

CHEMICAL ENGINEERING

Mass Transfer



Comprehensive Theory
with Solved Examples and Practice Questions





MADE EASY Publications Pvt. Ltd.

Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016 | **Ph. :** 9021300500

Email : infomep@madeeasy.in | **Web :** www.madeeasypublications.org

Mass Transfer

© Copyright by MADE EASY Publications Pvt. Ltd.
All rights are reserved. No part of this publication may be reproduced, stored in or introduced into a retrieval system, or transmitted in any form or by any means (electronic, mechanical, photo-copying, recording or otherwise), without the prior written permission of the above mentioned publisher of this book.



MADE EASY Publications Pvt. Ltd. has taken due care in collecting the data and providing the solutions, before publishing this book. In spite of this, if any inaccuracy or printing error occurs then **MADE EASY Publications Pvt. Ltd.** owes no responsibility. We will be grateful if you could point out any such error. Your suggestions will be appreciated.

EDITIONS

First Edition : 2021

Second Edition : 2022

Third Edition : 2023

Fourth Edition : 2024

CONTENTS

Mass Transfer

CHAPTER 1

Introduction and Basic Concepts..... 1-8

1.1	Introduction	1
1.2	Classification of Mass Transfer	2
1.3	Mechanism of Mass Transfer	3
1.4	Driving Force for Mass Transfer	3
1.5	Concentrations, Diffusion Velocities and Fluxes.....	4
1.6	Dimensionless Numbers used in Mass Transfer	7
	<i>Student's Assignments</i>	8

CHAPTER 2

Diffusion..... 9-45

2.1	Introduction	9
2.2	Molecular Diffusion.....	9
2.3	Fick's Law of Diffusion (Steady State) for a Binary System (A and B)	10
2.4	Diffusivity.....	10
2.5	Steady State Molecular Diffusion in Fluids under Stagnant and Laminar Flow Conditions.....	11
2.6	Multicomponent Diffusivity	19
2.7	Molecular Diffusion in Liquids.....	20
2.8	Molecular Diffusion in Solids.....	20
2.9	Prediction of Diffusivities.....	21
2.10	Mass Transfer Coefficients	22
2.11	Convective Mass Transfer	27
2.12	Boundary Layer Theory	28
2.13	Analogies between Heat, Mass and Momentum.....	31
2.14	Mass Transfer Theories.....	34
2.15	Turbulent Diffusion.....	37
2.16	Interphase Mass Transfer	38
	<i>Student's Assignments</i>	42

CHAPTER 3

Absorption and Stripping..... 46-67

3.1	Introduction	46
3.2	Equilibrium Solubility in a Gas-Liquid System	46
3.3	Selection of Solvent for Absorption and Stripping.....	46
3.4	Design of Packed and Tray Tower for Absorption	47
3.5	Gas Absorption Equipment	53
3.6	Comparison between Packed Tower and Tray Tower	57
3.7	Absorption with Chemical Reaction.....	57
	<i>Student's Assignments</i>	64

CHAPTER 4

Distillation..... 68-112

4.1	Introduction	68
4.2	Boiling Point Diagram/Phase Diagram	70
4.3	Relative Volatility	71
4.4	Fractional Distillation	72
4.5	McCabe-Thiele Method.....	72
4.6	Reflux Ratio.....	86
4.7	Fenske Equation.....	89
4.8	Use of Open Steam/Live Steam.....	89
4.9	Flash/Equilibrium Distillation.....	89
4.10	Differential/Batch Distillation.....	90
4.11	Steam Distillation	91
4.12	Vacuum Distillation.....	92
4.13	Calculation of Diameter of Distillation Column.....	93
4.14	Effect of Pressure on Dia of Column	93
4.15	Plate Efficiency	94
4.16	Relation between Murphree and Point Efficiency.....	95
4.17	Multicomponent Distillation	96
4.18	Azeotropes.....	98
4.19	Azeotropic and Extractive Distillation.....	101
	<i>Student's Assignments</i>	108

CHAPTER 5

Liquid-Liquid Extraction, Leaching and Adsorption 113-131

5.1 Introduction 113
 5.2 Types of System 114
 5.3 Equilibrium Diagram in Liquid-Liquid Extraction..... 114
 5.4 Effect of Temperature on Liquid-Liquid Equilibria 116
 5.5 Batch/Single Stage Extraction Unit 117
 5.6 Counter Current Extraction 117
 5.7 Multistage Cross-Current Extraction 119
 5.8 Selection of Solvent 119
 5.9 Leaching 120
 5.10 Adsorption 121
 5.11 Applications of Adsorption Process 121
 5.12 Classification of Adsorption 122
 5.13 Types of Adsorbents and Their Characteristics 122
 5.14 Adsorption Equilibria 123
 5.15 Batch Adsorption 123
Student's Assignments 129

CHAPTER 6

Humidification 132-144

6.1 Introduction 132
 6.2 Vapour-Gas Mixture 132
 6.3 Important Terminology to Measure Humidity 133
 6.4 Humid Volume 134
 6.5 Humit Heat 134
 6.6 Enthalpy..... 134
 6.7 Dry Bulb Temperature 135

6.8 Wet Bulb Temperature..... 135
 6.9 Dew Point Temperature 137
 6.10 Adiabatic Saturation Temperature..... 137
Student's Assignments..... 142

CHAPTER 7

Drying..... 145-161

7.1 Introduction 145
 7.2 Movement of Moisture within Solid 146
 7.3 Drying Operations..... 146
 7.4 Important Definitions 146
 7.5 Rate of Drying Curve 147
 7.6 Drying Rate in Constant Rate Period with Help of Heat Transfer Coefficient 152
 7.7 Humid Volume..... 152
Student's Assignments..... 157

CHAPTER 8

Membrane Separation 162-175

8.1 Introduction 162
 8.2 Types of Membrane 163
 8.3 Some Basic Definition in Membrane Separation Process..... 164
 8.4 Microfiltration Membrane Processes..... 164
 8.5 Reverse-Osmosis Membrane Process 165
 8.6 Ultrafiltration Membrane Processes 169
 8.7 Nanofiltration..... 170
Student's Assignments..... 174

Introduction and Basic Concepts

LEARNING OBJECTIVES

The reading of this chapter will enable the students:

- To understand the classification of mass transfer.
- To understand the mechanism of mass transfer.
- To understand the concentrations, diffusion velocities and fluxes.

1.1 INTRODUCTION

- The mass transfer is the net movement of a component in a mixture from one location to the another location in presence of a difference in concentration or partial pressure. So, when there is a driving force then mass transfer will occur. The driving force over here is the concentration or partial pressure difference. Let us consider one common example of mass transfer. Suppose, if you have taken a lump of sugar added to a cup of tea which dissolves and then diffuses throughout the tea cup uniformly.
- So, another examples are the deliberate use of agarbatti, the fragrance generally spreads uniformly when we put agarbatti at home.
- The other examples is drying of clothes under the sun. Here, the drying occurs because the moisture diffuses into the air. So, the diffusion or the mass transfer is basically occurs with a particular driving force. Like, if we want to consider a movement of solid through the conveyer belts or movement of liquid through a pipe is not the mass transfer operations because it is not based on the concentration or partial pressure driving force.



Examples on Industrial Processes

- Separation of CO_2 from flue gas : Absorption Process
- Separation of a mixture of Ethanol and Water into its components : Distillation Process
- Separation of mixtures of Toluene and Water using Benzene as solvent : Extraction Process
- Drying of wet solid such as wood with the help of air : Drying Process

1.2 CLASSIFICATION OF MASS TRANSFER

1.2.1 Distillation

Distillation is a vapour-liquid operation in which the mixture components are separated by use of thermal energy. When liquid mixture is heated, different components exert different vapour pressure, expressed in terms of relative volatility. This pressure difference results in separation of components in such a way that the top product contains higher amount of light component and bottom products contains higher amounts of heavier components, as shown in figure. A distillation example is separation of crude petroleum into gasoline, kerosene, etc.

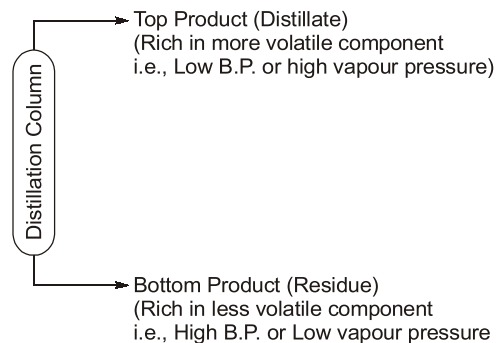


Fig. Schematic diagram of a distillation column

1.2.2 Absorption and Stripping

Gas absorption is a gas-liquid operation in which one or more constituents of a gas mixture are separated by using a suitable liquid solvent, i.e., component moves from gas phase to liquid phase as shown in figure. Example of gas absorption methods is ammonia washing from ammonia-air mixture by means of water. Stripping is opposite of absorption, i.e., a component moves from liquid phase to gas phase.

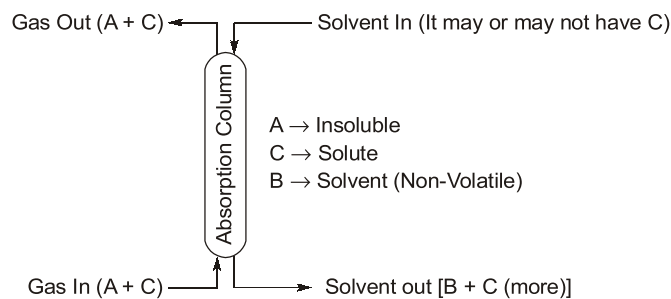


Fig. Schematic diagram of an absorption column

1.2.3 Liquid-Liquid Extraction

It is a liquid-liquid operation, also called as solvent extraction, in which components of a liquid mixture are separated by treating it with suitable solvent which dissolves one or more constituents of mixture more preferably. It is an efficient separation process in cases where separation is either not possible or not economical by using distillation. For example, separation of components of an azeotropic mixture.

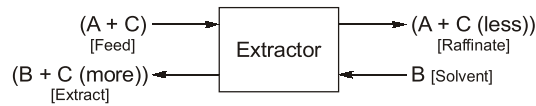


Fig. Schematic diagram of a Liquid-Liquid Extractor

1.2.4 Crystallization

It is liquid-solid operation in which we obtain uniform crystals of good purity. The saturated liquid is subjected to changes in temperature and pressure in such a way that crystals get separated from the feed liquor as shown in the figure.

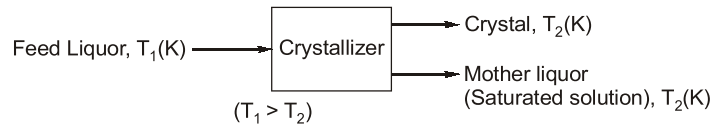


Fig. Schematic diagram of a crystallizer

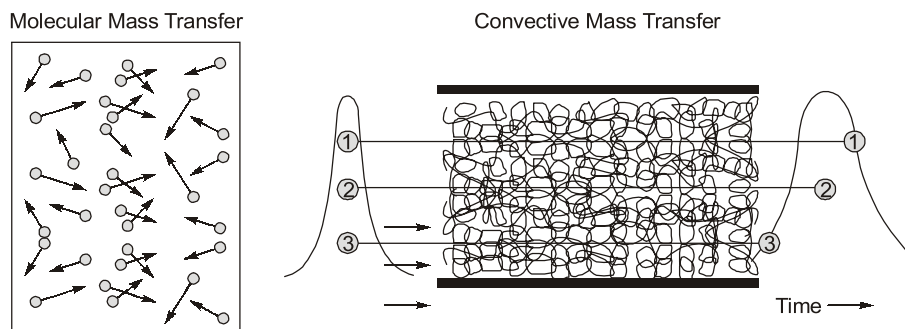
1.2.5 Drying

Drying is gas-solid operation in which a relatively small amount of water is removed from solid material, by contacting it with a continuous stream of gas (air) as shown in figure.



Fig. Schematic diagram of a dryer

1.3 MECHANISM OF MASS TRANSFER



In this case, eddy diffusion by random macroscopic fluid motion is the responsible for the convective mass transfer.

1.4 DRIVING FORCE FOR MASS TRANSFER

1.4.1 Two-Phase System

- Spontaneous alteration through molecular diffusion occurs.
- Ultimately brings the entire system to a state of equilibrium whereupon alteration stops.

- Mole fraction of component i (ideal gas mixture)

$$y_i = \frac{p_i}{p_t}$$

- Mole fraction of component i (gases)

$$y_i = \frac{C_i}{C}$$

- Sum of mole fractions

$$\sum_i x_i = 1; \sum_i y_i = 1$$

1.5.2 Diffusion Velocities

- Mass Average Velocity :
 - Defined in terms of mass concentrations :

$$V_{\text{mass-avg}} = \frac{\sum_{i=1}^n \rho_i v_i}{\sum_{i=1}^n \rho_i} = \sum_{i=1}^n \left(\frac{\rho_i}{\rho} \right) v_i = \sum_{i=1}^n \omega_i v_i$$

where,

v_i = absolute velocity of species i with respect to a fixed reference frame

ω_i = mass fraction of species i

- Molar Average Velocity :
 - Defined in terms of molar concentrations :

$$V_{\text{mol-avg}} = \frac{\sum_{i=1}^n C_i \times v_i}{\sum_{i=1}^n C_i} = \sum_{i=1}^n \left(\frac{C_i}{C} \right) v_i = \sum_{i=1}^n x_i v_i$$

where,

x_i = mole fraction of species i

Example 1.1

A gas mixture containing ($H_2 = 15\%$, $CO = 30\%$, $CO_2 = 5\%$ and $N_2 = 50\%$) flows through a tube of 1 inch diameter, at 15 bar total pressure. If the velocities of the respective components are 0.05 m/s, 0.03 m/s, 0.02 m/s and 0.03 m/s. Calculate the mass average and molar average velocities of the mixture.

Solution:

Rename $H_2 = 1$, $CO = 2$, $CO_2 = 3$ and $N_2 = 4$.

The volume average velocity (= molar average velocity) given by

$$\begin{aligned} V_{\text{mol-avg}} &= \frac{1}{C}(C_1 v_1 + C_2 v_2 + C_3 v_3 + C_4 v_4) \\ &= y_1 v_1 + y_2 v_2 + y_3 v_3 + y_4 v_4 \end{aligned}$$

Here, y_i is the mole fraction of component i in the gas mixture.

**Student's
Assignments**

- Q.1** Driving force for mass transfer in a binary system is
- (a) Temperature difference
 - (b) Chemical potential
 - (c) Concentration difference
 - (d) Pressure difference
- Q.2** Which of the following is/are a mechanism for mass transfer?
- (a) Difference in pressure
 - (b) Difference in solubility
 - (c) Difference in volatility
 - (d) Difference in viscosity
- Q.3** Which of the following is/are example of solid-liquid mass transfer operation?
- (a) Crystallization (b) Adsorption
 - (c) Leaching (d) Drying
- Q.4** Which of the following option(s) is/are example of gas-liquid operation?
- (a) Absorption (b) Distillation
 - (c) Drying (d) Humidification
- Q.5** Molecular mass transfer takes place either fluid is
- (a) stagnant or in laminar flow
 - (b) in turbulent flow
 - (c) forced to flow in bulk with eddies
 - (d) None of these
- Q.6** Molecular mass transfer (molecular diffusion) can be increased by
- (a) Increasing temperature
 - (b) Decreasing temperature
 - (c) Increasing pressure
 - (d) Decreasing pressure
- **ANSWERS** _____
- 1.** (b) **2.** (b, c) **3.** (a, b, c) **4.** (a, b, d)
5. (a) **6.** (a, d)



Diffusion

LEARNING OBJECTIVES

The reading of this chapter will enable the students

- To understand the concept of mass transfer coefficients.
- To understand the analogies between heat, mass and momentum.
- To understand the concept of mass transfer theories and interphase mass transfer.
- To understand the concept of molecular and convective diffusion.
- To understand the concept of steady state molecular diffusion in fluids under stagnant and laminar conditions.
- To understand the concept of molecular diffusion in liquids and solids.

2.1 INTRODUCTION

Diffusion is the movement, under the influence of a physical stimulus, of an individual component through a mixture. The most common cause of diffusion is a concentration gradient of the diffusing component. Although the usual cause of diffusion is a concentration gradient, diffusion can also be caused by an activity gradient, as in reverse osmosis, by a pressure gradient, by a temperature gradient, or by the application of an external force.

Diffusion is not restricted to molecular transfer through stagnant layers of solid or fluid. It also takes place in fluid phases by physical mixing and by the eddies of turbulent flow, just as heat flow may occur by convection. This is called **eddy diffusion**. Sometimes, the diffusion process is accompanied by bulk flow of the mixture in a direction parallel to the direction of diffusion, and it is often associated with heat flow.

A distinguishing feature of diffusion is that it results in mixing or mass transport, without requiring bulk motion (bulk flow). Thus, diffusion should not be confused with convection or advections, which are other transport phenomena that utilize bulk motion to move particles from one place to another. The driving force of molecular diffusion is concentration difference.

2.2 MOLECULAR DIFFUSION

The molecular diffusion is defined by the movement of individual molecules through a substance by virtue of their thermal energy. Let us take a beaker and keep some water, if we put a drop of dyes or any ink into a liquid drop then it will try to distribute throughout the solutions. The process by which it takes place without any external effect is known as the molecular diffusion and which is based on their thermal energy.

The rate of molecular diffusions is very slow. Thus we can change the rate of diffusion by two ways : one is to reduce the pressure. If we reduce the pressure like, if you can consider an example of pressure cooker releasing pressure at a certain time, it reduces the number of collisions because the number of molecules present in a per unit volume is less.

So, the collisions will be less and its diffusion will be higher. The other way to change the rate of molecular diffusion is to increase the temperature. If we increase the temperature of the system its molecular velocity will increase and hence, it will increase the rate of molecular diffusion.

Effect of Barrier on Molecular Diffusion :

- Rate of evaporation of water at 25°C into a complete vacuum is roughly 3.3 kg/(s m²) of water surface.
- Placing a layer of stagnant air at 1 std. atm. pressure and only 0.1 mm thick above the water surface reduces the rate by a factor of about 600.

2.3 FICK'S LAW OF DIFFUSION (STEADY STATE) FOR A BINARY SYSTEM (A AND B)

The molar flux of diffusing component A (diffusion flux of A) in z-direction in a binary mixture of A and B is proportional to the molar concentration gradient.

Mathematically, Fick's law is given by:

$$J_A = -D_{AB} \frac{dc_A}{dz} \quad \dots(2.1)$$

where J_A molar flux of A with respect to an observer moving with bulk velocity in z-direction.

c_A = Concentration of A [kmol/m³]

$\frac{dc_A}{dz}$ = Concentration gradient in z-direction

D_{AB} = Proportionality constant known as molecular diffusivity or diffusion coefficient, (m²/sec)

z = Distance in the direction of diffusion, (m)

"-ve" sign shows that diffusion occur in the direction of decreasing concentration.

2.4 DIFFUSIVITY

Diffusivity is defined as the ratio of the flux to the corresponding concentration gradient. It is denoted by D . Unit of diffusivity is m²/sec. It is a characteristic of the constituent and its environment (temperature, pressure, solution type, i.e., whether gas, liquid or solid solution). It is a measure of its diffusive mobility. Let D_{AB} = Diffusive mobility of A in mixture of A and B.

Then, D_{BA} = Diffusive mobility of B in mixture of A and B

Mathematical expression for molar flux with respect to a stationary observer.

$$N_A = J_A + x_A(N_A + N_B)$$

where J_A = Contribution due to concentration gradient, (diffusion flux)

$x_A(N_A + N_B)$ = Bulk motion contribution (flux due to bulk flow)

For a condition of steady state, the net flux is

$$N_A + N_B = N \quad \dots(2.2)$$

The movement of A is made up of two parts, namely, that resulting from the bulk motion N and the fraction x_A of N which is A and that resulting from diffusion J_A :

$$N_A = Nx_A + J_A \quad \dots(2.3)$$

$$N_A = (N_A + N_B) \frac{c_A}{C} - D_{AB} \frac{\partial c_A}{\partial z} \quad \dots(2.4)$$

The counterpart of Eq. (2.4) for B is

$$N_B = (N_A + N_B) \frac{c_B}{C} - D_{BA} \frac{\partial c_B}{\partial z} \quad \dots(2.5)$$

Adding these gives

$$-D_{AB} \frac{\partial c_A}{\partial z} = D_{BA} \frac{\partial c_B}{\partial z} \quad \dots(2.6)$$

or $J_A = -J_B$. If $c_A + c_B = \text{const}$, it follows that $D_{AB} = D_{BA}$ at the prevailing concentration and temperature.

2.5 STEADY STATE MOLECULAR DIFFUSION IN FLUIDS UNDER STAGNANT AND LAMINAR FLOW CONDITIONS

2.5.1 Steady State Diffusion through a Constant Area

- Assume steady state diffusion in the x -direction without any chemical reaction in a binary gaseous mixture of species A and B .
- For one dimensional diffusion of species A , molar flux :

$$N_A = -CD_{AB} \frac{dy_A}{dx} + y_A N \quad \text{where } N = N_A + N_B \quad \dots(2.7)$$

- Separating the variables in equation (2.7), it can be expressed as :

$$\frac{-dy_A}{N_A - y_A N} = \frac{dx}{CD_{AB}} \quad \dots(2.8)$$

- For the gaseous mixture, at constant pressure and temperature, C and D_{AB} are constant, independent of position and composition. Also, all the molar fluxes are constant in equation (2.8). Therefore, the equation (2.8) can be integrated between two boundary conditions as follows :

$$\frac{-dy_A}{N_A - y_A N} = \frac{dx}{CD_{AB}}$$

$$\text{at } x = x_1, \quad y_A = y_{A1}$$

$$\text{at } x = x_2, \quad y_A = y_{A2}$$

where, 1 indicates the start of the diffusion path and 2 indicates the end of the diffusion path.

Integration eqn. (2.8) with the above BC's :

$$\int_{y_{A1}}^{y_{A2}} \frac{-dy_A}{N_A - y_A N} = \int_{x_1}^{x_2} \frac{dx}{CD_{AB}}$$

Let

$$N_A - y_A N = Z$$

⇒

$$-dy_A N = dZ$$

⇒

$$-dy_A = \frac{dZ}{N}$$

Substituting the values in above equation,

$$\int_{z_1}^{z_2} \frac{dZ}{NZ} = \int_{x_1}^{x_2} \frac{dx}{CD_{AB}}$$

or,
$$\frac{1}{N}(\ln Z_2 - \ln Z_1) = \frac{1}{CD_{AB}}(x_2 - x_1)$$

$$\Rightarrow \ln\left(\frac{Z_2}{Z_1}\right) = \frac{N}{CD_{AB}}(x_2 - x_1)$$

$$\Rightarrow \ln\left(\frac{N_A - y_{A2}N}{N_A - y_{A1}N}\right) = \frac{N}{CD_{AB}}(x_2 - x_1)$$

$$\Rightarrow \ln\left[\frac{N\left(\frac{N_A}{N} - y_{A2}\right)}{N\left(\frac{N_A}{N} - y_{A1}\right)}\right] = \frac{N}{CD_{AB}}(x_2 - x_1)$$

Multiplying both sides with N_A in above equation,

$$\Rightarrow N_A = \frac{N_A}{N} \times \frac{CD_{AB}}{(x_2 - x_1)} \ln\left[\frac{\left(\frac{N_A}{N} - y_{A2}\right)}{\left(\frac{N_A}{N} - y_{A1}\right)}\right]$$

Therefore, after integration with the boundary conditions the equation for diffusion for the said condition can be expressed as

$$N_A = \frac{N_A(D)_{AB}}{N(x_2 - x_1)} \ln\left[\frac{\frac{N_2}{N} - y_{A2}}{\frac{N_4}{N} - y_{A1}}\right] \quad \dots(2.9)$$

2.5.2 Steady State Diffusion through Non-Diffusing B

- For steady state one dimensional diffusion of A through non-diffusing B, $N_B = 0$ and $N_A = \text{constant}$. Therefore,

$$\frac{N_A}{N} = \frac{N_A}{(N_A + N_B)} = 1 \quad (\text{As } N_B = 0)$$

- Hence, equation (2.9) becomes :

$$N_A = \frac{CD_{AB}}{x_2 - x_1} \ln\left[\frac{1 - y_{A2}}{1 - y_{A1}}\right] \quad \dots(2.10)$$

For, an ideal gas,
$$C = \frac{P_t}{RT}$$

For, mixture of ideal gases,
$$y_A = \frac{p_A}{P_t}$$

- The equation (2.10) can be expressed in terms of partial pressure as :

$$N_A = \frac{D_{AB}P_t}{(x_2 - x_1)RT} \ln \left[\frac{P_t - p_{A2}}{P_t - p_{A1}} \right] \quad \dots(2.11)$$

where,

P_t = Total pressure

p_{A1} and p_{A2} = Partial pressures of A at point 1 and 2 respectively

- For diffusion under turbulent conditions, the flux is usually calculated based on linear driving force. For this purpose, the equation (2.11) can be manipulated to rewrite it in terms of a linear driving force.
- Since for the binary gas mixture of total pressure P_t ,

$$P_t = p_A + p_B$$

$$P_t - p_{A2} = p_{B2}$$

and

$$P_t - p_{A1} = p_{B1} \Rightarrow p_{A1} - p_{A2} = p_{B2} - p_{B1}$$

- Then the equation (2.11) can be written as :

$$N_A = \frac{D_{AB}P_t}{RT(x_2 - x_1)} \left[\frac{p_{A1} - p_{A2}}{p_{B2} - p_{B1}} \right] \ln \left[\frac{p_{B1}}{p_{B2}} \right]$$

or,

$$N_A = \frac{D_{AB}P_t}{RT(x_2 - x_1)p_{BLM}} (p_{A1} - p_{A2}) \quad \dots(2.12)$$

where, p_{BLM} = Logarithmic mean partial pressure of species B which is defined as:

$$p_{BLM} = \frac{p_{B2} - p_{B1}}{\ln \left| \frac{p_{B2}}{p_{B1}} \right|}$$

- The component A diffuses by concentration gradient : $-\frac{dy_A}{dx}$
- Here, flux is inversely proportional to the distance through which diffusion occurs and the concentration of the stagnant gas (p_{BLM}) because with increase in x and p_{BLM} resistance increases and flux decreases.

Example 2.1

CO_2 is diffusing through non-diffusing air under steady state conditions at a total pressure of 1 atmosphere and temperature 300 K. The partial pressure of CO_2 is 20 kPa at one point and 5 kPa at other point. The distance between the points is 5 cm. Calculate the flux of CO_2 . Given that at 300 K and at 1 atm, $D_{CO_2-air} = 2 \times 10^{-5} \text{ m}^2/\text{s}$

Solution:

Assume ideal gas, let air = B

$$N_{CO_2} = \frac{D_{CO_2-air}}{RT(x_2 - x_1)} \times \frac{P_t}{p_{BLM}} (P_{CO_{2,1}} - P_{CO_{2,2}})$$

Given that

$$D_{CO_2-air} = 2 \times 10^{-5} \text{ m}^2/\text{s} \text{ at } 300 \text{ K and } 1 \text{ atm}$$

$$P = 1 \text{ atm} = 101.3 \text{ kPa} = 1.013 \times 10^5 \text{ Pa}$$

$$T = 300 \text{ K}, x_2 - x_1 = 5 \text{ cm} = 0.05 \text{ m}$$

$$P_{CO_{2,1}} = 20 \text{ kPa} = 20,000 \text{ Pa}$$

$$P_{CO_{2,2}} = 5 \text{ kPa} = 5,000 \text{ Pa}$$

$$R = 8314 \text{ Pa m}^3/\text{kmol K}$$

$$p_{B,1} = P_t - P_{CO_{2,1}} = (101.3 - 20) \text{ kPa} = 81.3 \text{ kPa}$$

$$P_{B,2} = P_t - P_{CO_2,2} = (101.3 - 5) \text{ kPa} = 96.3 \text{ kPa}$$

$$P_{BLM} = \frac{P_{B,2} - P_{B,1}}{\ln\left(\frac{P_{B,2}}{P_{B,1}}\right)} = \frac{96.3 - 81.3}{\ln\left(\frac{96.3}{81.3}\right)} \text{ kPa} = 88.59 \text{ kPa} = 88590 \text{ Pa}$$

$$N_{CO_2} = \frac{2 \times 10^{-5} \text{ m}^2/\text{s} \times 1.013 \times 10^5 \text{ Pa}}{8314 \text{ Pa m}^3/\text{kmol K} \times 300 \text{ K} \times 0.05 \text{ m} \times 88590 \text{ Pa}} (20,000 - 5,000)$$

$$= \frac{2 \times 10^{-5} \times 1.013 \times 10^5 \times 15000}{8314 \times 300 \times 0.05 \times 88590} \text{ kmol/m}^2\text{s}$$

$$= 2.75 \times 10^{-6} \text{ kmol/m}^2\text{s}$$

2.5.3 Steady State Equimolar Counter Diffusion

This is the case for the diffusion of two ideal gases where an equal number of moles of the gases diffusing counter-current to each other.

In this case $N_B = -N_A = \text{constant}$ and $N_A + N_B = 0$.

This molar flux equation (2.7) at steady state can then be written as :

$$N_A = -CD_{AB} \frac{dy_A}{dx} + y_A N$$

where,

$$N = N_1 + N_2$$

⇒

$$N_A = -CD_{AB} \frac{dy_A}{dx}$$

For ideal gas,

$$C = \frac{P_t}{RT}$$

So,

$$N_A = -\frac{D_{AB} P_t}{RT} \cdot \frac{dy_A}{dx} \quad \dots(2.13)$$

Integrating the equation (2.13) with the boundary conditions : at $x \rightarrow x_1$, $y_A \rightarrow y_{A1}$, at $x \rightarrow x_2$, $y_A \rightarrow y_{A2}$, the equation of molar diffusion for steady-state equimolar counter diffusion can be represented as

$$N_A = \frac{D_{AB} P_t}{RT(x_2 - x_1)} (y_{A1} - y_{A2})$$

$$= \frac{D_{AB}}{RT(x_2 - x_1)} (p_{A1} - p_{A2})$$

Example 2.2

CO_2 is diffusing at steady state through a straight tube of 0.5 m long with an inside diameter of 0.05 m containing N_2 at 300 K and 1 atm pressure. The partial pressure of CO_2 at one end is 15 kPa and 5 kPa at the other end. Given that at 300 K and 1 atm. Given that $D_{CO_2-N_2} = 4 \times 10^{-5} \text{ m}^2/\text{s}$. Calculate the following for steady state equimolar counter diffusion :

(i) Molar flow rate of CO_2 ; (ii) Molar flow rate of N_2 .

Solution:

Assume ideal gas in equimolar counter diffusion for CO_2 flux :

$$N_{CO_2} = \frac{D_{CO_2-N_2}}{RT(x_2 - x_1)} (p_{CO_2,1} - p_{CO_2,2})$$



Student's Assignments

- Q1** A large tank filled with a mixture of gases *A* and *B* at 101 kPa and 298 K (25°C) is connected to another large tank filled with a mixture of *A* and *B* of different compositions of *A* and *B* at 101 kPa and 298 K (25°C). The tanks are connected by a tube of inner diameter of 50 mm and is 150 mm long. Calculate the steady state rate of transport of *A* through the tube when the concentration of *A* in one tank is 90 mole % and in the other is 5 mole % assuming uniformity in the composition in each tank and transfer takes by molecular diffusion. The diffusivity of *A* in *B* is $4.3 \times 10^{-3} \text{ m}^2/\text{s}$.
- Q2** Methane diffuses at steady state through a tube containing helium. At point 1, the partial pressure of methane is 55 kPa and at point 2 it is 15 kPa. The points 1 and 2 are 30 mm apart. The total pressure is 101.3 kPa and temperature is 298 K (25°C). Calculate the flux of CH_4 at steady state for equimolar counter diffusion. The diffusivity of methane at the prevailing conditions is $6.75 \times 10^{-5} \text{ m}^2/\text{s}$.
- Q3** Ammonia gas (*A*) diffuses through a non-diffusing nitrogen gas (*B*) under steady state conditions. The partial pressure of *A* at location 1 is $1.5 \times 10^4 \text{ Pa}$ and that at location 2 is $5 \times 10^3 \text{ Pa}$ (Pascal). The locations 1 and 2 are 0.15 m apart. The total pressure is $1.103 \times 10^5 \text{ Pa}$ and temperature is 298 K. Calculate the flux of diffusion of ammonia. Also calculate the flux of diffusion for equimolar counter diffusion considering that nitrogen is also diffusing. The diffusivity of ammonia at the prevailing conditions is $2.30 \times 10^{-5} \text{ m}^2/\text{s}$.
- Q4** Calculate the rate of diffusion of acetic acid (*A*) across a film of non-diffusing water (*B*) which is 1 mm thick at 290 K if the concentrations of acetic acid on the opposite sides of the film are 9% and 3% respectively. The densities of 9% and 3% solutions are 1012 kg/m^3 and 1003.2 kg/m^3 respectively. The diffusivity of acetic acid in water is $0.95 \times 10^{-9} \text{ m}^2/\text{s}$.
- Q5** Hydrogen gas at 202.6 kPa (2 atm) and 298 K (25°C) flows through a pipe made of unvulcanised neoprene rubber with i.d. and o.d. 25 and 50 mm, respectively. The diffusivity of hydrogen through rubber is $1.8 \times 10^{-6} \text{ cm}^2/\text{s}$. Calculate the rate of loss of hydrogen by diffusion per meter length of the pipe. The solubility of H_2 is $0.053 \text{ cm}^3 \text{ (NTP)}/\text{cm}^3 \text{ atm}$.
- Q6** In a gas mixture of hydrogen and oxygen, steady state equimolar counter diffusion is occurring at a total pressure of 100 kPa and temperature of 20°C. If the partial pressures of oxygen at two planes 0.01 m apart, and perpendicular to the direction of diffusion are 15 kPa and 5 kPa, respectively and the mass diffusion flux of oxygen in the mixture is $1.6 \times 10^{-5} \text{ kmol/m}^2 \text{ sec}$, calculate the molecular diffusivity for the system.
- Q7** In a gas mixture of oxygen and nitrogen, steady state equimolar counter diffusion is occurring at a total pressure of 101.325 kPa and temperature of 25°C. If the concentration of oxygen at two planes 0.16 m, apart and perpendicular to the direction of diffusion are 20 vol % and 15 vol % respectively. Calculate the rate of diffusion of the oxygen as expressed as $\text{kmol/m}^2\text{s}$. The molecular diffusivity of oxygen-nitrogen mixture at 101.325 kPa and 25°C is $1.56 \times 10^{-5} \text{ m}^2/\text{s}$.
- Q8** Pressurized hydrogen gas is stored at 358 K in a 4.8 m outer-diameter spherical container made of nickel. The shell of the container is 6 cm thick. The molar concentration of hydrogen in the nickel at the inner surface is determined to be 0.087 kmol/m^3 . The concentration of hydrogen in the nickel at the outer surface is negligible. Determine the mass flow rate of hydrogen by diffusion through the nickel container.
Data : $D_{AB} = 1.2 \times 10^{-12} \text{ m}^2/\text{s}$

Q.9 Consider a circular pipe of inner diameter ($D = 0.015$ m) whose inner surface is covered with a layer of liquid water as a result of condensation. In order to dry, the pipe, air at 300 K and 1 atm is forced to flow through it with an average velocity of 1.2 m/s. Using the analogy between heat and mass transfer, determine the mass transfer coefficient inside the pipe for fully developed flow. Data :

$$v = 1.58 \times 10^{-5} \text{ m}^2/\text{s};$$

$$D_{AB} = 2.54 \times 10^{-5} \text{ m}^2/\text{s}$$

Q.10 Which of the following assumption(s) is/are made in Chilton-Colburn analogy?

- Only a turbulent core is present.
- Velocity, temperature and concentration profiles are same.
- Schmidt number and Prandtl number are not equal to unity.
- Schmidt and Prandtl number are necessarily equal to one.

■ **ANSWERS**

- (1.95×10^{-6})
- (3.63×10^{-5})
- $(7.48 \times 10^{-7}, 6.19 \times 10^{-7})$
- (1.018×10^{-6})
- (7.72×10^{-2})
- (3.898×10^{-5})
- (2×10^{-7})
- (2.46×10^{-10})
- (0.00620)
- (a, b, c)

■ **Explanation**

1. (1.95×10^{-6})

Mole fraction of A in tank 1

$$x_{A1} = \frac{90}{100} = 0.90$$

Mole fraction of A in tank 2

$$x_{A2} = \frac{5}{100} = 0.05$$

$$D_{AB} = 4.3 \times 10^{-3} \text{ m}^2/\text{s}$$

$$z = 150 \text{ mm} = 0.15 \text{ m}$$

= Length of diffusion path

$$\text{Area} = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (0.015)^2$$

$$= 1.963 \times 10^{-3} \text{ m}^2$$

$$p_{A1} = x_{A1} \cdot P = 0.9 \times 101 = 90.9 \text{ kPa}$$

$$p_{A2} = x_{A2} \cdot P = 0.05 \times 101 = 5.05 \text{ kPa}$$

$$R = 8.31451 \text{ m}^3 \text{ kPa}/(\text{kmol} \cdot \text{K})$$

Rate of transport of A

$$= N_A \cdot A = \frac{D_{AB}(p_{A1} - p_{A2})}{RTz} \times A$$

$$= \frac{4.3 \times 10^{-3} \times (90.9 - 5.05) \times 1.963 \times 10^{-3}}{8.31451 \times 298 \times 0.15}$$

$$= 1.95 \times 10^{-6} \text{ kmol/s}$$

2. (3.63×10^{-5})

The flux for the equimolar counter diffusion is given by

$$N_A = \frac{D_{AB}}{RTz}(p_{A1} - p_{A2})$$

where, $D_{AB} = 6.75 \times 10^{-5} \text{ m}^2/\text{s}$

$$R = 8.31451 \text{ m}^3 \cdot \text{kPa}/(\text{kmol} \cdot \text{K})$$

$$T = 298 \text{ K}, z = 30 \text{ mm} = 0.03 \text{ m}$$

$$p_{A1} = 55 \text{ kPa}, p_{A2} = 15 \text{ kPa}$$

$$N_A = \frac{6.75 \times 10^{-5}(55 - 15)}{8.31451 \times 298 \times 0.03}$$

$$= 3.63 \times 10^{-5} \text{ kmol}/(\text{m}^2 \cdot \text{s})$$

3. $(7.48 \times 10^{-7}, 6.19 \times 10^{-7})$

1. The flux equation for the diffusion of A through a non-diffusing B is given by

$$N_A = \frac{D_{AB}P}{RTz p_{B,M}}(p_{A1} - p_{A2})$$

where, $D_{AB} = 2.30 \times 10^{-5} \text{ m}^2/\text{s}$

$$P = 1.103 \times 10^5 \text{ Pa}$$

$$p_{A1} = 1.5 \times 10^4 \text{ Pa}$$

$$p_{A2} = 5 \times 10^3 \text{ Pa}$$

$$p_{B1} = P - p_{A1}$$

$$= 1.103 \times 10^5 - 1.5 \times 10^4$$

$$= 8.63 \times 10^4 \text{ Pa}$$

$$p_{B2} = P - p_{A2}$$

$$= 1.103 \times 10^5 - 5 \times 10^3$$

$$= 9.63 \times 10^4 \text{ Pa}$$

$$p_{B,M} = \frac{p_{B2} - p_{B1}}{\ln\left(\frac{p_{B2}}{p_{B1}}\right)}$$

$$= \frac{(9.63 \times 10^4 - 8.63 \times 10^4)}{\ln\left(\frac{9.63 \times 10^4}{8.63 \times 10^4}\right)}$$

$$= 9.121 \times 10^4 \text{ Pa}$$

$$T = 298 \text{ K}, z = 0.15 \text{ m}$$

$$R = 8.31451 \text{ m}^3 \cdot \text{kPa}/(\text{kmol} \cdot \text{K})$$

$$= 8314.51 \text{ m}^3 \cdot \text{Pa}/(\text{kmol} \cdot \text{K})$$

$$N_A = \frac{2.30 \times 10^{-5} \times 1.103 \times 10^5 (1.5 \times 10^4 - 5 \times 10^3)}{8314.51 \times 298 \times 0.15 \times 9.121 \times 10^4}$$

$$= 7.484 \times 10^{-7} \text{ kmol}/(\text{m}^2 \cdot \text{s})$$

2. The flux equation for the equimolar counter diffusion is given by

$$N_A = \frac{D_{AB}}{RTz} (p_{A1} - p_{A2})$$

$$= \frac{2.30 \times 10^{-5} (1.5 \times 10^4 - 5 \times 10^3)}{8314.51 \times 298 \times 0.15}$$

$$= 6.19 \times 10^{-7} \text{ kmol}/(\text{m}^2 \cdot \text{s})$$

4. (1.018 × 10⁻⁶)

$$D_{AB} = 0.95 \times 10^{-9} \text{ m}^2/\text{s}$$

$$z = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

At location 1, on one side of the film :

$p_1 = 1012 \text{ kg}/\text{m}^2$, Acetic acid = 9% by weight

Mol. wt. of acetic acid (CH_3COOH) = 60,

Mol. wt. of water (H_2O) = 18

In 100 kg mixture there are 9 kg of acetic acid and 91 kg of water

$$\therefore x_{A1} = \frac{\text{Moles of A}}{\text{Total moles}} = \frac{\frac{9}{60}}{\frac{9}{60} + \frac{100-9}{18}}$$

$$= \frac{0.15}{0.15 + 5.05} = \frac{0.15}{5.2} = 0.0288$$

M_1 = Average molecular weight of solution

$$= \frac{\text{kg of solution}}{\text{kmol of solution}} = \frac{100}{5.2} = 19,21 \text{ kg}/\text{kmol}$$

Since $x_{A1} + x_{B1} = 1$

$$x_{B1} = 1 - 0.0288 = 0.9714$$

At location-2, on the other/opposite side of the film:

Weight % of acetic acid = 3%, $p_2 = 100.3 \text{ kg}/\text{m}^3$

$$\therefore x_{A2} = \frac{\frac{3}{60}}{\frac{3}{60} + \frac{100-3}{18}} = 0.0092$$

and $x_{AB} = 1 - 0.0092 = 0.9908$

$$M_2 = \frac{100}{\frac{3}{60} + \frac{100-3}{18}} = 18.40$$

$$C_{\text{avg}} = \frac{\frac{p_1}{M_1} + \frac{p_2}{M_2}}{2}$$

$$= \frac{\left(\frac{1012}{19.21}\right) + \left(\frac{1003.2}{18.40}\right)}{2}$$

$$= 53.6 \text{ kmol}/\text{m}^3$$

$$x_{B,M} = \frac{(x_{B2} - x_{B1})}{\ln\left(\frac{x_{B2}}{x_{B1}}\right)}$$

$$= \frac{(0.9908 - 0.9712)}{\ln\left(\frac{0.9908}{0.9712}\right)} = 0.980$$

The flux of acetic acid is

$$N_A = \frac{D_{AB} \cdot C_{\text{avg}}}{z \cdot x_{B,M}} (x_{A1} - x_{A2})$$

$$= \frac{0.95 \times 10^{-9}}{1 \times 10^{-3} \times 0.98} (0.0288 - 0.0092)$$

$$= 1.018 \times 10^{-6} \text{ kmol}/(\text{m}^2 \cdot \text{s})$$

5. (7.72 × 10⁻²)

Consider 1 m of the pipe,

$$z = r_2 - r_1 = \frac{\text{o.d.} - \text{i.d.}}{2}$$

$$= \frac{50 - 25}{2} = 12.5 \text{ mm}$$

$$= 0.0125 \text{ m}$$

$$D_A = 1.8 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$= 1.8 \times 10^{-10} \text{ m}^2/\text{s}$$

$$L = 1 \text{ m}$$

$$A_{\text{avg}} = \frac{2\pi L(r_2 - r_1)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi(1)(0.0125)}{\ln\left(\frac{0.025}{0.0125}\right)}$$

$$= 0.1133 \text{ m}^2$$

At 202.6 kPa (2 atm) hydrogen pressure, the solubility is

$$= 0.053 \times \frac{2}{1} = 0.106 \text{ cm}^3 (\text{NTP})/\text{cm}^3$$

$$= 0.106 \text{ m}^3 \text{ (NTP)/m}^3$$

∴ C_{A1} at the inner surface of the pipe

$$= \frac{0.106}{22.4} \times 1 = 4.73 \times 10^{-3} \text{ kmol H}_2/\text{m}^3$$

At the outer surface, $C_{A2} = 0$

The rate of loss of hydrogen per 1 m length of the pipe is

$$\begin{aligned} w &= N_A \cdot A_{\text{avg}} \\ &= \frac{D_A \cdot A_{\text{avg}} [C_{A1} - C_{A2}]}{z} \\ &= \frac{1.8 \times 10^{-10} \times 0.1133 (4.73 \times 10^{-3} - 0)}{0.0125} \\ &= 7.72 \times 10^{-2} \text{ kmol H}_2/\text{s per m} \end{aligned}$$

6. (3.898 × 10⁻⁵)

For equimolar countercurrent diffusion:

$$N_A = \frac{D_{AB}}{RTz} (\bar{p}_{A1} - \bar{p}_{A2})$$

where,

N_A = molar flux of A (1.6×10^{-5} kmol/m².sec);

D_{AB} = molecular diffusivity of A in B;

R = Universal gas constant (8.314 kJ/kmol.K);

T = Temperature in absolute scale (273 + 20 = 293 K);

z = distance between two measurement planes 1 and 2 (0.01 m);

\bar{p}_{A1} = partial pressure of A at plane 1 (15 kPa);

\bar{p}_{A2} = partial pressure of A at plane 2 (5 kPa)

Substituting these in eqn. (6.1);

$$1.6 \times 10^{-5} = \frac{D_{AB}}{(8.314)(293)(0.01)} (15 - 5)$$

Therefore, $D_{AB} = 3.898 \times 10^{-5}$ m²/sec

7. (2 × 10⁻⁷)

For steady state equimolar counter diffusion

$$N_A = \frac{D_{AB}}{RT\Delta z} (\bar{p}_{A1} - \bar{p}_{A2})$$

Data : $p_{A1} = 0.2 \times 101.325 = 20.265$ kPa

$p_{A2} = 0.15 \times 101.325 = 15.198$ kPa

$$R = 8.314 \frac{\text{m}^3 \text{kPa}}{\text{kmol.K}}, z = 0.16 \text{ m}$$

$$T = 273 + 25 = 298 \text{ K}$$

$$D_{AB} = 1.56 \times 10^{-5} \text{ m}^2/\text{s}$$

$$N_A = \frac{1.56 \times 10^{-5}}{8.314 \times 298 \times 0.16} (20.265 - 15.198)$$

$$N_A = 1.99 \times 10^{-7} \text{ kmol/m}^2\text{s} \approx 2 \times 10^{-7} \text{ kmol/m}^2\text{s}$$

8. (2.46 × 10⁻¹⁰)

The molar flow rate of hydrogen through this spherical shell by diffusion can be determined

$$\dot{N}_{\text{diff}} = 4\pi r_1 r_2 D_{AB} \left(\frac{C_{A1} - C_{A2}}{r_2 - r_1} \right)$$

$$r_2 = 2.40 \text{ m}, r_1 = 2.34 \text{ m}$$

$$\dot{N} = 4\pi (2.34)(2.40)(1.2 \times 10^{-12}) \frac{(0.087 - 0)}{(2.40 - 2.34)}$$

$$\dot{N} = 1.228 \times 10^{-10} \text{ kmol/s}$$

∴ The mass flow rate is determined by multiplying the molar flow rate by the molar mass of hydrogen, i.e., $M = 2$ kg/kmol.

$$\begin{aligned} \dot{m}_{\text{diff}} &= 2 \times 1.228 \times 10^{-10} \\ &= 2.46 \times 10^{-10} \text{ kg/s} \end{aligned}$$

9. (d)

The Reynolds number for this internal flow is

$$Re = \frac{V.D}{\nu} = \frac{1.2 \times 0.015}{1.58 \times 10^{-5}} = 1139$$

which is less than 2300, thus flow is laminar. Therefore, based on the analogy between heat and mass transfer, the Nusselt and the Sherwood numbers in this case $Nu = Sh = 3.66$. Using definition of Sherwood number, the MTC is

$$k_c = \frac{(Sh)D_{AB}}{D}$$

$$= \frac{(3.66)(2.54 \times 10^{-5})}{0.015}$$

$$k_c = 0.00620 \text{ m/s}$$

