MADE EASY PUBLICATIONS

POSTAL BOOK PACKAGE 2025

CIVIL ENGINEERING

CONVENTIONAL Practice Sets

CONTENTS

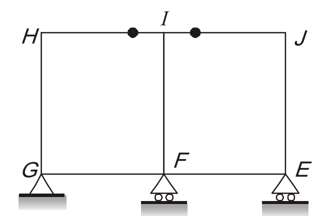
STRUCTURAL ANALYSIS

1
CHAPTER

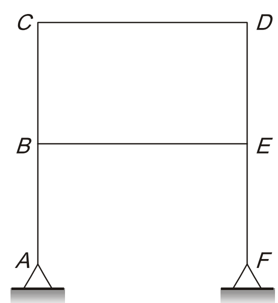
ILD & Rolling Loads and Determinacy

Q.1 (i) What do you understand by static indeterminacy and kinematic indeterminacy of a 2-*D* framed structure? Explain with an example of a fixed end beam.

(ii) The degree of static indeterminacy of the rigid frame having two internal hinges as shown in the figure below is



(iii) Consider the frame shown in the figure given below.



If the axial and shear deformations in different members of the frame are assumed to be negligible, then what would be the reduction in the kinematic indeterminacy.

Solution:

(i) Static indeterminacy (D_S): Those structures which cannot be analysed by using condition of static equilibrium alone are called indeterminate structures. To analyse these indeterminate structures extra equilibrium condition are required, called compatibility conditions and numbers of compatibility conditions needed to analyse structure is known as degree of static indeterminacy.

D_S = Total no. of reactions present (Both internal and external) – No. of available equilibrium equations.

For 2-*D* Rigid Frame: In two dimensional rigid member, each member has three internal reactions (viz. R_x, R_y and M_z) and at each joint three equilibrium conditions (viz. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_x = 0$) are available

Let there are r_e number of external support conditions.

∴ Total no. of reaction present,

$$R = \text{External reaction + Internal reaction}$$

$$R = r_e + 3\text{m}$$

and total no. of available equilibrium conditions,

$$E = 3j$$

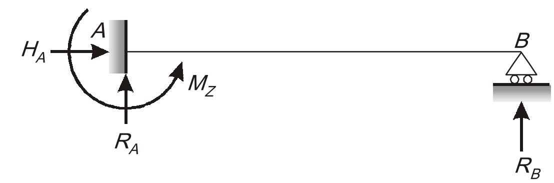
∴ $$D_S = R - E$$

$$D_S = r_e + 3\text{ m} - 3j \qquad \text{... when all joint are rigid}$$

$$D_S = r_e + 3\text{m} - 3j - r_r \qquad \text{... when some joints are hybrid}$$

where r_r = Number of released reactions

Example:



Here, $r_e = 3 + 1 = 4$

$m = 1$

$j = 2$

$D_S = r_e + 3m - 3j$

$= 4 + 3 \times 1 - 3 \times 2$

$= 1$ (indeterminate to 1st degree)

Kinematic Indeterminacy (D_k): It refers to the total no. of available degree of freedom at all joints. It is equal to total no. of unrestrained displacement component at all joints.

D_k = Total degree of freedom at all joints – degree of freedom restrained by supports

2-*D* Rigid Frames: At each joint there are three degree of freedom (*viz.* Δ_x, Δ_y and θ_z). Hence at all joint there will be $3j$ degree of freedoms. But at supports displacements are not available in the direction of reaction component.

$\therefore$ D_k = unrestrained displacement component

$\Rightarrow$ $\mathbf{D_k = 3j - r_e}$ ... members are axially flexible

$\mathbf{D_k = 3j - r_e - m''}$ ...m'' member are axially rigid

Example:

$D_k = 3j - r_e$

Here, $j = 2,\ r_e = 3$

$\therefore$ $D_k = 3 \times 2 - 3$

$\Rightarrow$ $D_k = 3$ (i.e., θ_B, Δ_{HB} & Δ_{VB})

(ii) **Method-I: (By Formula)**

The degree of **static indeterminacy** for a rigid hybrid frame is given by ,

$D_s = 3\,m + r_e - r_r - 3\,(j + j')$

Where, m = total number of members = 9

r_e = total number of external reactions

$= 2 + 1 + 1 = 4$

r_r = total number of released reactions at hybrid joint

$= \Sigma(m_j - 1) = (2 - 1) + (2 - 1) = 2$

j = total number of rigid joints = 6

j' = total number of hybrid joints = 2

$\therefore$ $D_S = (3 \times 9) + 4 - 2 - 3\,(6 + 2)$

$= 27 + 4 - 2 - 24 = 31 - 26 = 5$

Method-II: (By Loop Method)

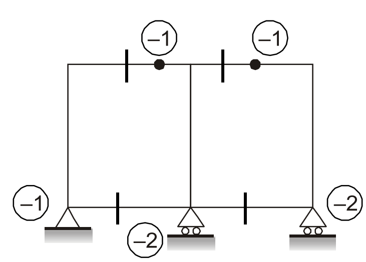
$\mathbf{D_{si} = 3C - r_r}$ where C = no. of closed loops

$= 3 \times 2 - 2 = 4$

$D_{se} = r_e - 3 = 1$

$D_s = D_{si} + D_{se} = 4 + 1 = 5$

Method-III:



D_s = 3 × Number of cuts to open-closed loops
– Reaction added to make stable cantilevers

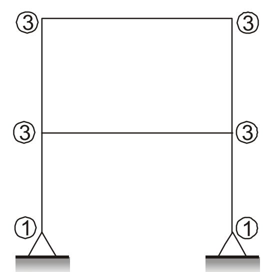
⇒ $D_s = (3 \times 4) - 1 - 1 - 2 - 2 = 5$

(iii) D_k (when inextensible) = D_k (when extensible) – Number of axially rigid members.

⇒ D_k(when extensible) – D_k(when inextensible)
= Number of axially rigid members
= 6

Key Point:

DOF of joints is shown below.

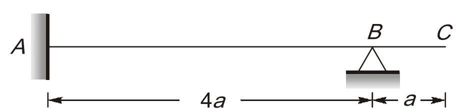


D_k (when extensible) = 3 + 3 + 3 + 1 + 1 = 14

D_k (when inextensible) = 14 − 6 = 8

Note: Reduction in $D_k = 6(\theta_A, \theta_B, \theta_C, \theta_D, \theta_E, \theta_F)$.

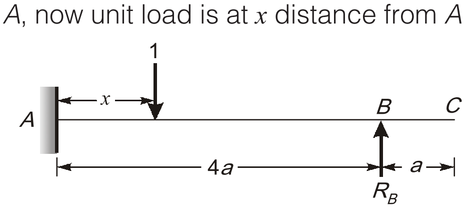
Q.2 State Muller Breslau principle. Derive the equation for influence line for the reaction R_B for the beams shown in the figure. EI is constant throughout.



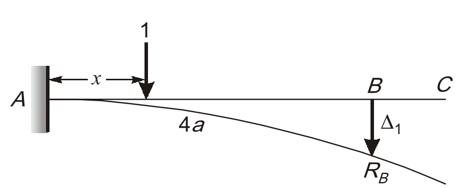
Solution:

Muller Breslau Principle: "The ILD for any stress function in a structure is represented by it's deflected shape obtained by removing the restraint offered by the stress function (SF, BM and reaction) and introducing a directly generalized unit displacement in the positive direction of that stress function".

Assume unit load travels from A, now unit load is at x distance from A

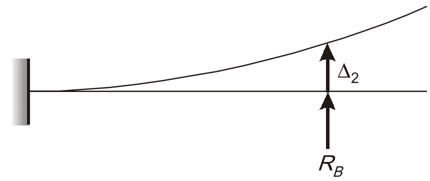


Case (i) $0 \le x \le 4a$



$$\Delta_1 = \frac{1 \cdot x^3}{3EI} + \frac{1 \cdot x^2}{2EI}(4a - x)$$ {Downward}

Where Δ_1 is deflection at B due to unit load.



$$\boxed{\Delta_2 = \frac{R_B(4a)^3}{3EI}} = \frac{64R_Ba^3}{3EI}$$

Where Δ_1 is deflection at B due to reaction R_B.
Since joint B is hinged, hence net deflection is zero

$$\boxed{\Delta_1 = \Delta_2}$$

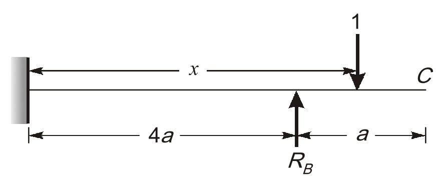
$$\frac{x^3}{3EI} + \frac{x^2(4a-x)}{2EI} = \frac{64R_Ba^3}{3EI}$$

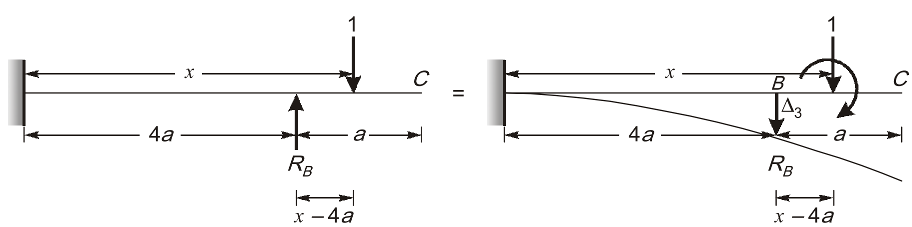
$$R_B = \frac{3EI}{64a^3}\left[\frac{x^3}{3EI} + \frac{2ax^2}{EI} - \frac{x^3}{2EI}\right]$$

$$= \frac{3EI}{64a^3}\left[\frac{2ax^2}{EI} - \frac{x^3}{6EI}\right] = \frac{3x^2}{32a^2} - \frac{1}{128}\frac{x^3}{a^3}$$

$$R_B = \frac{x^2(12a-x)}{128a^3} \qquad \text{(when } 0 \le x \le 4a\text{)}$$

Case (ii) $4a \le x \le 5a$

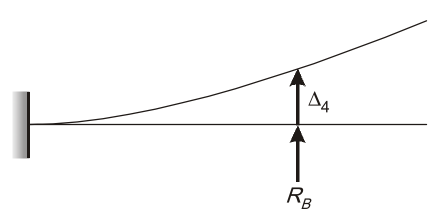




Deflection at B due to unit load at x is same as the deflection at x due to point load at B.

$$\therefore \qquad \boxed{\Delta_3 = \frac{1\cdot(4a)^3}{3EI} + \frac{(4a)^2(x-4a)\cdot 1}{2EI}}$$

When Δ_3 is the deflection at B due to unit load.



$$\boxed{\Delta_4 = \frac{R_B(4a)^3}{3EI}} = \frac{64R_Ba^3}{3EI}$$

When Δ_4 is the deflection at B due to reaction R_B.
Since joint B is hinged, hence net deflection is zero.

$$\boxed{\Delta_3 = \Delta_4}$$

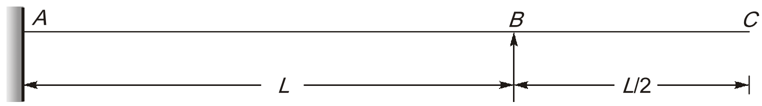
$$\frac{1\cdot(4a)^3}{3EI} + \frac{1\cdot(x-4a)(4a)^2}{2EI} = \frac{64R_Ba^3}{3EI}$$

$$R_B = \frac{3EI}{64a^3}\left[\frac{64a^3}{3EI} + \frac{16a^2(x-40)}{2EI}\right] = 1 + \frac{3EI \times 16a^2(x-4a)}{64a^2 \times 2EI}$$

$$= 1 + \frac{3}{8a}(x - 4a) = 1 + \frac{3x}{8a} - \frac{3}{2}$$

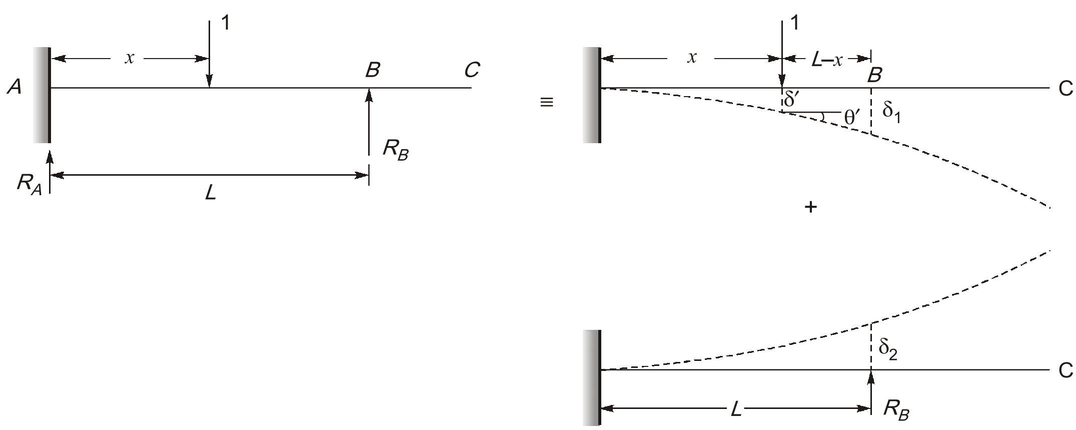
$$R_B = \frac{3x}{8a} - 0.5 \qquad \text{when } 4a \le x \le 5a$$

Q.3 A beam ABC as shown in below figure is fixed at A and is simply supported at B. Draw the qualitative diagram for influence line of vertical reaction at A.



Solution:

Case-I: When unit load lies between A and B (i.e. $\Delta \le x \le L$)



From unit load method:

$$\boxed{\delta_1 = \delta' + \theta'(L - x)}$$

$$= \frac{1.x^3}{3EI} + \frac{1.x^2}{2EI}(L - x) = \frac{x^2(3L - x)}{6EI}$$

$$\delta_2 = R_B \frac{L^3}{3EI}$$

$$\delta_B = \delta_1 - \delta_2 = 0$$

$$\Rightarrow \quad \delta_1 = \delta_2$$

$$\Rightarrow \quad R_B \frac{L^3}{3EI} = \frac{x^2(3L - x)}{6EI}$$

$$\Rightarrow \quad R_B = \frac{x^2(3L - x)}{2EI}$$

$$\therefore \quad R_A = 1 - R_B \qquad (\because \Sigma F_y = 0, \Rightarrow R_A + R_B = 1)$$

$$= \left\{1 - \frac{x^2(3L - x)}{2L^3}\right\} \qquad \text{...(Cubic)}$$

$$\frac{\partial R_A}{\partial x} = \frac{-2x(3L) - 3x^2}{2L^3}$$

$$= \frac{-3x(2L - x)}{2L^3} < 0 \qquad \text{...(decreasing slope)}$$

Hence, ILD for R_A between A and B is cubic with a decreasing slope.