RPSC 2024

Rajasthan Public Service Commission

Assistant Engineer

CIVIL ENGINEERING

Structural Analysis

MADE EASY
PUBLICATIONS

Note: This book contains copyright subject matter to MADE EASY Publications, New Delhi. No part of this book may be reproduced, stored in a retrieval system or transmitted in any form or by any means. Violators are liable to be legally prosecuted.

CONTENTS

■■■■

DETERMINACY AND INDETERMINACY

1

STATICALLY DETERMINATE STRUCTURES

1. Conditions of equilibrium are sufficient to analyse the structure.
2. Bending moment and shear force is independent of the cross-sectional areas of the components and flexural rigidity of the material.
3. No stresses are caused due to temperature changes.
4. No stresses are caused due to lack of fit.

STATICALLY INDETERMINATE STRUCTURES

1. Additional compatibility conditions are required.
2. Bending moment and shear force depends upon the cross-sectional area and EI of the material.
3. Stresses are caused due to temperature variation.
4. Stresses are caused due to lack of fit.

DEGREE OF INDETERMINACY

Degree of Indeterminacy can be Divided into

1. Static indeterminacy which can be classified as
   (a) External indeterminacy
   (b) Internal indeterminacy
2. Kinematic indeterminacy

1. Static Indeterminacy (D_S)

- Externally redundant structures are those which have redundant reactive restraint (redundant reactions), however internally redundant are those structures which have redundant members and are over stiff and the stress distribution depends not only on loading but also on the relative dimensions of their members and on properties of materials of which members are made.
- In general three reaction components are necessary for the external stability of a plane structure. This condition of three reaction components is necessary but not always sufficient. The arrangement of the three reaction components is very important from stability point of view. For example - If the lines of action of the three components are concurrent, the structure is externally unstable because the point of concurrency becomes the instantaneous centre of rotation giving a critical configuration. Similarly a structure will also be unstable if the three reaction components have parallel lines of action, since the structure doesn't have any resistance to horizontal motion.

(a) Degree of External Indeterminacy

- A structure is unstable if the total number of reaction components (R) are less than the total number of equilibrium condition or equations available (r).
- If number of equilibrium conditions or equations are equal to the number of reaction components, the structure is externally determinate.
- If the number of reaction components are more than the available equilibrium conditions or equations, the structure is statically indeterminate externally.
- The degree of external indeterminacy is given by

$$D_{SE} = r_e - r$$

© Copyright: Subject matter to MADE EASY Publications, New Delhi.

where,

D_{SE} = degree of external redundancy

r_e = Number of unknown reaction components

r = Total number of conditions or equilibrium equations available

r = '3' in general (for plane structures), unless additional conditions are available such as link or hinge present any where in the structure.

Example:

Here, $r_e = 6$, r = 3

Therefore, $D_{SE} = 6 - 3 = 3$

- For general loading system, a fixed beam is statically indeterminate to 3rd degree. However for vertical loading on the beam, the total reaction components are 4 only and two equilibrium equations are available hence beam is indeterminate by 2nd degree.
- It should be noted that in the case of continuous beams the shear and bending moment at any point in the beam are readily known once the reaction components are determined, hence these beams are statically determinate internally. The degree of indeterminacy of a beam is therefore equal to its external indeterminacy.

(b) Degree of Internal Indeterminacy

- A plane pin jointed two dimensional truss or a frame is said to be statically determinate internally if it has members given by

$$m = 2j - r$$

Where, j = number of joints

r = number of conditions or equations available

- If the number of members in a frame are more than given by equation, then the structure is said to be internally redundant.
- If the members are lesser than 'm' then the structure is unstable.
- The degree of internal redundancy (D_{SI}) is given by

$$D_{SI} = m - (2j - r)$$
$$= m - (2j - 3)\text{......(for plane frames)}$$

- Total Static Indeterminacy

$$D_S = D_{SE} + D_{SI} = (r_e - r) + m - (2j - r)$$
$$D_S = (m + r_e) - 2j$$

Where,

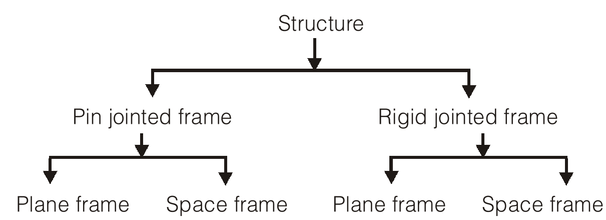
D_S = degree of static indeterminacy

m = Number of members

r_e = Number of unknown external reactions

j = Number of joints

- A Structure May be of Classified as



Total Static Indeterminacy

If, m = total number of members

r_e = number of unknown external reactions

j = total number of joints

(i) Pin jointed two dimensional (plane) frame

$$D_S = m + r_e - 2j$$

Do you know?

Truss member carry only axial forces (internal reactions) and each joint is subjected to only two equilibrium conditions. ($\Sigma F_x = 0$ and $\Sigma F_y = 0$). If the truss consist of m-members then total internal reactions will be m and if the truss consist j-joint then total available equilibrium condition will be 2j but out of these 2j equilibrium conditions, three equations are utilized to determine external reactions therefore total available equilibrium conditions for internal forces is (2j – 3).

We know that total internal static indeterminacy is the difference of total internal reactions and total available equilibrium conditions.

So we can say $D_{si} = m - (2j - 3)$. Also for 2-D support total external static indeterminacy $D_{se} = (r_e - 3)$. Therefore total degree of static

© Copyright: Subject matter to MADE EASY Publications, New Delhi. No part of this book may be reproduced or utilised in any form without the written permission.

indeterminacy is sum of external as well as internal static indeterminacy. Therefore $D_s = (m + r_e - 2j)$

(ii) Pin jointed space frame (3 dimensional):

$$D_S = m + r_e - 3j$$

Do you know?

For 3-D truss (pin-jointed space frame) system each joint is subjected to three equilibrium conditions ($\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma F_z = 0$).

(iii) Rigid jointed plane frame (2 dimensional):

$$D_S = 3m + r_e - 3j$$

- When hybrid joints are provided

$$D_S = 3m + r_e - 3j - r_r$$
$$= 3m + r_e - 3(j + j') - \Sigma(m_{j'} - 1)$$

Where,

j = Total number of rigid joints

j′ = Total number of hybrid joints

$m_{j'}$ = no. of members meeting at the hybrid joint

r_r = no. of released reaction

Do you know?

For rigid jointed plane flame (2-D). Each joint is subjected to three equilibrium conditions ($\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_z = 0$)

(iv) Rigid jointed space frame (3 dimensional)

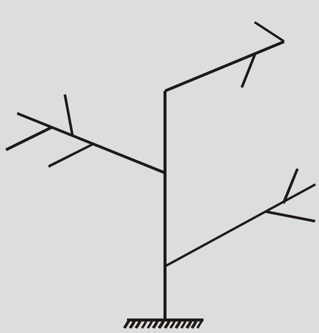
$$D_s = 6m + r_e - 6j$$

- When hybrid joints are provided

$$D_s = 6m + r_e - 6j - r_r$$
$$= 6m + r_e - 6(j + j') - \Sigma 3(m_{j'} - 1)$$

Do you know?

A rigid jointed frame is called internally determinate if its members form an open configuration like a tree structure.



© Copyright: Subject matter to MADE EASY Publications, New Delhi. No part of this book may be reproduced or utilised in any form without the written permission.

(v) Alternative method for rigid jointed closed Frames:

- Plane frame (2D)

$$D_{SE} = r_e - r$$
$$D_{SI} = 3C$$
$$= 3C - r_r \text{ .... (when hybrid joints are provided)}$$

where, $r_r = \Sigma(m_{j'} - 1)$

- Space frame (3D)

$$D_{SE} = r_e - r$$
$$D_{SI} = 6C$$
$$= 6C - r_r \text{ .... (when hybrid joints are provided)}$$

where, $r_r = \Sigma 3(m_{j'} - 1)$

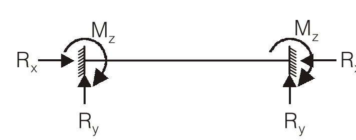
C = number of closed loops

2. Kinematic Indeterminacy (D_K)

- If the number of unknown displacement components are greater than the number of compatibility equations, than for these structures, additional equations based on equilibrium must be written in order to obtain sufficient number of equations for the determination of all the unknown displacement components. The number of these additional equations necessary is known as degree of kinematic indeterminacy or degree of freedom of the structure.

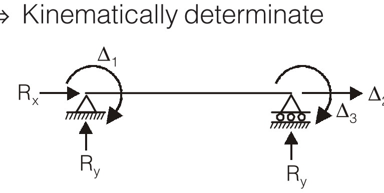
For Example:

- A fixed beam is kinematically determinate and a simply supported beam is kinematically indeterminate.



Degree of freedom = 0

⇒ Kinematically determinate



Degree of freedom = 3

⇒ Kinematically indeterminate

1. Each joint of pin jointed plane frame has 2 degrees of freedom. (Δ_x and Δ_y)
2. Each joint of pin jointed space frame has 3 degrees of freedom. (Δ_x, Δ_y and Δ_z)
3. Each joint of rigid jointed plane frame has 3 degrees of freedom. (Δ_x, Δ_y and θ_z)
4. Each joint of rigid jointed space frame has 6 degrees of freedom (Δ_x, Δ_y, Δ_z, θ_x, θ_y and θ_z)

- Degree of kinematic indeterminacy of
  - (a) Pin jointed plane frame (2 dimensional)
    $$D_k = 2j - r_e$$
  - (b) Pin jointed space frame (3 dimensional)
    $$D_k = 3j - r_e$$
  - (c) Rigid jointed plane frame
    $$D_k = 3j - r_e$$
  - (d) Rigid joint space frame
    $$D_k = 6j - r_e$$
  - (e) Continuous beams
    $$D_k = 3j - r_e$$

  where, j = number of joints and

  r_e is the total external reactions

Do you know?

In a pin-jointed plane frame each joint is subjected to two degree of freedom (Δx and Δy) therefore j-joint, is subjected to 2j degree of freedom. We know that kinetic indeterminaly is the difference of total possible displacement component and available reaction component. Therefore for a 2-D pin jointed plane frame :

$$D_k = 2j - r_e$$

Special Cases

(i) If a member is axially rigid then axial displacement for that member at one of the joint is not available, hence degree of freedom will be reduced.

(ii) If rigid frames carry hybrid joints, their internal reactions are released at hybrid joints. Hence, degree of freedom is increased. Thus,

(a) For plane hybrid frames with extensible members

$$D_K = [3\,(j + j') - r_e] + \Sigma(m_{j'} - 1)$$

(b) For plane hybrid frames with inextensible members

$$D_K = [3\,(j + j') - r_e] + \Sigma(m_{j'} - 1) - m$$

(c) For 3D hybrid frames with extensible members

$$D_K = [6\,(j + j') - r_e] + \Sigma 3(m_{j'} - 1)$$

(d) For 3D hybrid frames with inextensible members

$$D_K = [6\,(j + j') - r_e] + \Sigma 3(m_{j'} - 1) - m$$

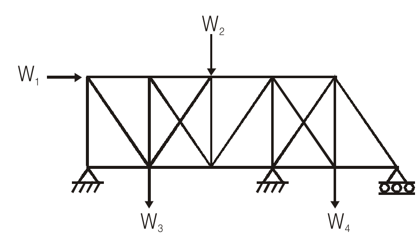
Here m = total numbers of inextensible members

Do you know?

If some of the joints in rigid frames are hybrid (hinged/shear roller etc.) then degree of freedom is increased.

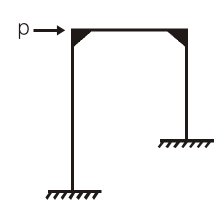
Practice Questions : Level-1

Q.1 The degree of static indeterminacy of the pin-jointed plane frame shown in figure is



(a) 1 (b) 2
(c) 3 (d) 4

Q.2



The portal frame shown in the above figure is statically indeterminate to the

(a) first degree (b) second degree
(c) third degree (d) None of the above

Q.3 A perfect plane frame having n number of members and j number of joints should satisfy the relation

(a) $n < (2j - 3)$ (b) $n = (2j - 3)$
(c) $n > (2j - 3)$ (d) $n = (3 - 2j)$

© Copyright: Subject matter to MADE EASY Publications, New Delhi. No part of this book may be reproduced or utilised in any form without the written permission.