CHEMICAL ENGINEERING

Plant Design and Economics

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Plant Design and Economics

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Plant Design and Economics

CHAPTER

1

Cost Estimation

LEARNING OBJECTIVES

The reading of this chapter will enable the students

- To understand the types of investment and different costs.
- To understand the cost index, their types and lang factor for cost estimation.

1.1 INTRODUCTION

An acceptable plant design must present a process that is capable of operating under conditions which will yield a profit. Since net profit equals total income minus all expenses, it is essential that the chemical engineer be aware of the many different types of costs involved in manufacturing processes. Capital must be allocated for direct plant expenses, such as those for raw materials, labor, and equipment. Besides direct expenses, many other indirect expenses are incurred, and these must be included if a complete analysis of the total cost is to be obtained. Some examples of these indirect expenses are administrative salaries, product-distribution costs, and costs for interplant communications.

A capital investment is required for any industrial process, and determination of the necessary investment is an important part of a plant-design project. The total investment for any process consists of fixed-capital investment for physical equipment and facilities in the plant plus working capital which must be available to pay salaries, keep raw materials and products on hand, and handle other special items requiring a direct cash outlay. Thus, in an analysis of costs in industrial processes, capital-investment costs, manufacturing costs, and general expenses including income taxes must be taken into consideration.

1.2 COST INDEXES

Most cost data which are available for immediate use in a preliminary or predesign estimate are based on conditions at some time in the past. Because prices may change considerably with time due to changes in economic conditions, some method must be used for updating cost data applicable at a past date to costs that are representative of conditions at a later time? This can be done by the use of cost indexes.

A cost index is merely an index value for a given point in time showing the cost at that time relative to a certain base time. If the cost at some time in the past is known, the equivalent cost at the present time can be determined by multiplying the original cost by the ratio of the present index value to the index value applicable when the original cost was obtained.

$$\text{Present cost} = \text{Original Cost}\left(\frac{\text{Index value at present time}}{\text{Index value at time original cost was obtained}}\right)$$

Cost indexes can be used to give a general estimate, but no index can take into account all factors, such as special technological advancements or local conditions. The common indexes permit fairly accurate estimates if the time period involved is less than 10 years.

The main indexes available for industries include:

- **Chemical Engineering Index, CE** composed of 4 major components – equipment, construction labour, buildings and engineering and supervision – the index is employed primary as a process plant construction index, was established using a base period of 1957-59 as 100. The CE index is updated monthly and it lags in time by about 3 months. The CE index was revised in 1982, to account for changes in labor productivity and again in 2002.
- **Marshall and Swift Cost Index, M and S** (originally known as Marshall and Stevens index): a composite of two major components – process-industry equipment average and all-industry equipment average - was established in 1926 with a value of 100. Some industries considered in the process-industry equipment average are chemicals, petroleum products, rubber and paper. The all-industry average encompasses 47 different types of industrial, commercial and housing equipment.
- **Intratec Chemical Plant Construction Index, IC** a process plant construction index developed by Intratec, a chemical consulting company. Although cost indexes do not usually forecast future escalation, the IC index stands out for presenting a smaller delay between release date and index date, besides a 12 months forecast.
- **Nelson-Farrar Indexes, NF** (originally known as the Nelson Refinery Construction Indexes) established in 1946 with a value of 100, the index is more suitable for petroleum or petrochemical business.

Estimating Equipment Costs by Scaling

It is often necessary to estimate the cost of a piece of equipment when no cost data are available for the particular size of operational capacity involved. Good results can be obtained by using the logarithmic relationship known as the **six-tenth factor** rule, if the new piece of equipment is similar to one of another capacity for which cost data are available. According to this rule, if the cost of a given unit at one capacity is known, the cost of a similar unit with X times the capacity of the first is approximately $(X)^{0.6}$ times the cost of the initial unit.

$$\text{Cost of equipment,} \quad a = \text{Cost of equipment } b\left(\frac{\text{Capacity equipment } a}{\text{Capacity equipment } b}\right)^{0.6}$$

1.3 LANG FACTOR FOR COST ESTIMATION

Factorial Method

(i) For chemical process plant chemical cost estimate are often based on an estimate of major process equipment purchase cost. Other cost being estimated as a factor of the equipment cost.

(ii) By using Lang factor we can make quick estimate of capital cost in the early stage of project design.

(iii) The project fixed capital is given by as a function of total purchase equipment cost by following relation:

Fixed capital investment or total capital investment to an existing plant = Lang factor × delivered equipment cost.

(iv) Lang factor for different type of plant are given below:

Types of Plant	Lang Factor	
	Fixed Capital Investment	Total Capital Investment
(i) Solid processing plant	3.9	4.6
(ii) Solid-fluid processing plant	4.1	4.9
(iii) Fluid-processing plant	4.8	5.7

1.4 CAPITAL INVESTMENTS

Before an industrial plant can be put into operation, a large sum of money must be supplied to purchase and install the necessary machinery and equipment. Land and service facilities must be obtained and the plant must be erected complete with all piping, controls and services. In addition, it is necessary to have money available for the payment of expenses involved in the plant operation.

The capital needed to supply the necessary manufacturing and plant facilities is called the **fixed capital** investment, while that necessary for the operation of the plant is termed the **working capital**. The sum of the fixed-capital investment and the working capital is known as the total capital investment. The fixed capital portion may be further subdivided into manufacturing fixed capital investment and non-manufacturing fixed capital investment.

1.4.1 Fixed Capital Investment (F.C.I.)

1. Fixed capital means total cost of plant needed for start up. It is the cost paid to the contractors. It is a once – only cost that is not recovered at the end of the project life, other than scrap value. Fixed capital investment can be divided into:
   (a) Manufacturing fixed capital investment.
   (b) Non-manufacturing fixed capital investment.
2. Manufacturing fixed capital investment represents the capital necessary for installed process equipment with all auxiliaries that are needed for complete process operation. Examples are piping, instrument, insulation, foundation and site operations.
3. Fixed capital required for construction, overhead and all plant components that are not directly related to the process operation is called non-manufacturing. Examples of fixed capital investment are land, processing building, administration and other office warehouses, laboratories, transportation, shipping and receiving facility utility and waste disposal facilities. Construction overhead cost consists of field office and supervision expenses, home office expenses, engineering expenses, miscellaneous construction cost, contractors fees and contingencies.
4. In some cases, construction overhead is proportioned between manufacturing and non-manufacturing fixed capital investment.

1.4.2 Working Capital Investment

1. Working capital is the additional investment needed over and above the fixed capital to start the plant up and operate plant to the point where profit is earned.

2. The working capital for an industrial plant consists of the total amount of money invested in:
   (a) Raw materials and supplies carried in stock.
   (b) Finished product in stock and semi-finished product in the process of being manufactured.
   (c) Accounts receivable.
   (d) Cash kept on hand for monthly payment of operating expenses such as salaries, wages and raw material purchase.
   (e) Accounts payable and taxes payable.
   (f) Start up cost, catalyst charges.
3. The ratio of working capital to total capital investment varies with different companies but most chemical plant use an initial working capital amount 10 to 20 percent of the total capital investment.
4. Most of the working capital is recovered at the end of the project.

1.5 DIRECT COSTS

1. Purchased equipment:
   - All equipment listed on a complete flow sheet.
   - Spare parts and non installed equipment spares.
   - Surplus equipment, supplies and equipment allowance.
   - Inflation cost allowance.
   - Freight charges.
   - Taxes, insurance, duties.
   - Allowance for modification during start-up.
2. Purchased equipment installation:
   - Installation of all equipment listed on complete flow sheet.
   - Structural supports.
   - Equipment insulation and painting.
3. Instrumentation and controls:
   - Purchase, installation, calibration, computer control with supportive software.
4. Piping:
   - Process piping utilizing suitable structural materials
   - Pipe hangers, fitting, valves
   - Insulation
5. Electrical systems:
   - Electrical equipments.
   - Electrical material and labor.
6. Building (including services):
   - Process buildings
   - Auxiliary buildings
   - Maintenance shops
   - Building services

7. Yard Improvements:
   Site development-site clearing. Grading, roads, walkways etc.
8. Service facilities:
   - **Utilities:** Steam, water, power, refrigeration, compressed air, fuel, waste disposal.
   - **Facilities:** Boiler plant incinerator, wells, river intakes, water treatment, cooling tower, storage tank etc.
   - **Non-process equipments:** Office furniture, cafeteria installment, medical equipment, etc.
   - **Distribution and packaging:** Law material and product storage and handling equipment, product packaging etc.
9. Land:
   - Surveys and fees
   - Property cost

1.6 INDIRECT COST

1. Engineering and supervision:
   - Engineering cost: Administrative, process, design and general engineering etc.
2. Legal expenses
3. Construction expenses:
   - Construction, operation and maintenance of temporary facilities, offices, etc.
   - Construction tools and equipment
   - Construction supervision
   - Warehouse expenses
   - Safety expenses
   - Permits and licenses
   - Taxes, insurance, interest
4. Contractor's fees
5. Contingency

Example 1.1 Direct costs component of the fixed capital consists of

(a) contingency
(b) onsite and offsite costs
(c) labour costs
(d) raw material costs

Solution: (b)

Direct costs component of the fixed capital consists of onsite and offsite costs.

Example 1.2 For a solid processing plant, the delivered equipment cost is Rs. 10 lakhs. Using lang multiplication method, the total capital investment, in lakhs of rupees, is

(a) 46
(b) 57
(c) 100
(d) 200

Solution: (a)

For solid processing plant,

$$\text{Lang factor} = 4.6$$

$$\text{Total capital investment} = 10 \times 4.6 = \text{Rs. 46 lakhs}$$

Example 1.3 Which relation gives total capital investment?

(a) Total capital investment = Fixed capital investment + Working capital

(b) Total capital investment = Fixed capital investment + scrap value

(c) Total capital investment = Working capital + Depreciation

(d) Total capital investment = Salvage value + Depreciation

Solution: (a)

Example 1.4 In a desalination plant, an evaporator of area 200 m^2 was purchased in 1996 at a cost of Rs. 3,00,000. In 2002, another evaporator of area 50 m^2 was added. What was the cost of the second evaporator (in Rs.)? Assume that the cost of evaporators scales as $(\text{capacity})^{0.54}$. The Marshall and Swift index was 1048.5 in 1996 and 1116.9 in 2002.

Solution: (151166)

$$\text{Cost of 200 m}^2 \text{ evaporator in year 2002} = \text{Cost in year 1996}\left(\frac{\text{Cost Index in year 2002}}{\text{Cost Index in year 1996}}\right)$$

$$= (3{,}00{,}000)\left(\frac{1116.9}{1048.5}\right)$$

Now use capacity scale,

$$\text{The cost of 50 m}^2 \text{ evaporator in 2002} = (3{,}00{,}000)\left(\frac{1116.9}{1048.5}\right)\left(\frac{50}{200}\right)^{0.54} = \text{Rs. 151166}$$

Example 1.5 If cost of a distillation column in the year 2000 is Rs. x. What is the cost of the column in Rs. in the year 2010? Given the cost indices for the years 2000 and 2010 are 480 and 520 respectively.

Solution:

$$\text{Cost in year 2010} = \text{Cost in year 2000} \times \left(\frac{\text{Cost Index in year 2010}}{\text{Cost Index in year 2000}}\right)$$

$$\Rightarrow \quad \text{Cost in year 2010} = x\left(\frac{520}{480}\right) = x\left(\frac{13}{12}\right)$$

Example 1.6 The purchased cost of a shell-and-tube heat exchanger with 10 m^2 of heating surface was Rs. 4200 in 1990. What will be the purchased cost of a similar heat exchanger with 100 m^2 of heating surface in 2000? Use both Marshal and Swift Index and Chemical Engineering Plant Cost Index for comparison.

Student's Assignments

Q.1 A heat exchange of area 10 m^2 costed Rs. 50,000 in the year 1985. What is estimated cost of a 15 m^2 exchanger in 1988. Assume that the cost index in 1985 was 270 and in 1988 it is 320.

Q.2 If the delivered cost of equipments of a fluid processing plant is 4×10^6. What is the capital cost of the plant in lakh?

Q.3 In a desalination plant, an evaporator of area 200 m^2 was purchased in 1996 at a cost of \$3,00,000. In 2002, another evaporator of area 50 m^2 was added. What was the cost of the second evaporator (in \$)? Assume that the cost evaporators scales as $(\text{capacity})^{0.54}$. The marshall and swift index was 1048.5 in 1996 and 1116.9 in 2002.
(a) 1,30,500 (b) 1,39,100
(c) 1,41,900 (d) 1,51,200

Q.4 The cost of a drum dryer is Rs. 10 lakhs. The cost of a drum dryer with double the surface area in lakhs of rupees is
(a) 2×10 (b) $3^{0.6} \times 10$
(c) $5^{0.6} \times 10$ (d) $2^{0.6} \times 10$

Q.5 Which of the following cost is related to Non-Manufacturing Fixed Capital Investment?
(a) Site preparation
(b) Land
(c) Taxes payable
(d) Insulation

Q.6 Which type of cost estimate is used when we have no design information about the project and estimate is based on the similar previous cost data?
(a) Definitive estimate
(b) Order of magnitude estimate
(c) Preliminary estimate
(d) Study estimate

Q.7 The purchased cost of a 50-gal glass-lined, jacketed reactor (without drive) was \$8350 in 1981. Estimate the purchased cost of a similar 300-gal, glass-lined, jacketed reactor (without drive) in 1986. Use the annual average Marshall and Swift equipment-cost index (all industry) to update the purchase cost of the reactor.

Q.8 The cost of a Shell and Tube Heat Exchanger with 100 ft^2 heating surface was Rs. 3000 in 1980. The cost (in Rs.) of a Heat Exchanger with 1000 ft^2 of heating surface in 1985 is _______.
Data : Cost index in 1985 = 813
Cost index in 1980 = 675

Q.9 In the year 1995, the cost of a Shell and Tube heat exchanger with 70 m^2 heat transfer area was Rs. 10 lakh. Chemical Engineering Index for cost in 1995 was 381.1 and the index in 2002 was 390.4. Based on index of 0.6 for capacity scaling, the cost (in Lakhs of Rupees) of a similar heat exchanger having 90 m^2 heat transfer area in 2002 will be _______.

Q.10 What will be the purchased cost of a similar 360 gal Jacketed reactor in 2013. If the purchased cost of 50 gal glass lined jacketed reactor was Rs. 7000 in 2002.
Cost index for 2002 : 710 and 2013 : 780
(a) 20110 (b) 20250
(c) 25140 (d) 22330

Q.11 Present cost relation is

(a) $$\text{P.C.} = \text{Original cost} \times \frac{\text{Index value at present time}}{\text{Index value at time original cost was obtained}}$$

(b) $$\text{P.C.} = \frac{\text{Index value at present time}}{\text{Index value at time original cost was obtained}} \times \frac{1}{\text{Original cost}}$$

(c) $$\text{P.C.} = \text{Original cost} \times \frac{\text{Index value at time original cost was obtained}}{\text{Index value at present time}}$$

(d) $\text{P.C.} = \dfrac{\begin{array}{c}\text{Index value at}\\ \text{time original cost}\\ \text{was obtained}\end{array}}{\begin{array}{c}\text{Index value at}\\ \text{present time}\end{array}} \times \dfrac{1}{\text{Original cost}}$

ANSWERS

1. (75580.71)	**2.** (22.8)	**3.** (d)	**4.** (d)
5. (b)	**6.** (b)	**7.** (24300)	
8. (17278.10)	**9.** (11.9)	**10.** (d)	**11.** (a)

Explanation

1. (75580.71)

Capacity of heat exchanger $(HE)_1 = 10\ m^2$
Capacity of $(HE)_2 = 15\ m^2$
Hence from six – tenth rule we know that

$$\frac{\text{Cost of }(HE)_1}{\text{Cost of }(HE)_2} = \left(\frac{\text{Capacity of }(HE)_1}{\text{Capacity of }(HE)_2}\right)^{0.6}$$

$$\text{Cost of }(HE)_2 \text{ in 1985} = 50000 \times \left(\frac{15}{10}\right)^{0.6}$$

$$= 63771.23$$

Cost index in 1985 = 270
Cost index 1987 = 320
Hence, we know from given relation:

$$\text{Present cost} = \left(\frac{\begin{array}{c}\text{Index value}\\ \text{at present time}\end{array}}{\begin{array}{c}\text{Index value at}\\ \text{time original cost}\\ \text{was obtained}\end{array}}\right)$$

$$\text{Present cost} = 63771.23 \times \left(\frac{320}{270}\right)$$

$$= \text{Rs. } 75580.71$$

2. (22.8)

We know that:
Fixed capital investment or total capital investment = Lang Factor × delivered cost
Total capital cost of the plant = $(4 \times 10^6) \times 5.7$
$= 22.8 \times 10^6$
Total capital cost of the plant = Rs. 22.8 lakh

3. (d)

Evaporator cost of 50 m^2 in 1996

$$= \$\left(\frac{50}{200}\right)^{0.54} \times 300{,}000$$

$$= \$141909$$

Evaporated cost of 50 m^2 in 2002

$$= \$141909 \times \frac{1116.9}{1048.5}$$

$$= \$151166 \simeq \$151200$$

4. (d)

By six tenth rule

$$\frac{C_2}{C_1} = \left(\frac{S_2}{S_1}\right)^{0.6}$$

$$\frac{C_2}{10} = (2)^{0.6}$$

$$\Rightarrow \quad C_2 = 10 \times (2)^{0.6}$$

7. (24300)

Marshall and Swift equipment-cost index (all industry)
(From Table 3) For 1981 : 721
(From Table 3) For 1986 : 798
From Table 5, the equipment vs. capacity exponent is given as 0.54:
In 1986, cost of reactor

$$= (\$8350)\left(\frac{798}{721}\right)\left(\frac{300}{50}\right)^{0.54}$$

$$= \$24{,}300$$

8. (17278.10)

Cost of H.E. with 1000 ft^2 = Cost of H.E. with

$$100\ ft^2 \times \frac{\text{C.I. 1985}}{\text{C.I. 1980}} \times (\text{Capacity Ratio})$$

$$= 3000 \times \frac{813}{675} \times \left(\frac{400}{100}\right)^{0.6} \times \left(\frac{1000}{400}\right)^{0.8}$$

$$= \text{Rs. } 17278.10$$

CHAPTER

2

Interest and Investment Cost

LEARNING OBJECTIVES

The reading of this chapter will enable the students

- To understand the concept of interest and types of interests
- To understand about present worth, future worth, annuity and capitalized cost

2.1 INTEREST

Interest is considered to be the compensation paid for the use of borrowed capital. A fixed rate of interest is established at the time the capital is borrowed; therefore, interest is a definite cost if it is necessary to borrow the capital used to make the investment for a plant. Although the interest on borrowed capital is a fixed charge, there are many persons who claim that interest should not be considered as a manufacturing cost. It is preferable to separate interest from the other fixed charges and list it as a separate expense under the general heading of management or financing cost. Annual interest rates amount to 5 to 10 percent of the total value of the borrowed capital.

2.2 TYPES OF INTEREST

Simple Interest

In economic terminology, the amount of capital on which interest is paid is designated as the **principal** and **rate of interest** is defined as the amount of interest earned by a unit of principal in a unit of time. The time unit is usually taken as one year. For example, if Rs. 100 were the compensation demanded for giving someone the use of Rs. 1000 for a period of one year, the principal would be Rs. 1000, and the rate of interest would be $\frac{100}{1000}$ = 0.1 or 10 percent/year.

The simplest form of interest requires compensation payment at a constant interest rate based only on the original principal. Thus, if Rs. 1000 were loaned for a total time of 4 years at a constant interest rate of 10 percent/year, the simple interest earned would be

$$1000 \times 0.1 \times 4 = \text{Rs. } 400$$

If P represents the principal, n the number of time units or interest periods, and i the interest rate based on the length of one interest period, the amount of simple interest Z during n interest periods is

$$Z = Pin \quad \text{...(2.1)}$$

The principal must be repaid eventually; therefore, the entire amount S of principal plus simple interest due after n interest periods is

$$S = P + Z = P(1 + in) \quad \text{...(2.2)}$$

Ordinary and Exact Simple Interest

The time unit used to determine the number of interest periods is usually 1 year, and the interest rate is expressed on a yearly basis. When an interest period of less than 1 year is involved, the ordinary way to determine simple interest is to assume the year consists of twelve 30-day months, or 360 days. The exact method accounts for the fact that there are 365 days in a normal year. Thus, if the interest rat is expressed on the regular yearly basis and d represents the number of days in an interest period, the following relationships apply

$$\text{Ordinary simple interest} = Pi\frac{d}{360} \quad \text{...(2.3)}$$

$$\text{Exact simple interest} = Pi\frac{d}{365} \quad \text{...(2.4)}$$

Ordinary interest is commonly accepted in business practices unless there is a particular reason to use the exact value.

Interest, like all negotiable capital, has a time value. If the interest were paid at the end of each time unit, the receiver could put this money to use for earning additional returns. Compound interest takes this factor into account by stipulating that interest is due regularly at the end of each interest period. If payment is not made, the amount due is added to the principal, and interest is charged on this converted principal during the following time unit. Thus, an initial loan of Rs. 1000 at an annual interest rate of 10 percent would require payment of Rs. 100 as interest at the end of the first year. If this payment were not made, the interest for the second year would be (Rs. 1000 + Rs. 100)(0.10) = Rs. 110, and the total compound amount due after 2 years would be

Rs. 1000 + Rs. 100 + Rs. 110 = Rs. 1210

Therefore, the total amount of principal plus compounded interest due after n interest periods and designated as S is

$$S = P(1 + i)^n \quad \text{...(2.5)}$$

The term $(1 + i)^n$ is commonly referred to as the discrete single-payment compound-amount factor. Values for this factor at various interest rates and basis.

Consider an example in which the interest rate is 3 percent per period and the interest is compounded at half-year periods. A rate of this type would be referred to as "6 percent compounded semi-annually." Interest rates stated in this form are known as nominal interest rates. The actual annual return on the principal would not be exactly 6 percent but would be somewhat larger because of the compounding effect at the end of the semi-annual period.

It is desirable to express the exact interest rate based on the original principal and the convenient time unit of 1 year. A rate of this type is known as the effective interest rate. In common engineering practice, it is usually preferable to deal with effective interest rates rather than with nominal interest rates. The only time that nominal and effective interest rates are equal is when the interest is compounded annually.

If nominal interest rates are quoted, it is possible to determine the effective interest rate by proceeding from equation (2.5).

$$S = P(1 + i)^n$$

In this equation, S represents the total amount of principal plus interest due after n periods at the periodic interest rate i. Let r be the nominal interest rate under conditions where there are m conversions or interest periods per year. Then the interest rate based on the length of one interest period is r/m, and the amount S after 1 year is

$$S_{\text{after 1 year}} = P\left(1+\frac{r}{m}\right)^m \quad ...(2.6)$$

Designating the effective interest rate as i_{eff}, the amount S after 1 year can be expressed in an alternate form as

$$S_{\text{after 1 year}} = P(1 + i_{\text{eff}}) \quad ...(2.7)$$

By equating Eq. (2.6) and (2.7), the following equation can be obtained for the effective interest rate in terms of the nominal interest rate and the number of periods per year:

$$\text{Effective annual interest rate} = i_{\text{eff}} = \left(1+\frac{r}{m}\right)^m - 1 \quad ...(2.8)$$

Similarly, by definition,

$$\text{National annual interest rate} = m\left(\frac{r}{m}\right) = r \quad ...(2.9)$$

The extreme case, of course, is when the time interval becomes infinitesimally small so that the interest is compounded continuously.

The symbol r represents the nominal interest rate with m interest periods per year. If the interest is compounded continuously, m approaches infinity, and Eq. (2.6) can be written as

$$S_{\text{after } n \text{ years}} = P\lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{mn} = P\lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{(m/r)(rn)} \quad ...(2.10)$$

The fundamental definition for the base of the natural system of logarithms ($e = 2.71828$) is

$$\lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{m/r} = e = 2.71828 \quad ...(2.11)$$

Thus, with continuous interest compounding at a nominal annual interest rate of r, the amount S an initial principal P will compound to in n years is

$$S = Pe^{rn} \quad ...(2.12)$$

Similarly, from Eq. (2.8), the effective annual interest rate i_{eff}, which is the conventional interest rate that most executives comprehend, is expressed in terms of the nominal interest rate r compounded continuously as

$$I_{\text{eff}} = e^r - 1 \quad ...(2.13)$$

$$r = \ln(i_{\text{eff}} + 1) \quad ...(2.14)$$

Therefore,

$$e^{rn} = (1 + i_{\text{eff}})^n \quad ...(2.15)$$

and

$$S = Pe^{rn} = P(1 + i_{\text{eff}})^n \quad ...(2.16)$$

Example 2.1 For the case of a nominal annual interest rate of 20.00 percent, determine:

(a) The total amount to which one dollar of initial principal would accumulate after one 365-day year with daily compounding.

(b) The total amount to which one dollar of initial principal would accumulate after one year with continuous compounding.

(c) The effective annual interest rate if compounding is continuous.

Solution:

(a) $P = \$1.0,\ r = 0.20,\ m = 365$

$$S_{\text{after 1 year}} = P\left(1+\frac{r}{m}\right)^m = (1.0)\left(1+\frac{0.20}{365}\right)^{365} = \$1.2213$$

(b) $S = Pe^{rn} = (1.0)(e)^{(0.20)(1)} = \$1.2214$

(c) $I_{\text{eff}} = e^r - I = 0.2214 \text{ or } 22.14\%$

The present worth (or present value) of a future amount is the present principal which must be deposited at a given interest rate to yield the desired amount at some future date?

In Eq. (2.5), S represents the amount available after n interest periods if the initial principal is P and the discrete compound-interest rate is i. Therefore, the present worth can be determined by merely rearranging Eq. (2.5).

$$\text{Present worth} = P = S\frac{1}{(1+i)^n} \qquad ...(2.17)$$

The factor $\frac{1}{(1+i)^n}$ is commonly referred to as the discrete single-payment present-worth factor.

Similarly, for the case of continuous interest compounding, Eq. (2.12) gives

$$\text{Present worth} = P = S\frac{1}{e^{rn}} \qquad ...(2.18)$$

Example 2.2 A bond has a maturity value of \$1000 and is paying discrete compound interest at an effective annual rate of 3 percent. Determine the following at a time four years before the bond reaches maturity value:

(a) Present worth

(b) Discount

(c) Discrete compound rate of effective interest which will be received by a purchaser if the bond were obtained for \$700.

(d) Repeat part (a) for the case where the nominal bond interest is 3 percent compounded continuously.

Solution:

(a) $$\text{Present worth} = \frac{S}{(1+i)^n} = \frac{1000}{(1+0.03)^4} = \$\ 888$$

(b) $$\text{Discount} = \text{Future value} - \text{Present worth} = 1000 - 888 = \$\ 112$$

(c) $$\text{Principal} = \$\ 700 = \frac{S}{(1+i)^n} = \frac{1000}{(1+i)^4}$$

$$i = \left(\frac{1000}{700}\right)^{1/4} - 1 = 0.0935 \text{ or } 9.35\%$$

(d) By Eq. (18), $$\text{Present worth} = \frac{S}{e^{rn}} = \frac{1000}{e^{(0.03)(4)}} = \$\ 869$$

Annuities

An annuity is a series of equal payments occurring at equal time intervals. The common type of annuity involves payments which occur at the end of each interest period. This is known as an ordinary annuity. Interest is paid on all accumulated amounts, and the interest is compounded each payment period. An annuity term is the time from the beginning of the first payment period to the end of the last payment period. The amount of an annuity is the sum of all the payments plus interest if allowed to accumulate at a definite rate of interest from the time of initial payment to the end of the annuity term.

By definition, the amount of the annuity is the sum of all the accumulated amounts from each payment; therefore,

$$S = R(1+i)^{n-1} + R(1+i)^{n-2} + R(1+i)^{n-3} + ... + R(1+i) + R \quad ...(2.19)$$

To simplify Eq. (2.19), multiply each side by (1 + i) and subtract Eq. (2.19) from the result. This gives

$$Si = R(1+i)^n - R \quad ...(2.20)$$

or

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] \quad ...(2.21)$$

Note : The term $\frac{[(1+i)^n - 1]}{i}$ is commonly designated as the discrete uniform series compound amount factor or the series compound-amount factor.

Present Worth of an Annuity

The present worth of an annuity is defined as the principal which would have to be invested at the present time at compound interest rate i to yield a total amount at the end of the annuity term equal to the amount of the annuity. Let P represent the present worth of an ordinary annuity. Combining Eq. (2.5) with Eq. (2.21) gives, for the case of discrete interest compound,

$$P = R\frac{(1+i)^n - 1}{i(1+i)^n} \quad ...(2.22)$$

Note : The expression $\frac{[(1+i)^n - 1]}{[i(1+i)^n]}$ is referred to as the discrete uniform-series present-worth factor or the series present-worth factor, while the reciprocal $\frac{[i(1+i)^n]}{[(1+i)^n - 1]}$ is often called the capital-recovery factor.

For the case of continuous cash flow and interest compounding,

$$P = R\frac{e^{rn} - i}{re^{rn}} \quad ...(2.23)$$

Perpetuities and Capitalized Costs

A perpetuity is an annuity in which the periodic payments continue indefinitely. Consider the example in which the original cost of a certain piece of equipment is \$12,000. The useful-life period is 10 years, and the scrap value at the end of the useful life is \$2000. The engineer reasons that this piece of equipment, or its replacement will be in use for an indefinitely long period of time, and it will be necessary to supply \$10,000 every 10 years in order to replace the equipment. He, therefore, wishes to provide a fund of sufficient size so that it will earn enough interest to pay for the periodic replacement. If the discrete annual interest rate is 6 percent, this fund would need to be \$12650. At 6 percent interest compounded annually, the fund would amount to (\$12650)(1 + 0.06) = \$22650 after 10 years. Thus, at the end of 10 years, the equipment can be replaced for \$10000 and \$12650 will remain in the fund. This cycle could now be repeated indefinitely. If the equipment is to perpetuate itself, the theoretical amount of total capital necessary at the start would be \$12000 for the equipment plus \$12650 for the replacement fund. The total capital determined in this manner is called the **capitalized cost**. Engineers use time. Let P be the amount of present principal (i.e., the present worth) which can accumulate to an amount of S during n interest periods at periodic interest rate i. Then, by Eq. (2.5),

$$S = P(1 + i)$$

If perpetuation is to occur, the amount S accumulated after n periods minus the cost for the replacement must equal the present worth P. Therefore, letting C_R represent the replacement cost,

$$P = S - C_R \quad \text{...(2.24)}$$

Combining eqs. (2.5) and (2.24),

$$P = \frac{C_R}{(1+i)^n - 1} \quad \text{...(2.25)}$$

The capitalized cost is defined as the original cost of the equipment plus the present value of the renewable perpetuity. Designating K as the capitalized cost and C_V as the original cost of the equipment,

$$K = C_V + \frac{C_R}{(1+i)^n - 1} \quad \text{...(2.26)}$$

Example 2.3 If one borrows INR 1000 at a monthly interest rate of 2%, what will the total amount of principal plus simple interest due after 2 years if no intermediate payments are made?

Solution:

The length of one interest period = 1 month

The number of interest periods in 2 years = 24

For simple interest, the total amount due after n periods at a periodic (in this case monthly) interest rate of i is given by :

$$F = P(1 + i^n) = 1000[1 + (0.02)(24)]$$
$$= \text{INR } 1480$$

Example 2.4 If one borrows INR 1000 at a monthly interest rate of 2%, what will the total amount of principal plus compounded interest due after 2 years if no intermediate payments are made?

Solution:

The length of one interest period = 1 month

The number of interest periods in 2 years = 24

$$F = P(1 + i)^n$$

For compound interest, the total amount due after n periods at a periodic (in this case monthly) interest rate of i is given by :

$$F = P(1 + i)^n = 1000[1 + (0.02)]^{24} = \text{INR } 1608$$

Example 2.5 If one borrows INR 1000 at a monthly interest rate of 2%, (1) What is the nominal interest rate when the interest is compounded monthly? (2) What is the effective interest rate when the interest is compounded monthly?

Student's Assignments

Q.1 A sum of Rs. 50,000 was taken as a loan from a loan agency. If the agency charges a quarterly interest rate of 3%, then the effective interest rate (in percent), when the interest is compounded quarterly, is equal to ________.

Q.2 The total amount to which a sum of Rs. 70,000 would accumulate after 3 years at a nominal interest rate of 15% per year with continuous compounding, is equal to ________. Also the effective annual interest rate in this case is equal to ________.
(a) Rs. 106461 and 15%
(b) Rs. 109782 and 17.18%
(c) Rs. 106461 and 17.18%
(d) Rs. 109782 and 16.18%

Q.3 The cost of installation of a reactor is Rs. 30 lakhs and is expected to have a working life of 8 years. Let *R* be the amount deposited annually in an annuity at an interest rate of 12%. In order to obtain sufficient funds to replace the reactor at the end of 8 years, the value of *R* should approximately be ________.
(a) Rs. 3.75 lakhs
(b) Rs. 6.04 lakhs
(c) Rs. 7.04 lakhs
(d) Rs. 5.75 lakhs

Q.4 Chetan deposits Rs. 2000 at the end of each year in an account earning 10% compounded annually. How much interest did he earn after 25 years?
(a) 100,000 (b) 154,653.24
(c) 146,694.12 (d) None of these

Q.5 What lump sum deposited today would allow payments of Rs. 2000/year for 7 years at 5% compounded annually?
(a) 12572.81 (b) 11572.71
(c) 14572.71 (d) None of these

Q.6 An amount of Rs. 1500 is deposit in SBI bank at an annual interest rate of 4.3% compounded quarterly. Find total amount after 6 years by using effective interest rate.
(a) 1600 (b) 1800
(c) 1939 (d) 1539

Q.7 If any loan which given by any bank applied 1% interest per month then calculate nominal inter rate and effective interest rate (in %).
(a) 12%, 12.68%
(b) 14%, 14.68%
(c) 12.68%, 12%
(d) None of these

Q.8 For the cases of a nominal annual interest rate of a 15%, calculate total amount to which Rs. 100 of initial principal would accumulated after one year of 365 days with daily compounding.
(a) Rs. 116.18 (b) Rs. 130.15
(c) Rs. 148.15 (d) Rs. 100.15

Q.9 A sale contract signed by a chemical manufacturer is expected to generate a net cash flow of Rs. 2,50,000/- per year at the end of each year for a period of three years the applicable discount rate (interest rate) is 10%. The net present worth of the total cash flow is (in Rs.)
(a) 7,50,000 (b) 6,83,750
(c) 6,21,500 (d) 3,32,750

Q.10 What will be the total amount to which Rs. 150 of initial principal would accumulate after one year with continuous compounding if the nominal annual interest rate is 10%.
(a) Rs. 165.77
(b) Rs. 150
(c) Rs. 180
(d) Rs. 155.67

■ ANSWERS

1. (12.5) **2.** (d) **3.** (b) **4.** (c)
5. (b) **6.** (c) **7.** (a) **8.** (a)
9. (c) **10.** (a)

Explanation

1. (12.5)

$$P = 50,000$$
$$i = 4 \times 3 = 12\%$$
$$i_{eff} = \left(1+\frac{i}{m}\right)^m - 1$$
$$i_{eff} = (1 + 0.03)^4 - 1$$
$$i_{eff} = 0.125 = 12.5\%$$

2. (d)

For continuous compounding,

$$S = P.e^{in}$$
$$S = 70000 \times e^{0.15 \times 3}$$
$$S = \text{Rs. } 109781.85$$
$$i_{eff} = e^i - 1$$
$$= e^{0.15} - 1 = 0.1618 = 16.18\%$$

3. (b)

$$P = \frac{R[(1+i)^n - 1]}{i(1+i)^n}$$
$$30 \times 10^5 = \frac{R[(1+0.12)^8 - 1]}{0.12 \times (1.12)^8}$$
$$R = \text{Rs. } 6.039 \times 10^5$$

4. (c)

$$S = 2000\left[\frac{(1+0.1)^{25} - 1}{0.1}\right]$$
$$S = 2000\left(\frac{9.834706}{0.1}\right)$$
$$= \text{Rs. } 196,694.12$$
$$I = S - (25 \times 2000)$$
$$I = \text{Rs. } 146,694.12$$

5. (b)

$$P = 2000\left[\frac{(1+0.05)^7 - 1}{0.05(1+0.05)^7}\right]$$
$$= 2000\left(\frac{0.407100}{0.0703552}\right)$$
$$= \text{Rs. } 11,572.71$$

6. (c)

Principal amount, P = Rs. 1500

Total sum, $S = P\left(1+\frac{r}{m}\right)^m$

$$r = \text{Nominal interest rate}$$
$$= 4.3\% = 0.043$$
$$m = \frac{\text{Number of interest period}}{\text{Year}} = 4$$
$$S = 1500\left(1+\frac{0.043}{4}\right)^{6 \times 4}$$
$$S = \text{Rs. } 1938.84$$

7. (a)

Nominal interest rate $= r = 12 \times \frac{1}{100}$

$= 12\%$ yearly $= 0.12$ yearly

Effective interest rate i_{eff}

$$i_{eff} = \left(1+\frac{r}{m}\right)^m - 1$$
$$i_{eff} = \left(1+\frac{0.12}{12}\right)^{12} - 1 = 12.68\%$$
$$i_{eff} = 12.68\% \text{ or } 0.1268$$

8. (a)

$$S_{\text{after 1 year}} = P\left(1+\frac{r}{m}\right)^m$$
$$= 100\left(1+\frac{0.15}{365}\right)^{365} = \text{Rs. } 116.18$$

9. (c)

Net cash flow = Rs. 2,50,000/per year

Discount rate = 10%

$$\text{Net present worth} = \frac{250000}{(1+i)} + \frac{250000}{(1+i)^2} + \frac{250000}{(1+i)^3}$$
$$= 250000\left[\frac{1}{1.1} + \frac{1}{(1.1)^2} + \frac{1}{(1.1)^3}\right] = \text{Rs. } 621500$$

10. (a)

$$S = Pe^{rn}$$
$$= 150e^{0.10 \times 1} = \text{Rs. } 165.77$$

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