RPSC 2024

Rajasthan Public Service Commission

Assistant Engineer

CIVIL ENGINEERING

Theory of Structures (SOM)

MADE EASY PUBLICATIONS

CONTENTS

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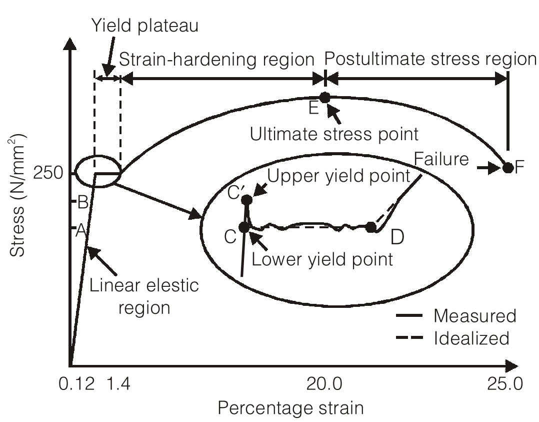
PROPERTIES OF METALS, SIMPLE STRESS-STRAIN AND ELASTIC CONSTANTS

1

STRESS-STRAIN CURVE

Simple Tension Test for Mild Steel



- **A is Limit of proportionality:** Beyond A linear variation ceases.
- **B is Elastic limit:** The maximum stress up to which a specimen regains its original length on removal of applied load. For mild steel B is very near to A. However, for other materials B may be greater than A.
- **C′ is Upper yield point:** The magnitude of the stress corresponding to C′ depends on the cross-sectional area, shape of the specimen and the type of the equipment used to perform the test. It has no practical significance.
- **C is Lower yield point:** The stress at C is the yield stress (σ_y) with a typical value of σ_y = 250 N/mm^2 for mild steel. The yielding begins at this stress.
- **CD represents perfectly plastic region:** It is the strain which occurs after the yield point C, without any increase in stress. The strain corresponding to point D is about 1.4% and corresponding to C is about 0.12% for mild steel. Hence, plastic strain is 10 to 15 times of elastic strain.
- **DE represents strain hardening:** In this range further addition of stress gives additional strain. However, strain increases with faster rate in this region. The material in this range undergoes change in its atomic and crystalline structure, resulting in increased resistance to further deformation. This portion is not used for structural design.
- **E is Ultimate point:** The stress corresponding to this point is ultimate stress (σ_y) and the corresponding strain is about 20% for mild steel.
- **F is fracture point:** Stress corresponding to this is called breaking stress and strain is called fracture strain. It is about 25% for mild steel.
- Region between E and F is the necking region, in which area of cross-section is drastically decreased.

Do you know?

1. Strain that occurs before the yield point is called elastic strain and that which occurs after yield point with no increase in stress is called plastic strain. For mild steel, plastic strain is 10 to 15 times of elastic strain.
2. Ideal curve for tension is shown in the figure. However, actual behaviour is different and indicates apparently reduced yield stress in compression for mild steel. The divergence between tension and compression results is explained by Bauschinger and is called **Bauschinger effect**.

3. The stress defined as the ratio of load to original area (A_o) is known as 'engineering stress' or 'stress' or 'nominal stress'.

$$\text{Engineering stress or Nominal stress} = \frac{P}{A_o}$$

- On the contrary when changing area is taken into account, the ratio of load to actual area (A) is called 'true stress'.

$$\text{True stress} = \frac{P}{A}$$

PROPERTIES OF METALS

1. Ductility

- Ductility is the property by which material can be stretched. Large deformations are thus possible in ductile materials before the absolute failure or rupture takes place. Some of the examples are mild steel, aluminium, copper, manganese, lead, nickel, brass, bronze, monal metal etc.

2. Brittleness

- Brittleness is the lack of ductility i.e. material cannot be stretched. In brittle materials, failure takes place with a relatively smaller deformation. This property is undesirable. For brittle materials fracture point and ultimate points are same, and after proportional limit very small strain is seen. Some of the examples are cast iron, concrete and glass.
- To distinguish between these two types of materials, materials with strain less than 5% at fracture point are regarded as brittle and those having strains greater than 5% at fracture point are called ductile (this value for mild steel at fracture is about 25%).

3. Malleability

- The property by which a material can be uniformly extended in a direction without rupture. A malleable material possess a high degree of plasticity. This property is of great use in operations like forging, hot rolling, drop (stamping) etc.

4. Toughness

- The property which enables material to absorb energy without fracture. This property is very desirable in case of cyclic loading or shock loading.
- The **modulus of toughness** is measured as area under entire stress-strain curve and is the energy absorbed by material of the specimen per unit volume upto fracture stage.

$$\text{Modulus of toughness} = \left[\frac{\sigma_y + \sigma_u}{2}\right]\epsilon_f$$

Where, σ_y = Yield tensile strength
σ_u = Ultimate tensile strength
ϵ_f = Strain at fracture point

- The modulus of toughness will depend upon ultimate tensile strength and strain at failure (fracture strain). Hence the material which is very ductile will exhibit a higher modulus of toughness as the case with mild steel.
- The **modulus of resilience** is the maximum elastic energy per unit volume that can be absorbed without attaining plastic stage.

$$\text{Modulus of resilience (u)} = \frac{\sigma_y^2}{2E}$$

- **The modulus of resilience** depends upon yield strength and hence a material with higher yield strength will have higher modulus of resilience.
- Higher toughness is a desirable property in materials used for gears, chains, crane hooks, freight car etc. Higher resilience is desirable in springs.

5. Hardness

- Hardness is defined as the resistance to indentation or scratching or surface abrasion.
- There are two methods of hardness measurement:
  1. Scratch hardness - commonly measured by Mohs' test.
  2. Indentation hardness (abrasion) measured by
     - Brinell hardness method

- Rockwell hardness
- Vickers hardness
- Knoop hardness
- It should be noted that ductile materials are tough and brittle materials are hard.

6. Fatigue

- It has been found that material behave differently under the static loading and dynamic loading.
- The behaviour of material under variable loads (dynamic loads) is referred to as **fatigue**. In recent past several failures of structures have been noted due to fatigue.
- Factors affecting fatigue are:
  1. Loading conditions
  2. Frequency of loading
  3. Corrosion
  4. Temperature
  5. Stress concentration

7. Creep and Stress Relaxation

- At any temperature, a material will progressively deform with the passage of time under constant loading, even if the stress is below yield point. This phenomenon is called creep. However, such deformation is negligibly small at lower temperature.
- At higher temperature, due to greater mobility of atoms, most of the materials loose their strength and elastic constants also get reduced. Hence, greater deformations at elevated temperature results even under constant loading. Therefore, creep is more pronounced at higher temperature, and thus it must be considered for design of engines and furnaces.
- The temperature at which the creep becomes very appreciable is half of the melting point temperature on absolute scale and is known as **homologous temperature**.
- If a wire of metal is stretched between two immovable supports, so that it has an initial tension stress σ_o. The stress in the wire gradually diminishes, eventually reaching a constant value. This process, which is a manifestation of creep, is called **stress relaxation**.

Do you know?
For metallic minerals creep becomes an important consideration at half of the melting point temperature on absolute scale.

SIMPLE STRESS AND STRAIN

- It is the internal resistance offered by the body against external loading or deformation. Stresses may be of following type:
  1. Direct stresses or normal stresses which may be tensile or compressive
  2. Shear or Tangential stresses
  3. Transverse or Bending stresses
  4. Torsional or Twisting stresses
- For direct stresses, if area under consideration is original area, then it is known as engineering stress or nominal stress or simply stress. But, if the area taken is actual area, then stress is known as true stress.

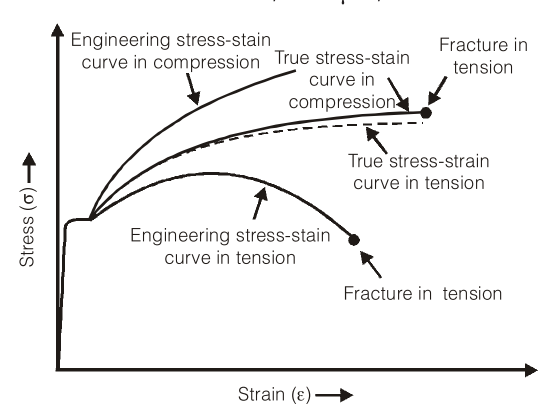
$$\text{Nominal Stress} = \frac{P}{A_o}$$

$$\text{True Stress} = \frac{P}{A}$$

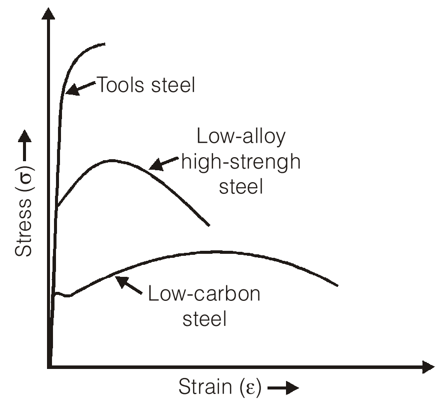
where,

A_o = Original area of specimen
A = Actual area of specimen

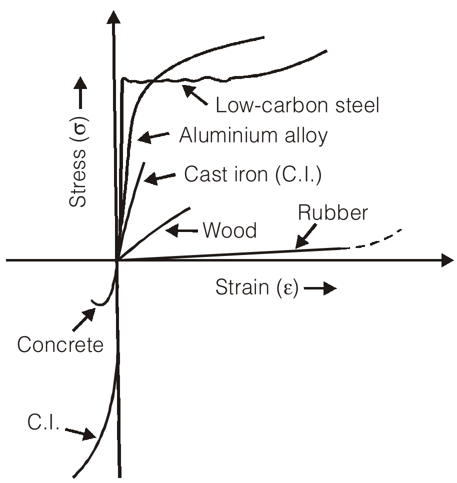
- When a prismatic bar is subjected to axial load, it undergoes a change in length. This change is length is usually called deformation. The deformation per unit length of the bar is termed as strain. Since strain is deformation per unit length, it is a dimensionless quantity. But sometimes in practice, strain is recorded in forms such as mm/m or µm/m etc.



Tension and compression monotonic Stress-strain diagrams



Tensile stress-strain diagram for different steels



Stress-strain diagrams for different materials

1. Hooke's Law

- It states that within elastic limit stress for most of the metals is proportional to strain. i.e.

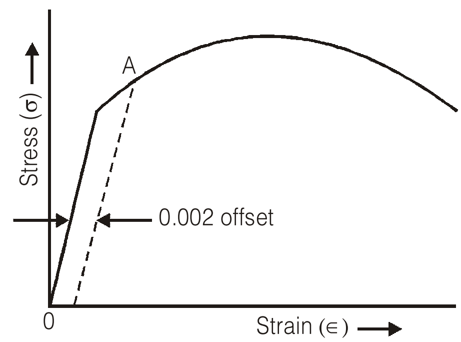
  Stress ∝ strain

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant} = E$$

  Where, E = Young's modulus of elasticity
- Hooke's law is valid up to limit of proportionality. However for mild steel proportional limit and elastic limit are almost equal. But for other metals and materials elastic limit may be higher than proportional limit. e.g. rubber.
- The slope of stress-strain curve is called modulus of elasticity (E). The modulus of elasticity (E) is the constant of proportionality which is defined as the intensity of stress that causes unit strain. Thus, modulus of elasticity (E) has the units same as units of stress.

2. Proof Stress

- When a material such as aluminium does not have an obvious yield point and yet undergoes large strains after the proportional limit is exceeded, an arbitrary yield stress may be determined by the offset method.
- A line parallel to initial linear part is drawn, which is offset by some standard amount of strain such as 0.2%. The intersection of the off set point (A) defines the yield stress or off set yield stress, which is slightly above the proportional limit and is called **proof stress**.



Stress-strain curve for aluminium

ELONGATION OF BARS

1. A bar of uniform cross-sectional area

$$\Delta L = \frac{PL}{AE}$$

2. Non-uniform bar

$$\Delta L = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \ldots\ldots\right]$$

3. Tapering bar of circular cross-section whose diameter changes from 'd_1' at one end to 'd_2' at the other end

$$\Delta L = \frac{4PL}{\pi E d_1 d_2}$$

   where P is the load applied and E is the Young's modulus of elasticity. L is the length of the specimen.

4. Tapering bar of rectangular cross-section having uniform thickness 't' but width of the bar varies from 'a' to 'b'.

determining similar to composite bars. The final stress will be algebraic sum of ((iv) and (v).

Practice Questions : Level-1

Q.1 The ratio of Young's modulus to modulus of rigidity for a material having Poisson's ratio 0.2 is

(a) $\frac{12}{5}$ (b) $\frac{5}{12}$

(c) $\frac{5}{14}$ (d) $\frac{14}{5}$

Q.2 A straight wire 15 m long is subjected to tensile stress of 2000 kgf/cm^2. Elastic modulus is 1.5×10^6 kgf/cm^2. Coefficient of linear expansion for the material is 16.66×10^{-6}/°F. The temperature change (in °F) to produce the same elongation as due to the 2000 kgf/cm^2 tensile stress in the material is:

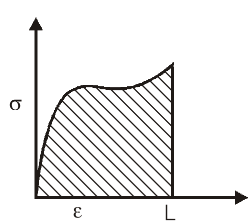
(a) 40 (b) 80

(c) 120 (d) 160

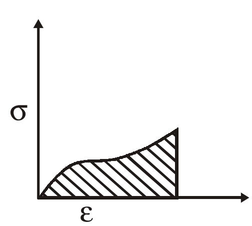
Q.3 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I **List-II**

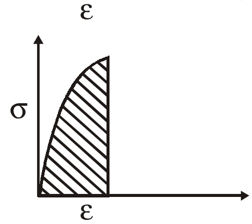
1. Soft & Weak **A.**



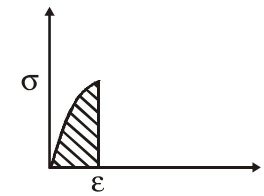
2. Hard brittle **B.**



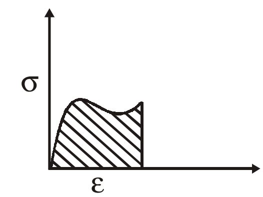
3. Hard strong **C.**



4. Soft tough **D.**



5. Hard tough **E.**

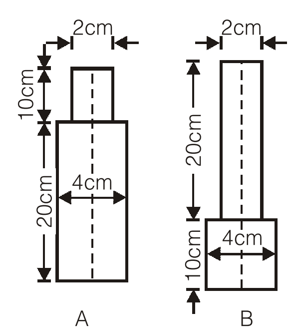


Codes:

	1	2	3	4	5
(a)	E	D	C	B	A
(b)	A	B	C	D	E
(c)	E	C	B	D	A
(d)	A	D	C	B	E

Q.4 Two similar round bars A and B are each 30 cm long as shown in the given figure. The ratio of strain energies stored by the bars A and B, $\frac{U_B}{U_A}$ is

(Assume both bars to be axially loaded equally)



(a) 3/2 (b) 1.0

(c) 5/8 (d) 2/3

Q.5 Match **List-I** (Various test stages) with **List-II** (Observation) and select the correct answer using the codes given below the lists:

List-I	List-II
A. I-Stage	1. Yield point
B. II-Stage	2. Limit of proportionality
C. III-Stage	3. Breaking stress
D. IV-Stage	4. Ultimate stress