder of 4 is the last digit. To extract the next last digit, you again move the decimal point left by one digit and see what drops out.

$$123/10 = 12 + 3/10$$

The remainder of 3 is the next last digit. You repeat this process until there is nothing left. Then you stop in summary, you do the following:

| | Quotient | Remainder |
|---|---|---|
| 1234/10 | 123 | 4 |
| 123/10 | 12 | 3 |
| 12/10 | 1 | 2 |
| 1/10 | 0 | 1 |

(Stop when quotient is 0)

1 2 3 4

Now, let's try a non-trivial example. Let's express a decimal number 1341 in binary notation. Note that the desired base is 2, so we repeatedly divide the given decimal number by 2.

| | Quotient | Remainder |
|---|---|---|
| 1341/2 | 670 | 1 |
| 670/2 | 335 | 0 |
| 335/2 | 167 | 1 |
| 167/2 | 83 | 1 |
| 83/2 | 41 | 1 |
| 41/2 | 20 | 1 |
| 20/2 | 10 | 0 |
| 10/2 | 5 | 0 |
| 5/2 | 2 | 1 |
| 2/2 | 1 | 0 |
| 1/2 | 0 | 1 |

(Stop when the quotient is 0)
(BIN; Base 2)

1 0 1 0 0 1 1 1 1 0 1

Let's express the same decimal number 1341 in hexadecimal notation.

| | Quotient | Remainder |
|---|---|---|
| 1341/16 | 83 | 13 |
| 83/16 | 5 | 3 |
| 5/16 | 0 | 5 |

(Stop when the quotient is 0)
(HEX; Base 16)

5 3 D

## Conclusion:

In conclusion, the easiest way to convert fixed point numbers to any base is to convert each part separately. We begin by separating the number into its integer and fractional part. The integer part is converted using the remainder method, by using a successive division of the number in the base until a zero is obtained. At each division, the reminder is kept and then the new number in the base *r* is obtained by reading the remainder from the last to remainder upwards.

The conversion of the fractional part can be obtained by successively multiplying the fraction with the base. If we iterate this process on the remaining fraction, then we will obtain successive significant digit. This methods form the basis of the multiplication methods of converting fractions between bases.

**Example 1.1** Convert $(13)_{10}$ to binary.

**Solution :**

| | Quotient | Remainder | |
|---|---|---|---|
| $13 \div 2$ | 6 | 1 | LSB |
| $6 \div 2$ | 3 | 0 | |
| $3 \div 2$ | 1 | 1 | |
| $1 \div 2$ | 0 | 1 | MSB |

$\therefore$ $(13)_{10} \Rightarrow (1101)_2$

**Example 1.2** Convert $(0.65625)_{10}$ to an equivalent base-2 number.

**Solution :**

| | | | | |
|---|---|---|---|---|
| 0.65625 | 0.31250 | 0.62500 | 0.25000 | 0.50000 |
| × 2 | × 2 | × 2 | × 2 | × 2 |
| 1.31250 | 0.62500 | 1.25000 | 0.50000 | 1.00000 |
| ↓ 1 | ↓ 0 | ↓ 1 | ↓ 0 | ↓ 1 |

Thus, $(0.65625)_{10} = (0.10101)_2$

**Example 1.3** Convert $(3287.5100098)_{10}$ into octal.

**Solution :**

For integral part:

| | Quotient | Remainder |
|---|---|---|
| $3287 \div 8$ | 410 | 7 |
| $410 \div 8$ | 51 | 2 |
| $51 \div 8$ | 6 | 3 |
| $6 \div 8$ | 0 | 6 |

$\therefore$ $(3287)_{10} = (6327)_8$

Now for fractional part:

| | | | |
|---|---|---|---|
| 0.5100098 | 0.0800784 | 0.6406272 | 0.1250176 |
| × 8 | × 8 | × 8 | × 8 |
| 4.0800784 | 0.6406272 | 5.1250176 | 1.0001408 |
| ↓ 4 | ↓ 0 | ↓ 5 | ↓ 1 |

$\therefore$ $(0.5100098)_{10} = (0.4051)_8$

Finally, $(3287.5100098)_{10} = (6327.4051)_8$

**Example 1.4** Convert $(675.625)_{10}$ into Hexadecimal.

**Solution :**

For Integral Part:

## Student's Assignments

**Q.1** If we convert a binary sequence, $(1100101.1011)_2$ into its octal equivalent as $(X)_8$, the value of 'X' will be
(a) (145.13) (b) (145.54)
(c) (624.54) (d) (624.13)

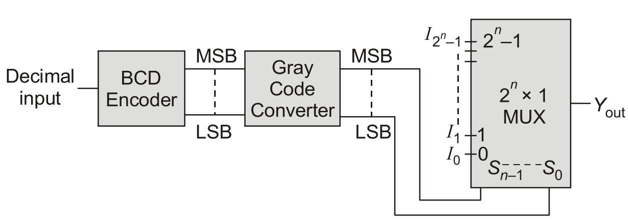
**Q.2** A binary $(11011)_2$ may be represented by following ways:
**1.** $(33)_8$ **2.** $(27)_{10}$
**3.** $(10110)_{GRAY}$ **4.** $(1\,B)_H$
Which of these above is/are correct representation?
(a) 1, 2 and 3 (b) 2 and 4
(c) 1, 2, 3 and 4 (d) 2 only

**Q.3** The decimal equivalent of hexadecimal number of '2A0F' is
(a) 17670 (b) 17607
(c) 17067 (d) 10767

**Q.4** The sign-magnitude form and 2's complement form of a signed binary number $(10111)_2$ are:
(a) –23 and –25 (b) –23 and –9
(c) –7 and –23 (d) –7 and –9

**Q.5** Given that $292_{10}$ = 1204 in some number system. Which of the following represents the base of the that system?
(a) 5 (b) 6
(c) 7 (d) 8

**Q.6** Consider the circuit given below:




If the decimal input is 92 then $Y_{out}$ corresponds to $I_m$, then value of $m$ is ______.

**Q.7** Consider the addition of numbers with different bases $(X)_7 + (Y)_8 + (W)_{10} + (Z)_5 = (K)_9$
If $X = 36$, $Y = 67$, $W = 98$ and $K = 241$ then $Z$ is ________.

**Q.8** Which one of the following is the correct sequence of numbers represented in the series $(2)_3$, $(3)_4$, $(14)_5$, $(15)_6$?
(a) 2, 5, 10, 12 (b) 2, 3, 9, 11
(c) 3, 7, 10, 14 (d) 3, 8, 13, 17

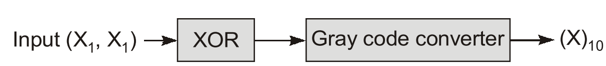
**Q.9** Which of the following statement is **incorrect** for the range of $n$ bits binary numbers?
(a) Range of unsigned numbers is 0 to $2^n - 1$.
(b) Range of signed numbers is $-2^{n-1} + 1$ to $2^{n-1} - 1$
(c) Range of signed 1's complement numbers is $-2^{n-1} + 1$ to $2^{n-1}$
(d) Range of signed 2's complement numbers is $-2^{n-1}$ to $2^{n-1} - 1$

**Q.10** The base of the number system for the addition 13 + 24 = 42 to be true will be _________.

**Q.11** Decimal equivalent of $(1000)_2 = -2^n$
Decimal equivalent of $(10000)_2 = -2^m$
So, $(n + m)_2$ would be
(a) 1 1 1 (b) 0 1 1
(c) 0 0 1 (d) 1 0 1

**Q.12** When $(-89)_{10}$ is converted in binary, the sum of bits in binary will be _________.

**Q.13** Consider the input $X_1$ = 10101010 and $X_2$ = 11111111 is feeded as input in the diagram:




Which of the following represent the value of $X$?
(a) +127 (b) –127
(c) –255 (d) +255

**Q.14** The $r$'s compliment of an $n$-digit decimal number $N$ in base $r$ is defined for all values of $N$ except for $N = 0$. If the given number is $(247)_9$, then its 9's compliment will be equal to $( \_\_\_\_\_ )_9$.

**Q.15** The maximum positive and negative decimal numbers that can be represented in two's complement using $n$-bits are
(a) $(2^{n-1} - 1)$ and $(2^{n-1} - 1)$
(b) $(2^{n-1} - 1)$ and $-2^{n-1}$
(c) $2^{n-1}$ and $-2^{n-1}$
(d) $2^{n-1}$ and $-(2^{n-1} - 1)$

**Q.16** Consider the arithmetic operation performed in a particular number system whose radix is equal to '$r$'.

$$(23)_r + (44)_r + (14)_r + (32)_r = (223)_r$$

The value of radix '$r$' is equal to ________.

**Q.17** A quadratic equation is formed in some number system with radix $r$ as $x^2 - 11x + 22 = 0$. The roots of this equation are equal to $x = (3)_r$ and $x = (6)_r$ where $r$ is the base of the number. Then, the value of $r$ = ________.

**Q.18** The representation of the value of 20 bit signed integer in 2's complement form is P = $(A72E5)_{16}$. Which of the following represents 16 × P in 1's complement representation?

(a) $(72E4F)_{16}$ (b) $(72E50)_{16}$
(c) $(72E4E)_{16}$ (d) None of the above

**Q.19** The representation of the value of a 16-bit unsigned integer X in hexadecimal number system is A72E. The representation of the value of X in octal number system is

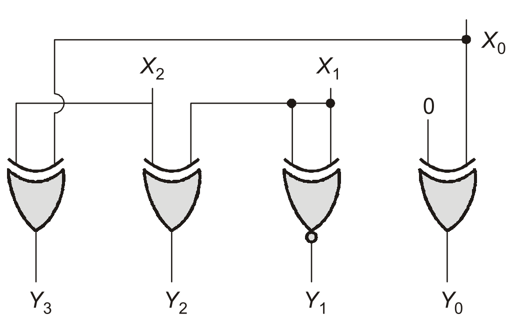
(a) 12346 (b) 123456
(c) 125756 (d) 10634

**Q.20** If $(504)_X$ in base-X is equal to $(2320)_4$. Then what will be the value of base-X (in decimal) ________?

**Q.21** Let A = 11111010 and B = 00001111 be two 8-bit 2's complement numbers. Their product in 2's complements is

(a) 01011010 (b) 10100110
(c) 10010010 (d) 11010101

**Q.22** Consider the circuit shown in the figure below:




If the three bit input to the circuit is $(X_2X_1X_0)$ = 111 then the decimal equivalent of the corresponding output of the circuit $(Y_3Y_2Y_1Y_0)$ will be equal to ________.

**Q.23** Which of the following is not true?

(a) The $r$'s complement of a positive number N in base $r$ is $(r^n - N)$.
(b) The $(r-1)$'s complement of a positive number N in base $r$ is $(r^n - N - 1)$.
(c) The $(r-1)$'s complement of a positive number N having $n$ digits and $m$ digits in integer and fraction respectively in base $r$ is $(r^n - r^{-m} - N)$.
(d) The $(r-1)$'s complement of a positive number N having $n$ digit and $m$ digits in integer and fraction part respectively in base $r$ is $(r^n - r^m - N)$.

**Q.24** How many minimum number of decimal digits is required to represent 19 bit of binary data. The number of decimal digit will be ________.

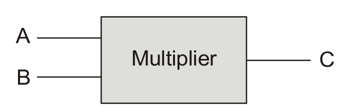
**Q.25** The minimum decimal equivalent of the number $(1AC)_x$ is equal to ________.

**Q.26** Consider the following arithmetic equation:

$$\frac{302}{20} = 12.1$$

The minimum possible non-zero base for the given system is ________.

**Q.27** Consider a 3-bit number A and 2 bit number B are given to a multiplier. The output of multiplier is realized using AND gate and one bit full adders. If minimum number of AND gates required are $X$ and one bit full adders required are $Y$, then $X + Y$ = ________.




***Answer Key :***

| | | | | |
|---|---|---|---|---|
| **1.** (b) | **2.** (c) | **3.** (d) | **4.** (d) | **5.** (b) |
| **6.** (219) | **7.** (34) | **8.** (b) | **9.** (c) | **10.** (5) |
| **11.** (a) | **12.** (5) | **13.** (a) | **14.** (642) | **15.** (b) |
| **16.** (5) | **17.** (8) | **18.** (a) | **19.** (b) | **20.** (6) |
| **21.** (b) | **22.** (3) | **23.** (d) | **24.** (6) | **25.** (311) |
| **26.** (4) | **27.** (9) | | | |

## Student's Assignments Explanations

**1. (b)**

We have, binary sequence $(1100101.1011)_2$

In order to convert binary number into octal equivalent we need to group the bits into triplets.

$\underline{001}\ \underline{100}\ \underline{101}.\underline{101}\ \underline{100} = (145.54)_8$

**2. (c)**

- In decimal $= 2^0 \times 1 + 2^1 \times 1 + 2^2 \times 0 + 2^3 \times 1 + 2^4 \times 1 = 1 + 2 + 8 + 16 = (27)_{10}$
- In octal $= \underbrace{011}_{3}\ \underbrace{011}_{3} = (33)_8$
- In hexadecimal $= \underbrace{0001}_{(1B)_{16}}\ \underbrace{1011}_{3}$
- Gray code: 1 1 0 1 1

  (1 0 1 1 0)$_{GRAY}$

Hence, all the options are true.

**3. (d)**

We have, hexadecimal number 2A0F to convert it into decimal number, we can do:

$$\begin{array}{cccc} 2 & A & 0 & F \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 10 & 0 & 15 \end{array}$$

$16^3 \times 2 + 16^2 \times 10 + 16^1 \times 0 + 16^0 \times 15$

Which is equals to 10767.

**4. (d)**

In sign-magnitude from 10111 can be defined as $\underbrace{1}_{\text{sign}}\ \underbrace{0111}_{\text{magnitude}}$ i.e. –7.

In 2's complement from 10111 can be defined as – 9.

**5. (b)**

Let the base be $x$, then

$$\begin{aligned} 292_{10} &= 1204\,x \\ &= 1 \times x^3 + 2 \times x^2 + 0 \times x^1 + 4 \times x^0 \\ &= 292_{10} = x^3 + 2x^2 + 4 \\ &= 6 \text{ (By substitution)} \end{aligned}$$

**6. (219)**

$$\begin{aligned} \text{Decimal input} &= 92 \\ \text{BCD} &= 10010010 \end{aligned}$$

Output of Gray code converter = 11011011

$Y_0$ corresponds to $I_m$ with $(S_n \ldots\ldots S_0)$ is

$$\begin{aligned} &= (11011011)_2 \\ m &= 219 \end{aligned}$$

**7. (34)**

$$\begin{aligned} (36)_7 &= (27)_{10} \\ (67)_8 &= (55)_{10} \\ (98)_{10} &= (98)_{10} \\ (Z)_5 &= (Z)_5 \\ (241)_9 &= (199)_{10} \\ \therefore \quad (Z)_5 &= (199)_{10} - (27)_{10} - (55)_{10} - (98)_{10} \\ (Z)_5 &= (19)_{10} \end{aligned}$$

Converting $(19)_{10} = (34)_5$

$\therefore \quad Z = 34$

**8. (b)**

Converting into decimal,

$$\begin{aligned} (2)_3 &= 2 \times 3^\circ = 2 \\ (3)_4 &= 3 \times 4^\circ = 3 \\ (14)_5 &= 1 \times 5^1 + 4 \times 5^\circ = 9 \\ (15)_6 &= 1 \times 6^1 + 5 \times 6^\circ = 11 \end{aligned}$$

**9. (c)**

Range of signed 1's complement number is $-2^{n-1} + 1$ to $2^{n-1} - 1$.

**10. (5)**

Let base be $x$, then

$$\begin{aligned} (13)\,x + (24)\,x &= (42)\,x \\ (1x^1 + 3x^0) + (2x^1 + 4x^0) &= 4x^1 + 2x^0 \\ 3x^1 + 7x^0 &= 4x^1 + 2x^0 \\ x &= 5 \end{aligned}$$

**11. (a)**

Decimal equivalent of $(1000)_2 = -2^3$

$\Rightarrow \quad n = 3$

Decimal equivalent of $(10000)_2 = -2^4$

$\Rightarrow \quad m = 4$

So, $\quad (n + m)_2 = (3 + 4)_2 = (7)_2 = 111$

**12. (5)**

Binary representation of $(89)_{10} = (01011001)$

$(-89)_{10}$ = 2's compliment of (01011001)

CHAPTER

2

# Boolean Algebra and Minimization Techniques

## 2.1 INTRODUCTION

- The binary operations performed by any digital circuit with the set of elements 0 and 1, are called logical operation or logic function. The algebra used to symbolically represent the logic function is called Boolean algebra. It is a two state algebra invented by George Boole in 1854.
- Boolean algebra is a system of mathematics logic for the analysis and designing of digital systems.
- A variable or function of variables in Boolean algebra can assume only two values, either a '0' or a '1'. Hence, (unlike another algebra) there are no fractions, no negative numbers, no square roots, no cube roots, no logarithms etc.

## 2.2 LOGIC OPERATIONS

In Boolean algebra, all the algebraic functions performed are logical. They actually represent logical operations. The AND, OR and NOT are the basic operations in Boolean algebra. In addition to these operations, there are some derived operation such as NAND, NOR, EX-OR, EX-NOR in Boolean algebra.

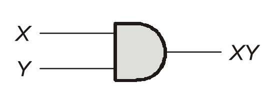
### 2.2.1 AND Operation

The AND operation in Boolean algebra is similar to the multiplication in ordinary algebra. It is a logical operation performed by AND gate.

AND operation:

Truth table for AND gate

| $X$ | $Y$ | $X \cdot Y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



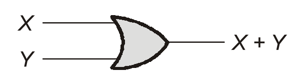

### 2.2.2 OR Operation

The OR operation in Boolean algebra is performed by OR-gate.

OR operation:

Truth table for OR gate

| $X$ | $Y$ | $X + Y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |




### 2.2.3 NOT Operation

The NOT operation in Boolean algebra is similar to the complementation or inversion in ordinary algebra. The NOT operation is indicated by a bar (¯) or (') over the variable.

*Example:* $A \xrightarrow{\text{NOT}} \bar{A}$ or $A'$ (complementation law)

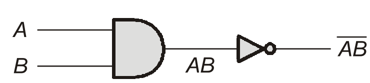
and $\bar{\bar{A}} = A \Rightarrow$ double complementation law.

NOT operation is performed by NOT gate.

### 2.2.4 NAND Operation

The NAND operation is AND operation followed by NOT operation i.e. the negation of AND gate operation is performed by the NAND gate.

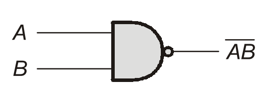
Thus, NAND gate can be represented as:




Truth table for NAND gate

| $A$ | $B$ | $\overline{AB}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

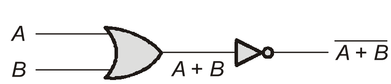
NAND gate can also be represented as:




### 2.2.5 NOR Operation

The NOR operation is OR operation followed by NOT operation i.e. the negation of OR gate operation is performed by the NOR gate.

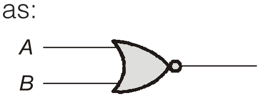
Thus, NOR gate can be represented as




Truth table for NOR gate

| $A$ | $B$ | $\overline{A+B}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

NOR gate can also be represented as:

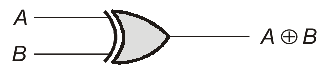


### 2.2.6 EX-OR Operation

EX-OR or XOR gate gives a true output when the number of true input(s) is/are odd.

| $A$ | $B$ | $A \oplus B$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

EX-OR gate can be represented as




## 2.3 LAWS OF BOOLEAN ALGEBRA

The Boolean algebra is governed by certain well defined rules and laws.

### 2.3.1 Commutative Laws

- The commutative law allows change in position of AND or OR variables.
  (*i*) $A + B = B + A$
  Thus, the order in which the variables are ORed is immaterial.
  (*ii*) $A \cdot B = B \cdot A$
  Thus, the order in which the variables are ANDed is immaterial.
- This law can be extended to any number of variables.

### 2.3.2 Associative Laws

- The associative law allows grouping of variables.
  (*i*) $(A + B) + C = A + (B + C)$
  Thus, the way the variables are grouped and ORed is immaterial.
  (*ii*) $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
  Thus, the way the variables are grouped and ANDed is immaterial.
- This law can be extended to any number of variables.

### 2.3.3 Distributive Laws

- The distributive law allows factoring or multiplying out of expressions.
  (*i*) $A(B + C) = AB + AC$
  (*ii*) $A + BC = (A + B)(A + C)$
- This law is applicable for single variable as well as a combination of variables.

### 2.3.4 Idempotence Laws

- Idempotence means the same value.
  (*i*) $A \cdot A = A$ i.e. ANDing of a variable with itself is equal to that variable only.
  (*ii*) $A + A = A$ i.e. ORing of a variable with itself is equal to that variable only.

  Proof: We may prove either part (i) or part (ii) of this theorem. Suppose we prove part (ii):

$$\begin{aligned} a + a &= (a + a)\,1 \\ &= (a+a)(a+\bar{a}) \\ &= a \cdot a + a\bar{a} + aa + a\bar{a} \\ &= aa + a\bar{a} = a \end{aligned}$$

## Summary

- With '$n$' number of variables the maximum possible minterms or maxterms are equal to '$2^n$'.
- With '$n$' number of variables, the maximum possible logical expression is equal to '$2^{2^n}$' i.e. Boolean expression = $2^{2^n}$.
- For '$n$' number of variable the maximum possible self dual expression = $2^{2^{n-1}}$.
- K-map will provide minimized expressions but not necessarily unique.
- The two K-maps are said to be equal if '1's are placed in the same position on both the maps. Thus, the logical expression is also same.
- The two K-maps are said to be complemented if one K-map has 1's and another K-map has 0's on the same location.

## Student's Assignments

**Q.1** What would be the number of PI and EPI for the following function as, $f(A, B, C) = \Sigma m(0, 1, 5, 7)$.

(a) 3 and 5 (b) 3 and 2
(c) 2 and 3 (d) 2 and 5

**Q.2** Find the minimize form of the logical expression $Z = ABC + ABD + \bar{A}B\bar{C} + CD + B\bar{D}$, by using K-map technique.

(a) $B + CD$ (b) $\bar{B} + CD$
(c) $B + \bar{C}D$ (d) $B + \overline{CD}$

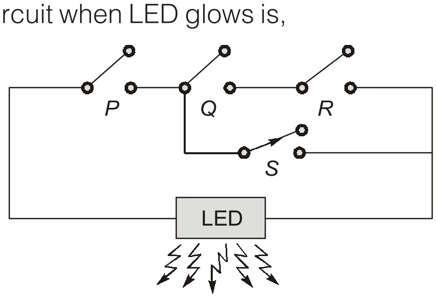
**Q.3** For an 8-bit microprocessor, the maximum possible number of self dual-functions equals to

(a) $(16)^8$ (b) $(16)^{16}$
(c) $(16)^{32}$ (d) $(16)^{64}$

**Q.4** For a 3-variable function given that
$f(A, B, C) = \Pi M(0, 1, 2, 3, 4, 5, 6, 7)$
The minimized Boolean function is

(a) $A\bar{B}C$ (b) $(A + B + C)$
(c) 0 (d) 1

**Q.5** For the switching circuit shown below, taking open as '0' and closed as '1', the expression for the circuit when LED glows is,



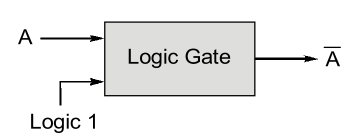

(a) $P + (Q + R)S$ (b) $P(QR + S)$
(c) $P + QR + S$ (d) LED can not glow

**Q.6** A combinational circuit has input $A$, $B$, and $C$ and its K-map is as shown in figure. The output of the circuit is given by

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | 1 | | 1 |
| 01 | 1 | | 1 | |

(a) $(\bar{A}B + A\bar{B})\bar{C}$ (b) $(AB + \bar{A}\bar{B})\bar{C}$
(c) $\bar{A}B\bar{C}$ (d) $A \oplus B \oplus C$

**Q.7** Identify the logic gate shown below:




(a) EX-NOR gate (b) EX-OR gate
(c) NAND gate (d) Either (b) or (c)

**Q.8** The minimal logic expression corresponding to the K-map shown below is

| WX \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | 1 | |
| 01 | 1 | 1 | 1 | |
| 11 | | 1 | 1 | 1 |
| 10 | | 1 | | |

(a) $XZ$
(b) $\bar{W}X\bar{Y} + \bar{W}YZ + W\bar{Y}Z + WXY$
(c) $\bar{W}X\bar{Y} + \bar{W}YZ + W\bar{Y}Z + WX\bar{Y}$
(d) $XZ + \bar{W}YZ + \bar{W}X\bar{Y} + WXY + W\bar{Y}Z$

**Q.9** Consider the following boolean expression:

$F = \left[x + z\{\bar{y} + (\bar{z} + x\bar{y})\}\right]\left[\{\bar{x} + z(x + y)\}\right] = 1$

If $x = 1$ in above expression then the value of $z$ is ________.

**Q.10** A Boolean function of two variables $X$ and $Y$ is defined as follows:

$f(0, 0) = f(0, 1) = f(1, 1) = 1$ and $f(1, 0) = 0$

Assume complement of $X$ and $Y$ are not available, then the minimum cost solution for implement $f$ using 2 input NAND gate and 2 input OR gate is (Total cost) ________. (Let each 2 input OR or NAND gate have 2 unit cost).

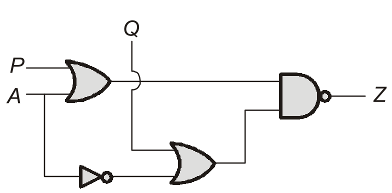
**Q.11** Consider the Boolean function $f(A, B, C, D) = \Sigma m(0, 1, 2, 5, 7, 8, 10, 12, 14, 15)$. Function is having how many number of essential prime implicants?

(a) 2 (b) 3
(c) 4 (d) 5

**Q.12** The Boolean function can be expressed in canonical SOP and POS forms. So, for $Y = A\bar{B} + B\bar{C}$, the SOP and POS forms will be

(a) $Y = \Sigma(0, 2, 4, 6);\ Y = \pi(1, 3, 7)$
(b) $Y = \Sigma(1, 2, 5, 7);\ Y = \pi(0, 3, 4, 6)$
(c) $Y = \Sigma(2, 4, 5, 6);\ Y = \pi(0, 1, 3, 7)$
(d) $Y = \Sigma(1, 2, 4, 5);\ Y = \pi(0, 3, 6)$

**Q.13** The circuit shown below is used to implement the function $z = f(A, B) = \bar{A} + B$. The values of $P$ and $Q$ are




(a) $P = A,\ Q = B$ (b) $P = B,\ Q = \bar{A}$
(c) $P = \bar{B},\ Q = O$ (d) $P = O,\ Q = \bar{B}$

**Q.14** Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

| **List-I** | **List-II** |
|---|---|
| **A.** $(A \oplus B) \oplus (B \oplus C)$ | **1.** $(A \odot C)$ |
| **B.** $AB + \bar{A}C + BC$ | **2.** $(A + B) \odot (A + C)$ |
| **C.** $(A \odot B) \odot (B \odot C)$ | **3.** $AB + \bar{A}C$ |
| **D.** $A + (B \odot C)$ | **4.** $(A \oplus C)$ |
| | **5.** $\bar{A}B \oplus AC$ |

**Codes:**

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 4 | 3 | 1 | 2 |
| (b) | 3 | 4 | 1 | 2 |
| (c) | 2 | 3 | 1 | 2 |
| (d) | 4 | 3 | 5 | 2 |

**Q.15** The maximum number of Boolean expressions that can be formed for the function $f(x, y, z)$ satisfying the relation $f(\bar{x}, y, \bar{z}) = f(x, y, z)$ is ______.

**Q.16** The expression $f = \overline{\bar{A}\bar{B} + \bar{A} + AB}$ when reduces, gives

(a) $\bar{A}$ (b) $\overline{AB}$
(c) $\overline{A + B}$ (d) 0

**Q.17** If the logical expressions of the outputs in the circuits shown in figures *A* and *B* are same, then select the correct combination of signals to be connected to the inputs of multiplexer (i.e. $I_0$, $I_1$, $I_2$, $I_3$) using the codes given below the figures.

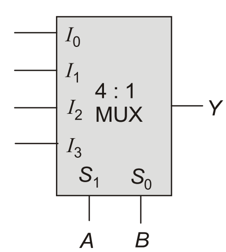



Figure *A*

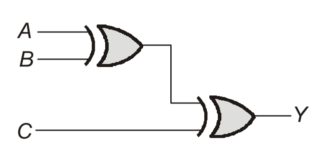



Figure *B*

**Codes:**

| | $I_0$ | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|---|
| (a) | 0 | $C$ | $\bar{C}$ | 1 |
| (b) | $\bar{C}$ | $C$ | $C$ | $\bar{C}$ |
| (c) | $C$ | $\bar{C}$ | $\bar{C}$ | $C$ |
| (d) | 1 | $C$ | $C$ | $\bar{C}$ |

**Q.18** The boolean expression

$F = C(B + C)(A + B + C)$ when simplified will be

(a) $\bar{A} + \bar{B} + \bar{C}$ (b) $C$
(c) $AB + BC + CA$ (d) $ABC$

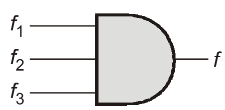
**Q.19** The minimum number of 2 input NAND gates required to implement the Boolean function $F = (\bar X + \bar Y)(Z + W)$ is ______.

**Q.20** For the K-map shown below, the minimized logical expression in SOP form is

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | | 1 |
| 01 | | | | |
| 11 | | | 1 | 1 |
| 10 | 1 | | 1 | 1 |

(a) $\bar A\bar B\bar C + AC + \bar BCD + \bar A\bar B\bar D$
(b) $\bar A\bar B\bar C + \bar B\bar D + AC$
(c) $\bar A\bar B\bar C + AC + \bar A\bar BCD + A\bar B\bar CD$
(d) $\bar A\bar B\bar C + AC + \bar A\bar B\bar D$

**Q.21** Consider the logical functions given below:
$f_1(A, B, C) = \Sigma(2, 3, 4)$
$f_2(A, B, C) = \pi(0, 1, 3, 6, 7)$




If $f$ is logic zero, then maximum number of possible minterms in function $f_3$ are ______.

**Q.22** Consider an $n$-variable boolean function $f(A, B, C, \ldots..)$. If the boolean function is represented as $f(A, B, C, \ldots..) = A + \bar A B + \bar A\bar B C + \ldots.$, then the alternative representation of the above function can be given as
(a) $A + B + C + D \ldots\ldots$
(b) $\bar A + \bar B + \bar C + \bar D + \ldots\ldots$
(c) 1
(d) 0

**Q.23** A three variable boolean function is defined as, $f(A, B, C) = \Sigma m(1, 2, 5, 6)$. If $\overline{f(A, B, C)}$ denotes the compliment of the function $f(A, B, C)$, then the simplified expression of $\overline{f(A, B, C)}$ can be given as
(a) $A \odot B$ (b) $B \oplus C$
(c) $B \odot C$ (d) $A \oplus B$

**Q.24** A logical function is given as $F(A, B, C, D) = \Sigma m\,(0, 4, 5, 10, 11, 13, 15)$. The number of Essential Prime Implicants in the given function will be ______.

**Q.25** A boolean function is represented as $f = XY + XZ$. The POS representation of the function in K-map can be best described by (Assuming no don't care condition is present)

(a)

| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | 0 | 0 | 0 |

(b)

| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 0 | 0 | 0 |
| 1 | | | 0 | |

(c)

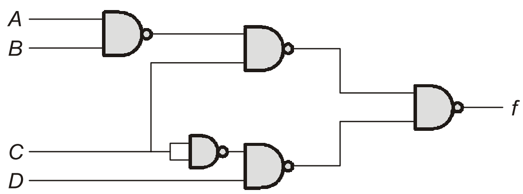
| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | | | |

(d)

| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | | 0 | |
| 1 | 0 | 0 | 0 | 0 |

**Q.26** Given the function $F = AB'C + ABC$ where $F$ is a function in three Boolean variables $A$, $B$ and C consider the following statements:
$S_1: F = \Sigma\,(5, 7)$
$S_2: F = \Sigma\,(0, 1, 2, 3, 4, 6)$
$S_3: F = \Pi\,(5, 7)$
$S_4: F = \Pi\,(0, 1, 2, 3, 4, 6)$
Which of the following is true?
(a) $S_1$ - False, $S_2$ - True, $S_3$ - True, $S_4$ - False
(b) $S_1$ - True, $S_2$ - False, $S_3$ - False, $S_4$ - True
(c) $S_1$ - False, $S_2$ - False, $S_3$ - True, $S_4$ - True
(d) $S_1$ - True, $S_2$ - True, $S_3$ - False, $S_4$ - False

**Q.27** Consider the circuit shown in the figure below:




The circuit consists of NAND gates only. Then which one of the following options represents the output of the logic circuit?

(a) $f = (\bar{A}+\bar{B})C + \bar{C}D$

(b) $f = \overline{(A+\bar{B})\bar{C}} + C\bar{D}$

(c) $f = \overline{\overline{A\bar{B}\bar{C}} + \bar{D}C}$

(d) $f = (\bar{A}+\bar{B})\bar{C} + \bar{D}$

**Q.28** A boolean function is given as,

$$f = B\bar{D} + \bar{B}CD + ABC + AB\bar{C}D + \bar{B}\bar{D}$$

Then the minimum number of two input NAND gates required to implement this function is ________.

**Q.29** Two functions $f_1$ and $f_2$ are given as follows:

$f_1(A, B, C, D) = \Sigma m(0, 1, 5, 6, 7, 8, 15)$

$f_2(A, B, C, D) = \Pi M(1, 3, 5, 9, 10, 11, 12)$

Then the number of essential prime implicants of the function $f_3 = f_1 f_2$ is equal to ________.

**Q.30** Which one of the following statement is true about the cyclic prime implicant K-map function?

(a) The function is having two minimal forms, with one common prime implicant.

(b) The function is having two minimal forms, with two common prime implicants.

(c) The function is having two minimal forms, with no common prime implicants.

(d) The function is having two minimal forms and one essential prime implicant.

**Q.31** Consider the following boolean expression for F.

$$F(A, B, C, D) = \bar{C}D + \bar{A}\bar{C}\bar{D} + AB\bar{C}\bar{D}$$

The minimal sum-of-products for the function F(A, B, C, D) is

(a) $\bar{A}+\bar{B}+\bar{C}+\bar{D}$ (b) $\bar{A}B+\bar{B}C+\bar{C}D$

(c) $\bar{A}\bar{C}+\bar{C}D+B\bar{C}$ (d) $\bar{C}D+\bar{B}+\bar{D}$

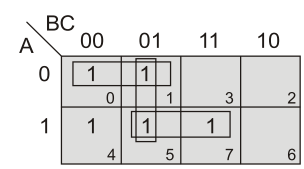
***Answer Key:***

| | | | | |
|---|---|---|---|---|
| **1.** (b) | **2.** (a) | **3.** (c) | **4.** (c) | **5.** (b) |
| **6.** (d) | **7.** (d) | **8.** (b) | **9.** (1) | **10.** (4) |
| **11.** (a) | **12.** (c) | **13.** (d) | **14.** (a) | **15.** (16) |
| **16.** (d) | **17.** (c) | **18.** (b) | **19.** (4) | **20.** (b) |
| **21.** (6) | **22.** (a) | **23.** (c) | **24.** (2) | **25.** (c) |
| **26.** (b) | **27.** (a) | **28.** (6) | **29.** (3) | **30.** (c) |
| **31.** (c) | | | | |

## Student's Assignments Explanations

**1. (b)**

We have, $f(A, B, C) = \Sigma m(0, 1, 5, 7)$




**Prime implicant:** Prime implicant is a smallest possible product term on the given function, removing any one of the literal from which is not possible.

Here, number of prime implicants are 3 that is (0, 1), (1, 5) and (5, 7).

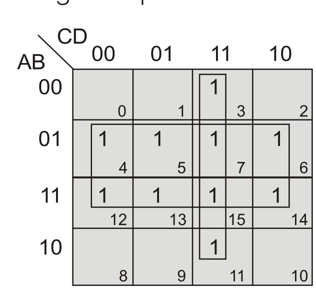
**Essential prime implicant:** Essential prime implicant is a prime implicant it must cover atleast one minterm, which is not covered by any other prime implicant.

Here, number of essential prime implicants are 2 that is (0, 1) and (5, 7).

**2. (a)**

$$Z = ABC + ABD + \bar{A}B\bar{C} + CD + B\bar{D}$$

constructing K-map.




Hence, minimize form is $B + CD$.

**3. (c)**

Self dual functions:

Hence number of self dual functions are $2^{2^7}$ i.e. $2^{128}$

Since, $2^{128}$ can also be written as $(16)^{32}$.

Thus, option (c) is correct.

**4. (c)**

We have, $f(A, B, C) = \Pi M(0, 1, 2, 3, 4, 5, 6, 7)$

Minimizing above expression using K-map:

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 3 | 2 |
| 1 | 4 | 5 | 7 | 6 |

Hence, $f(A, B, C) = 0$

**5. (b)**

Since, $QR$ is attached serially and $S$ is just parallel to $QR$ while $P$ is in serial with former $QR + S$. Thus,

$\Rightarrow$ LED will glow when $P(QR + S)$.

Hence, option (b) is true.

**6. (d)**

Given, K-map is like:

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $_0$ | $1_1$ | $_3$ | $1_2$ |
| 1 | $1_4$ | $_5$ | $1_7$ | $_6$ |

We can see there is no pair, no quad and no octet.

Thus, minimized expression can only be written by writing the expression for each minterm.

Output = $\bar{A}\bar{B}C + A\bar{B}\bar{C} + \bar{A}B\bar{C} + ABC$

**7. (d)**

- When we do EX-OR between A and 1 then, $A \oplus 1 = A' \cdot 1 + A \cdot 0 = A'$
- When we do EX-NOR between A and 1 then, $A \odot 1 = A \cdot 1 + A' \cdot 0 = A$
- When we do NAND operation between A and 1 then, $A \uparrow 1 = \overline{A \cdot 1} = \bar{A}$.

**8. (b)**

| WX \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | 1 | |
| 01 | 1 | 1 | 1 | |
| 11 | | 1 | 1 | 1 |
| 10 | | 1 | | |

$$Z = \bar{W}X\bar{Y} + WXY + \bar{W}YZ + W\bar{Y}Z$$

**9. (1)**

Given boolean expression

$$\left[x + z\{\bar{y} + (\bar{z} + x\bar{y})\}\right]\left[\{\bar{x} + z(x + y)\}\right] = 1$$

Put $x = 1$ and $\bar{x} = 0$

$$\underbrace{\left[1 + z\{\bar{y} + (\bar{z} + \bar{y})\}\right]}_{1}\underbrace{\left[0 + z(1 + y)\right]}_{z} = 1$$

So minimum expression is $[1][z([1]] = 1$

Then to satisfy equation $z$ must be 1.

**10. (4)**

The Boolean function of two variables $X$ and $Y$ are $f(0, 0) = f(0, 1) = f(1, 1) = 1$ and $f(1, 0) = 0$

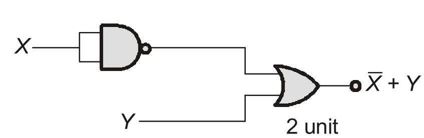
Truth table is:

| *X* | *Y* | *F* |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Function $f$ boolean expression is

$$= \bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + XY = \bar{X} + Y$$

Since function implement using 2 input NAND gate or OR gate.




So total cost = (2 + 2) unit = 4 unit

**11. (a)**

EPI = Essential Prime Implicant [which cover a minterm not covered by any other prime implicants]

NEPI = Non Essential Prime Implicant. Number of EPI's = 2, number of NEPI's = 5.