# CHEMICAL ENGINEERING

## Instrumentation and Process Control

Comprehensive Theory
*with* Solved Examples *and* Practice Questions

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## Instrumentation and Process Control



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CHAPTER

1

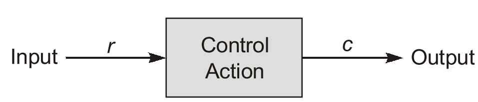
# Introduction

**Control System:**
Control system is a means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in accordance with a desired manner.
Control system can also be defined as the combination of elements arranged in a planned manner wherein each element causes an effect to produce a desired output.

Control systems are classified into two general categories as open-loop and close-loop systems.

## 1.1 OPEN LOOP CONTROL SYSTEMS

An open loop control system is one in which the control action is independent of the output.




*Open-loop control system*

This is the simplest and most economical type of control system and does not have any feedback arrangement.

Some common examples of open-loop control systems are
(a) Traffic light controller
(b) Electric washing machine
(c) Automatic coffee server
(d) Bread toaster

### Advantages of Open Loop Control Systems

(a) Simple and economic
(b) No stability problem

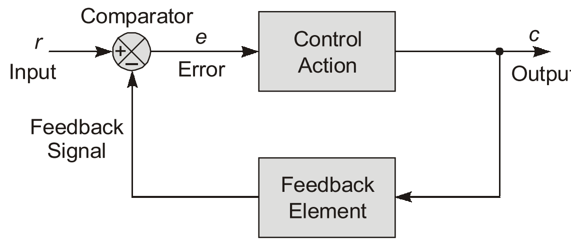
### Disadvantages of Open Loop Control Systems

(a) Inaccurate
(b) Unrealisable
(c) The effect of parameter variation and external noise is more

**Note:** Open loop control systems does not require performance analysis.

## 1.2 CLOSED LOOP CONTROL SYSTEMS

A *closed loop control system* is one in which the control action is some how dependent on the output.




*Closed loop control system*

The closed loop system has same basic features as of open loop system with an additional feedback feature. The actual output is measured and a signal corresponding to this measurement is feedback to the input section, where it is with the input to obtain the desired output.

Some common examples of closed loop control systems are:

(a) Electric iron
(b) DC motor speed control
(c) A missile launching system (direction of missile changes with the location of moving target)
(d) Radar tracking system
(e) Human respiratory system
(f) Autopilot system
(g) Economic inflation

### Advantages of Closed Loop Control Systems

(a) Accurate and reliable
(b) Reduced effect of parameter variation
(c) Bandwidth of the system can be increased with negative feedback
(d) Reduced effect of non-linearities

### Disadvantages of Closed Loop Control Systems

(a) The system is complex and costly
(b) System may become unstable
(c) Gain of the system reduces with negative feedback

**Remember**

- Feedback is not used for improving stability
- An open loop stable system may also become unstable when negative feedback is applied
- Except oscillators, in positive feedback, we have always unstable systems.

## 1.3 COMPARISON BETWEEN OPEN LOOP AND CLOSED LOOP CONTROL SYSTEMS

| Open Loop System | Closed Loop System |
|---|---|
| 1. So long as the calibration is good, open-loop system will be accurate | 1. Due to feedback, the close-loop system is more accurate |
| 2. Organization is simple and easy to construct | 2. Complicated and difficult |
| 3. Generally stable in operation | 3. Stability depends on system components |
| 4. If non-linearity is present, system operation degenerates | 4. Comparatively, the performance is better than open-loop system if non-linearity is present |

**Example 1.1** Match List-I (Physical action or activity) with List-II (Category of system) and select the correct code:

List-I
A. Human respiration system
B. Pointing of an object with a finger
C. A man driving a car
D. A thermostatically controlled room heater

List II
1. Man-made control system
2. Natural including biological control system
3. Control system whose components are both man-made and natural

Codes:

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 2 | 2 | 3 | 1 |
| (b) | 3 | 1 | 2 | 1 |
| (c) | 3 | 2 | 2 | 3 |
| (d) | 2 | 1 | 3 | 3 |

**Solution: (a)**

## 1.4 LAPLACE TRANSFORMATION

In order to transform a given function of time $f(t)$ into its corresponding Laplace transform first multiply $f(t)$ by $e^{-st}$, $s$ being a complex number ($s = \sigma + j\omega$). Integrate this product with respect to time with limits from zero to ∞. This integration results in Laplace transform of $f(t)$, which is denoted by $F(s)$ or $\mathcal{L} f[(t)]$.

The mathematical expression for Laplace transform is,

$$\mathcal{L} f[(t)] = F(s),\ t \geq 0$$

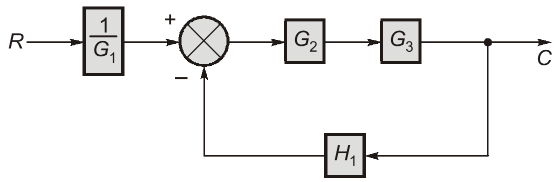
where,
$$F(s) = \int_0^{\infty} f(t) \cdot e^{-st} dt$$

The original time function f(t) is obtained back from the Laplace transform by a process called inverse Laplace transformation and denoted as $\mathcal{L}^{-1}$

Thus,
$$\mathcal{L}^{-1}[L f(t)] = \mathcal{L}^{-1}[F(s)] = f(t)$$

## Student's Assignments

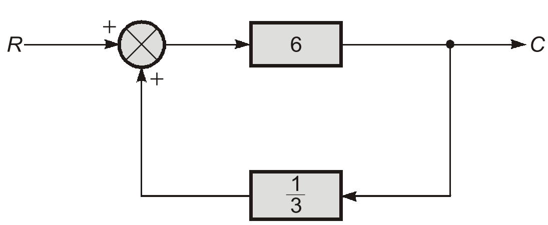
**Q.1** A feedback control system is shown below. Find the transfer function for this system.



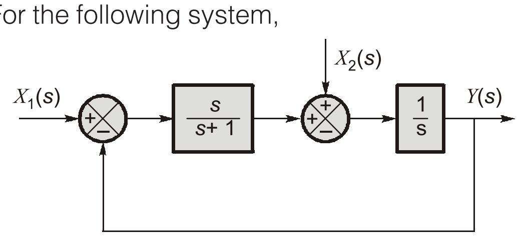

**Q.2** The step response of a system is given as $y = 1 - \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$. If the transfer function of this system is $\frac{(s+a)}{(s+b)(s+c)(s+d)}$ then $a + b + c + d$ is ________.

**Q.3** A system has the transfer function $\frac{(1-s)}{(1+s)}$. Its gain at $\omega = 1$ rad/sec is ________.

**Q.4** The close loop gain of the system shown below is




**Q.5** For the following system,

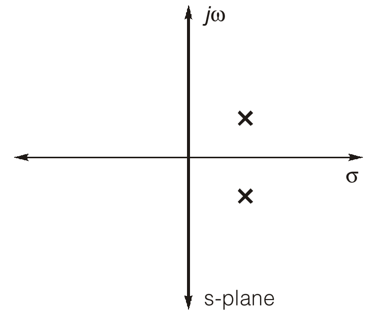



when $X_1(s) = 0$, the transfer function $\frac{Y(s)}{X_2(s)}$ is

(a) $\frac{s+1}{s^2}$  (b) $\frac{1}{s+1}$

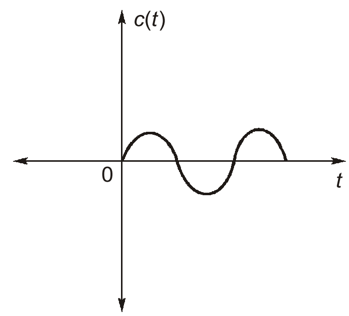
(c) $\frac{s+2}{s(s+1)}$  (d) $\frac{s+1}{s(s+2)}$

**Q.6** If closed-loop transfer function poles shown below

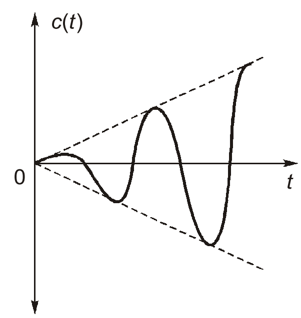



Impulse response is

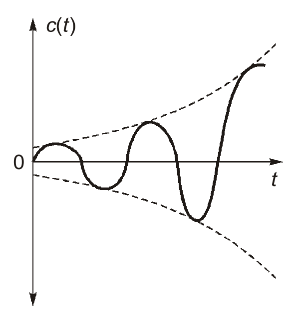
(a)



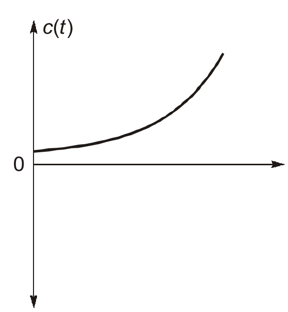

(b)




(c)




(d)



**Q.7** The impulse response of several continuous systems are given below. Which is/are stable?

1. $h(t) = te^{-t}$
2. $h(t) = 1$
3. $h(t) = e^{-t}\sin 3t$
4. $h(t) = \sin \omega t$

(a) 1 only (b) 1 and 3
(c) 3 and 4 (d) 2 and 4

**Q.8** Ramp response of the transfer function $F(s) = \dfrac{s+1}{s+2}$ is

(a) $\dfrac{1}{4} - \dfrac{1}{4}e^{-2t} + \dfrac{1}{2}t$ (b) $\dfrac{1}{4}e^{-2t} + \dfrac{1}{4} + \dfrac{1}{2}t$
(c) $\dfrac{1}{2} - \dfrac{1}{2}e^{-2t} + t$ (d) $\dfrac{1}{2}e^{-2t} + \dfrac{1}{2} - t$

**Q.9** Which of the following statements are correct?

1. Transfer function can be obtained from the signal flow graph of the system.
2. Transfer function typically characterizes to linear time invariant systems.
3. Transfer function gives the ratio of output to input in frequency domain of the system.

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

**Q.10** Which of the following is not a desirable feature of a modern control system?

(a) Quick response
(b) Accuracy
(c) Correct power level
(d) Oscillations

**Q.11** In regenerating feedback, the transfer function is given by

(a) $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$

(b) $\dfrac{C(s)}{R(s)} = \dfrac{G(s)\,H(s)}{1-G(s)\,H(s)}$

(c) $\dfrac{C(s)}{R(s)} = \dfrac{G(s)H(s)}{1+G(s)H(s)}$

(d) $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1-G(s)H(s)}$

**Q.12** Consider the following statements regarding the advantages of closed loop negative feedback control systems over open-loop systems:

1. The overall reliability of the closed loop systems is more than that of open-loop system.
2. The transient response in the closed loop system decays more quickly than in open-loop system.
3. In an open-loop system, closing of the loop increases the overall gain of the system.
4. In the closed-loop system, the effect of variation of component parameters on its performance is reduced.

Of these statements:

(a) 1 and 3 are correct
(b) 1, 2 and 4 are correct
(c) 2 and 4 are correct
(d) 3 and 4 are correct

**Q.13** Match **List-I** (Time function) with **List-II** (Laplace transforms) and select the correct answer using the codes given below lists:

| | List-I | | List-II |
|---|---|---|---|
| A. | $\left[af_1(t) + bf_2(t)\right]$ | 1. | $aF_1(s) + bF_2(s)$ |
| B. | $\left[e^{-at}f(t)\right]$ | 2. | $sF(s) + f(0)$ |
| C. | $\left[\dfrac{df(t)}{dt}\right]$ | 3. | $\dfrac{1}{s}F(s)$ |
| D. | $\left[\int_0^t f(x)\,dx\right]$ | 4. | $sF(s) - f(0^-)$ |
| | | 5. | $F(s+a)$ |

**Codes:**

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 5 | 2 | 3 | 4 |
| (b) | 1 | 5 | 4 | 3 |
| (c) | 2 | 1 | 3 | 4 |
| (d) | 1 | 5 | 3 | 4 |

**Q.14** If a system is represented by the differential equation, is of the form $\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} + 9y = r(t)$

(a) $k_1\, e^{-t} + k_2 e^{-9t}$ (b) $(k_1 + k_2)\, e^{-3t}$
(c) $ke^{-3t}\sin(t + \phi)$ (d) $te^{-3t}\, u(t)$

**Q.15** A linear system initially at rest, is subject to an input signal $r(t) = 1 - e^{-t}\,(t \geq 0)$. The response of the system for $t > 0$ is given by $c(t) = 1 - e^{-2t}$. The transfer function of the system is

(a) $\frac{(s+2)}{(s+1)}$ (b) $\frac{(s+1)}{(s+2)}$

(c) $\frac{2(s+1)}{(s+2)}$ (d) $\frac{(s+1)}{2(s+2)}$

## ANSWERS

**1.** (sol.) **2.** (15) **3.** (1) **4.** (–6) **5.** (d)
**6.** (c) **7.** (b) **8.** (a) **9.** (d) **10.** (*)
**11.** (d) **12.** (b) **13.** (*) **14.** (d) **15.** (c)

## Explanation

**1.** $\left(\frac{G_2G_3}{G_1(1+H_1G_2G_3)}\right)$

Multiply $G_2$ and $G_3$ and apply feedback formula and then again multiply with $\frac{1}{G_1}$

$$T(s) = \frac{G_2G_3}{G_1(1+G_2G_3H_1)}$$

**2.** **(15)**

$$y = 1-\frac{7}{3}e^{-t}+\frac{3}{2}e^{-2t}-\frac{1}{6}e^{-4t}$$

$$p(t) = \frac{dy}{dt}$$

$$= \frac{7}{3}e^{-t}+\frac{3}{2}\times(-2)\times e^{-2t}-\left(\frac{1}{6}\right)(-4)e^{-4t}$$

Laplace transform of $p(t)$

$$p(s) = \frac{\frac{7}{3}}{s+1}+\frac{-3}{s+2}+\frac{\frac{2}{3}}{s+4}$$

$$= \frac{s+8}{(s+1)(s+2)(s+4)}$$

$\Rightarrow$ $a + b + c + d = 15$

**3.** **(1)**

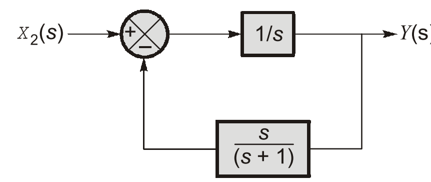
For all pass system, gain = '1' at all frequencies.

**4.** **(–6)**

$$\text{C.L.T.F.} = \frac{6}{1-6\times\frac{1}{3}} = \frac{6}{-1} = -6$$

**5.** **(d)**

Redrawing the block diagram with $X_1(s) = 0$




The transfer function

$$T(s) = \frac{Y(s)}{X_2(s)} = \frac{G(s)}{1+G(s)H(s)} \quad \text{...(i)}$$

Here, $G(s) = \frac{1}{s}$ and $H(s) = \frac{s}{s+1}$

$$\frac{Y(s)}{X_2(s)} = \frac{\frac{1}{s}}{1+\frac{1}{s}\times\frac{s}{s+1}} = \frac{(s+1)}{s(s+2)}$$

**6.** **(c)**

$$\text{T.F.} = \frac{1}{[s-(\sigma+j\omega)][s-(\sigma-j\omega)]}$$

$$= \frac{1}{[(s-\sigma)-j\omega)][(s-\sigma)+j\omega)]}$$

$$= \frac{1}{\left[(s-\sigma)^2-(j\omega)^2\right]} = \frac{1}{\left[(s-\sigma)^2+\omega^2\right]}$$

For impulse response, taking its inverse Laplace transformation we get,

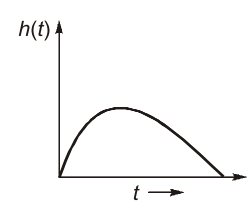
$$c(t) = e^{\sigma t}\sin\omega t$$

So, option (c) is correct.

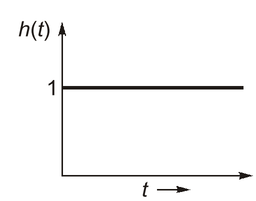
**7.** **(b)**

If the impulse response decays to zero as time approaches infinity, the system is stable.

(1)




(2)



CHAPTER

2

# Dynamic Behaviour of First and Second Order System

## 2.1 INTRODUCTION

The **time response** of a control system means how the response behaves in accordance with time when specified input test signal is applied to a system.

## 2.2 TRANSIENT AND STEADY STATE RESPONSE

In the initial part of time response of a control system, transients appear and during the post transient period, steady state is achieved. Theoretically, steady state means a state of the output of a control system as the time approaches to infinity (i.e. after the transient has died out) after initiation of the input. In actual practice the **transient period** and **steady state period** is identified in terms of time constants of a control system. Thus, the time response of a control system consists of two parts namely:

(a) Transient response
(b) Steady state response

$$c(t) = c_{tr}(t) + c_{ss}(t)$$

The transient part of time response reveals the nature of response (i.e. oscillatory or nature of damping) and also gives an indication about its speed.

The steady state part of time response reveals the accuracy or steady state error of a control system.

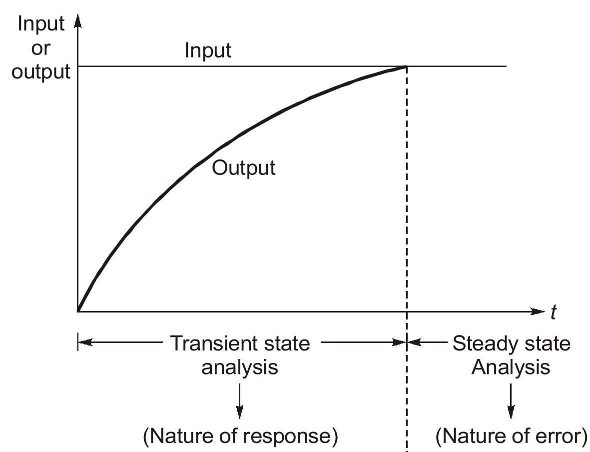
### Input Test Signals

Unlike electric networks and communication systems, the actual inputs of a control system may vary in random fashion with respect to time. For instance, in a radar tracking system for antiaircraft missiles, the position and speed of the target to be tracked may vary in an unpredictable manner, so that they cannot be predetermined. This poses a problem for the designers, since it is difficult to design a control system so that it will perform satisfactorily to all possible forms of input signals. For the purpose of analysis and design, it is necessary to assume some basic types of test inputs so that the performance of a system can be evaluated. By selecting these basic test signals properly, not only the mathematical treatment of the problem gets systematized, but also the response due to these inputs allows the prediction of the system's performance to other more complex inputs. This approach is particularly useful for linear systems, since the response to complex signals can be determined by superimposing those due to simple test signals.

The commonly used test input signals are step functions, ramp functions, acceleration functions, impulse functions and sinusoidal functions. In this chapter we use test signals such as step, ramp, parabolic and impulse signals. With these test signals, mathematical and experimental analysis of control systems can be carried out easily, since the signals are very simple functions of time.

If the inputs to a control system are gradually changing functions of time, then a ramp function of time may be a good test signal. Similarly, if a system is subjected to sudden disturbances, a step function of time may be a good test signal; and for a system subjected to shock inputs, an impulse function may be best. Once a control system is designed on the basis of test signals, the performance of the system in response to actual inputs is generally satisfactory.

**Standard Test Signal**




| | | | |
|---|---|---|---|
| (i) | Step signal | Sudden input | Time domain Analysis |
| (ii) | Ramp signal | Velocity type input | |
| (iii) | Parabolic Signal | Acceleration type input | |
| (iv) | Impulse signal | Sudden shocks → | Stability analysis |

Signal (i) and (iv) $\Rightarrow$ Bounded signal

Signal (ii) and (iii) $\Rightarrow$ Unbounded signal

- The steady state response depends on the type of control system.
- Transient state or nature of response of the system depends on order of the system.
- Order of the system is obtain from closed loop T.F. $\dfrac{G(s)}{1+G(s)H(s)}$.
- The highest power of the characteristic equation $1 + G(s)\,H(s) = 0$ determine the order of the control system.

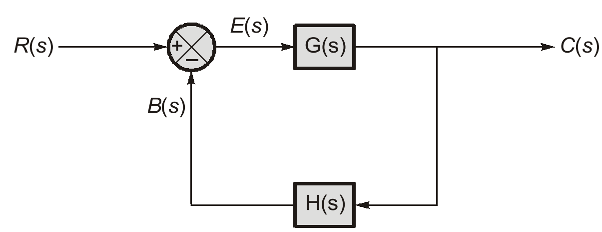
**Note:** Natural time constant of response only depends on poles of the system.

## 2.3 STEADY STATE ERROR

A desirable feature of a control system is the faithful following of its reference input by the output. However, if the actual output of a control system during steady state deviates from the reference input (i.e. desired output), the system is said to posses a steady state error.

These errors arise from the nature of inputs, type of system and from non-linearities of system components such as static friction, backlash, etc. and further aggravated by amplifier drifts, aging or deterioration.

Consider the control system,




*Block diagram of a closed-loop control system*

Where $R(s)$ is reference input, $C(s)$ is output, $B(s)$ is feedback signal, $H(s)$ is feedback gain, $G(s)$ is forward path gain (or plant gain) and the difference between the reference input and feedback signal is the error signal $E(s)$.

$\therefore$ $$E(s) = R(s) - B(s)$$
$$= R(s) - C(s)\,H(s)$$
$$= R(s) - E(s)\,G(s)\,H(s)$$

$\Rightarrow$ $$E(s) = \frac{R(s)}{1+G(s)\cdot H(s)} \quad \cdots \text{ Error ratio}$$

Steady state error, $$e_{ss} = \lim_{t\to\infty} e(t)$$
$$= \lim_{s\to 0} sE(s) \qquad \text{(Using final value theorem)}$$

Hence, $$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)\,H(s)}$$

Thus, the magnitude of the steady state error in a close-loop control system depends on its open loop transfer function, $G(s)\,H(s)$ and input signal, $R(s)$. More specifically, we can see that $e_{ss}$ depends on the number of poles that $G(s)$ has at $s = 0$ (i.e. type of the control system).

As the steady state error is an index of accuracy of a control system, therefore it should be minimum as far as possible.

**Example 2.1** When the time period of observation is large, the type of the error is

(a) Transient error
(b) Steady state error
(c) Half-power error
(d) Position error constant

**Solution : (b)**

Steady state error is the error at $t \to \infty$.

| Input function | Time response expression | Remarks |
|---|---|---|
| Unit ramp $\frac{1}{s^2}$ | $c(t) = t - T + Te^{-t/T}$ | Differentiate ↓ ↑ |
| Unit step $\frac{1}{s}$ | $c(t) = 1 - e^{-t/T}$ | |
| Unit impulse 1 | $c(t) = \frac{1}{T}e^{-t/T}$ | Integrate |

It is observed that step function is first derivative of a ramp function and impulse function is first derivative of a step function. From the derived time response expression it is concluded that the output time response also follows the same sequence as that of input functions.

**Example 2.2** A unit step is applied at $t = 0$ to a first order system without time delay. The response has a value of 1.264 units at $t = 10$ mins. and 2 units at steady state. The transfer function of the system is ________.

**Solution :**

$$\theta(t) = K[1 - e^{-t/\tau}]$$

at $t = \infty$,

$$2 = K[1 - e^{-\infty}]$$

∴ $K = 2$

at $t = 10$,

$$1.264 = 2[1 - e^{-10/\tau}]$$

$$0.632 = 1 - e^{-10/\tau}$$

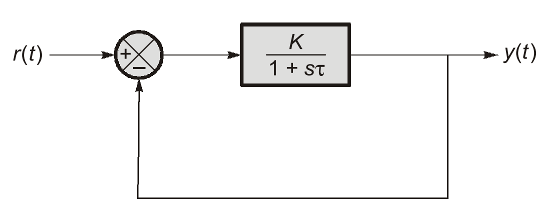
$$e^{-10/\tau} = 0.368$$

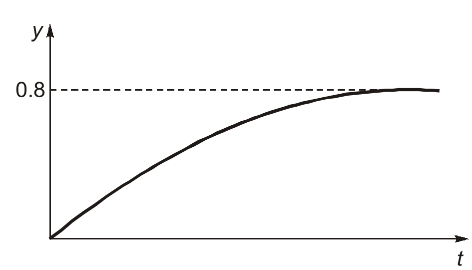
$$\frac{-10}{\tau} = \ln(0.368)$$

∴ $\tau = 10 \text{ min} = 600 \text{ sec}$

$$G(s) = \frac{K}{(1+s\tau)} = \left(\frac{2}{1+600s}\right)$$

**Example 2.3** If a first order system and its time response to a unit step input are as shown below, the gain $K$ is

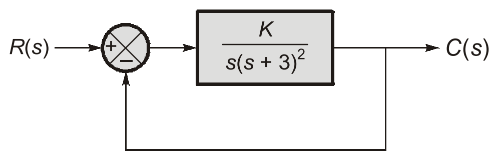



(a) 0.25
(b) 0.8
(c) 1
(d) 4

## Student's Assignments

**Q.1** By properly choosing the value of '$K$', the output $c(t)$ of the system as shown in the following figure can be made to oscillate sinusoidally at a frequency (in rad/sec) of ________ rad/sec.




**Common Data Question (2 and 3):**

The transfer function of a system is given as $\frac{100}{s^2+ks+100}$. Settling time is 4 sec. (Assuming 2% tolerance band)

**Q.2** The frequency of damped oscillation is ________ rad/sec.

**Q.3** Number of damped oscillation is ________.

**Q.4** The transfer function of a system is $\frac{6}{s^2+2s+4}$, the peak overshoot of the system to a step input of four is ________.

**Q.5** A unity feedback system has an open loop transfer function $G(s) = \frac{6}{s^2(s+2)}$. Find steady state error to an input $r(t) = 12t^2 + 4t + 2$.

**Q.6** O.L.T.F. of an unity feedback system is $G(s) = \frac{10}{s-2}$. The time constant of corresponding close loop system is ________ sec.

**Common Data Question (7 and 8):**

When the system shown in figure (1) given below is subjected to a unit-step input, the system output response is as shown below in figure (2). Then

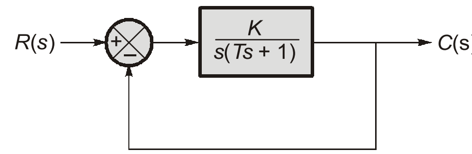



Fig. (1)

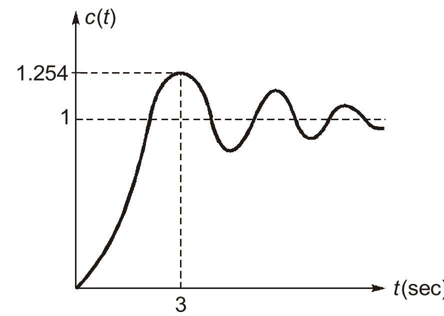



Fig. (2)

**Q.7** The value of $T$ is ________ sec.

**Q.8** The value of $K$ is ________.

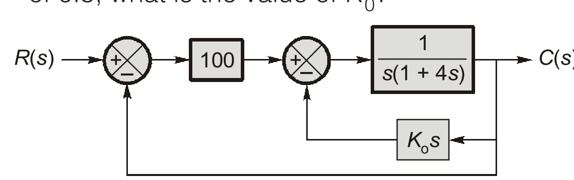
**Q.9** Consider the systems with the following open-loop transfer functions:

1. $\frac{36}{s+3.6}$ 2. $\frac{100}{s+5}$ 3. $\frac{6.25}{s+4}$

Find the correct sequence of these systems in increasing order of the time taken to settle down when subjected to an unit step input.

**Q.10** A unity-feedback control system has the open-loop transfer function $G(s) = \frac{4(1+2s)}{s^2(s+2)}$. If the input to the system is unit ramp, find the steady state error.

**Q.11** Output rate control is used to improve the damping of the system given below. If the closed-loop system has to have a damping factor of 0.5, what is the value of $K_0$?



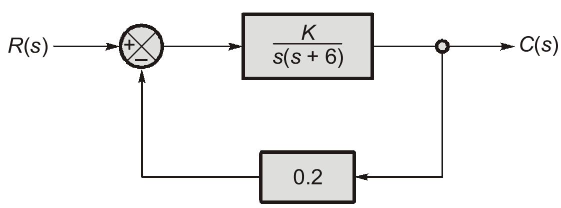

**Q.12** Find $e_{ss}$ for input $r(t) = 5 + 2t$ and open-loop transfer function $G(s)\,H(s) = \frac{100}{s(s+2)}$.

**Q.13** A system described by the following differential equation $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=x(t)$ is initially at rest. For input $x(t) = 2u(t)$, find the output $y(t)$.

**Q.14** The step response of the system is $c(t) = 1 - e^{-3t} + 3e^{-t}$, $(t > 0)$. Find the gain of the system transfer function in time constant form.

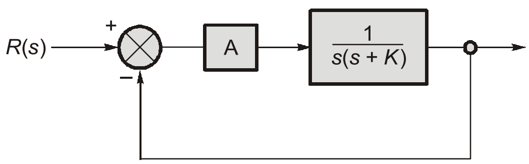
**Q.15** The unit impulse response of a system is
$h(t) = e^{-t}$, $t \geq 0$
For this system, find the steady-state value of the output for unit step input.

**Q.16** A closed loop system is shown in following figure:




The system is to have a damping ratio of 0.7. Determine the value of $K$ to satisfy this condition.

**Q.17** A step signal of magnitude 2 is applied as input to the following system. Determine the value of $K$ and A such that damping ratio is 0.6 and damped natural frequency is 8 rad/sec.




**Q.18** The open loop transfer function of unity feedback control system is given by $G(s)=\frac{25}{s(s+5)}$.
Determine the damped frequency of oscillation.

**Q.19** For the system, $\frac{C(s)}{R(s)}=\frac{16}{s^2+8s+16}$
The nature of the time response will be
(a) Overdamped (b) Underdamped
(c) Critically damped (d) None of these

**Q.20** Which of the following transfer function will have greatest maximum overshoot?
(a) $\frac{9}{s^2+2s+9}$ (b) $\frac{16}{s^2+2s+16}$
(c) $\frac{25}{s^2+2s+25}$ (d) $\frac{36}{s^2+2s+36}$

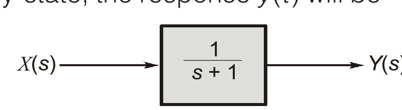
**Q.21** The system is originally critically damped. If the gain is doubled then it will become
(a) Remains same (b) Overdamped
(c) Underdamped (d) Undamped

**Q.22** The forward path transfer function of unity feedback control system is given by $G(s)=\frac{2}{s(s+3)}$. Obtain an expression for unit step response of the system.
(a) $1 + 2e^{-t} + e^{-2t}$
(b) $1 + e^{-t} - 2e^{-2t}$
(c) $1 - e^{-t} + 2e^{-2t}$
(d) $1 - 2e^{-t} + e^{-2t}$

**Q.23** Find the initial value and final values of the following function:
$$F(s) = \frac{12(s+1)}{s(s+2)^2(s+3)}$$
(a) 1, ∞ (b) 0, ∞
(c) ∞, 1 (d) 0, 1

**Q.24** The impulse response of an initially relaxed linear system is $e^{-2t}u(t)$. To produce a response of $te^{-2t}u(t)$, the input must be equal to
(a) $2e^{-t}u(t)$ (b) $\frac{1}{2}e^{-2t}u(t)$
(c) $e^{-2t}u(t)$ (d) $e^{-t}u(t)$

**Q.25** In the system shown below, $x(t) = \sin t\, u(t)$. In steady-state, the response $y(t)$ will be




(a) $\frac{1}{\sqrt{2}}\sin\left(t-\frac{\pi}{4}\right)$ (b) $\frac{1}{\sqrt{2}}\sin\left(t+\frac{\pi}{4}\right)$
(c) $\frac{1}{\sqrt{2}}e^{-t}\sin t$ (d) $\sin t - \cos t$

**Q.26** The response of a stable process is
(a) underdamped
(b) overdamped
(c) either undamped or overdamped
(d) always critically damped

**Q.27** In case of a stable process, once the value of the variable is disturbed
(a) The output returns back to the desired level.
(b) The output never returns back to the desired level.
(c) The output may or may not come to the desired level.
(d) None of the above

**Q.28** Find the time constant of the following process

$$T(s) = \frac{1}{(4s+2)}$$

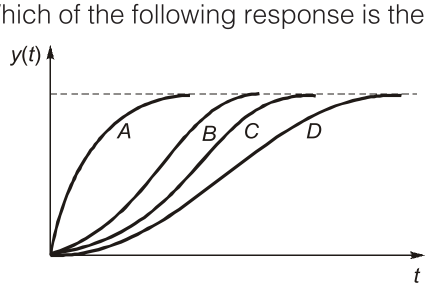
(a) 2 sec (b) 4 sec
(c) 1 sec (d) $\frac{1}{2}$ sec

**Q.29** The dynamics of a second order system is given as:

$$9\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 1 = 0$$

Then the system is
(a) overdamped
(b) underdamped
(c) critically damped
(d) overdamped with ($\zeta = 1.5$)

**Q.30** Which of the following response is the fastest?




(a) *D* (b) *C*
(c) *B* (d) *A*

**Q.31** The response of interacting capacities is always
(a) underdamped
(b) overdamped
(c) critically damped
(d) may be underdamped or overdamped

**Q.32** Liquid is transmitted from one point of a cylindrical pipe of length 2 m. If the velocity of the fluid inside the pipe is 0.5 m/sec. Find the dead time in the process.
(a) 4 sec (b) 8 sec
(c) 2 sec (d) 16 sec

**Q.33** $u(t)$ represents the unit step function. The Laplace transfer of $u(t-\tau)$ is
(a) $\frac{1}{s\tau}$ (b) $\frac{1}{s-\tau}$
(c) $\frac{e^{-s\tau}}{s}$ (d) $e^{-s\tau}$

## ANSWERS

| | | | | |
|---|---|---|---|---|
| **1.** (3) | **2.** (9.95) | **3.** (6.33) | **4.** (0.978) | **5.** (8) |
| **6.** (0.125) | **7.** (1.09) | **8.** (1.42) | **9.** (2,3,1) | **10.** (0) |
| **11.** (19) | **12.** (0.04) | **13.** (Sol) | **14.** (–2) | **15.** (1) |
| **16.** (91.8) | **17.** (12, 100) | | **18.** (4.3) | **19.** (c) |
| **20.** (d) | **21.** (c) | **22.** (d) | **23.** (d) | **24.** (c) |
| **25.** (a) | **26.** (a) | **27.** (a) | **28.** (a) | **29.** (b) |
| **30.** (d) | **31.** (b) | **32.** (a) | **33.** (c) | |

## Explanation

**1. (3)**

The characteristic equation, $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s+3)^2} = 0$$

$$s^3 + 6s^2 + 9s + K = 0$$

Now, the Routh's table is

| | | |
|---|---|---|
| $s^3$ | 1 | 9 |
| $s^2$ | 6 | $K$ |
| $s^1$ | $\frac{54-K}{6}$ | |
| $s^0$ | $K$ | |

For the system to oscillate sinusoidally,

$$\frac{54-K}{6} = 0$$

$\therefore \quad K = 54$

$\therefore$ Auxiliary equation becomes

$$6s^2 + K = 0$$
$$6s^2 + 54 = 0$$
$$s^2 = -9$$
$$\therefore \quad s = \pm 3j$$

$\therefore$ Frequency of oscillations is 3 rad/sec.

**2. (9.95)**

$$\frac{4}{\xi\omega_n} = 4$$
$$\Rightarrow \quad \xi\omega_n = 1$$
$$\Rightarrow \quad \xi = \frac{1}{10} = 0.1$$
$$\omega_n^2 = 100, \ \omega_n = 10 \text{ rad/sec.}$$
$$\omega_d = \omega_n\sqrt{1-\xi^2} = 10\sqrt{1-(0.1)^2}$$
$$= 9.95 \text{ rad/sec}$$

**3. (6.33)**

$$f_d = \frac{\omega_d}{2\pi} = \frac{9.95}{2\pi} = 1.583 \text{ Hz}$$

Number of cycles = 1.583 × 4 sec

$$= 6.3342 \approx 6.33$$

**4. (0.978)**

$$\text{Peak overshoot} = C_{ss} = e^{-\pi\xi/\sqrt{1-\xi^2}}$$
$$2\xi\omega_n = 2, \ \xi = 0.5$$
$$C_{ss} = \lim_{s\to 0} s\cdot\frac{6}{s^2+2s+4}\times\frac{4}{s} = 6$$

and $e^{-\xi\pi/\sqrt{1-\xi^2}} = e^{-0.5\pi/\sqrt{1-(0.5)^2}} = 0.1630$

$$\text{Peak overshoot} = 6 \times 0.1630 = 0.9782$$

**5. (8)**

$$e_{ss} = \lim_{s\to 0}\frac{s\,R(s)}{1+G(s)H(s)}$$
$$= \lim_{s\to 0} s\cdot\frac{\left[\frac{12\times 2}{s^3}+\frac{4}{s^2}+\frac{2}{s}\right]}{1+\frac{6}{s^2(s+2)}}$$
$$e_{ss} = \lim_{s\to 0} s\cdot\frac{\frac{24}{s^2}+\frac{4}{s}+2}{\frac{s^2(s+2)+6}{s^2(s+2)}}$$
$$= \lim_{s\to 0}\frac{(24+4s+2s^2)(s+2)}{s^2(s+2)+6}$$
$$e_{ss} = \frac{24\times 2}{6} = 8$$

**6. (0.125)**

$$1 + G(s)H(s) = 1 + \frac{10}{s-2} = 0$$
$$\Rightarrow \quad s + 8 = 0$$
$$\text{C.L.T.F} = \frac{10}{s+8} = F(s)$$

Taking inverse Laplace, $f(t) = 10e^{-8t}$ or $10e^{-t/\tau}$

$$\tau = \frac{1}{8} = 0.125 \text{ sec}$$

**7. (1.09) & 8. (1.42)**

$$0.254 = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}, \ \xi \approx 0.4 \text{ (or) } 0.399$$
$$t_p = 3 = \frac{\pi}{\omega_n\sqrt{1-0.4^2}}$$
$$\omega_n = 1.14 \text{ rad/sec}$$

From the block diagram given in problem

$$\frac{C(s)}{R(s)} = \frac{K}{Ts^2+s+k}, \ \omega_n = \sqrt{\frac{K}{T}}$$
$$2\xi\omega_n = \frac{1}{T}$$
$$T = \frac{1}{2\xi\omega_n} = \frac{1}{2\times 0.4\times 1.14} = 1.09 \text{ sec}$$
$$K = \omega_n^2 T = 1.14^2 \times 1.09 = 1.42$$

**9. (2, 3, 1)**

$$\frac{C(s)}{R(s)} = \text{T.F.}$$
$$\Rightarrow \quad C(s) = R(s)\times \text{T.F.}$$
$$C(s) = \frac{1}{s}\times \text{T.F.}$$
$$\text{(i)} \quad C(s) = \frac{1}{s}\times\frac{36}{s+3.6}$$
$$= \frac{36}{3.6}\left(\frac{1}{s}-\frac{1}{s+3.6}\right)$$
$$\Rightarrow \quad c(t) = 10\,(1-e^{-3.6t})$$