CHEMICAL ENGINEERING

Heat Transfer

Comprehensive Theory
with Solved Examples *and* Practice Questions

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1

Introduction and Basic Concepts

LEARNING OBJECTIVES

The reading of this chapter will enable the students:

- To understand how thermodynamics and heat transfer are related to each other.
- To understand the various modes of heat transfer.
- To understand the physical mechanisms of different modes of heat transfer and the basic laws that govern the process of heat transfer in different modes.

1.1 INTRODUCTION

- Before 18th century, heat was defined as calorific fluid when it get added in any system, system get heated and when it released from any system, system get cooled.
- The definition of **'heat'** is provided by classical thermodynamics. It is defined as an energy that flows due to difference in temperature.
- Heat flows in a direction from higher temperature to lower temperature.
- Heat energy can neither be observed nor be measured directly. However, the effects produced by the transfer of this energy are amenable to observations and measurements.

1.1.1 Difference between Thermodynamics and Heat Transfer

- Thermodynamics deals with the amount of heat transfer as a system undergoes a process from one equilibrium state to another, and makes no reference to how long the process will take.
- Where as the science of heat transfer deals with the rate of heat transfer, which is the main quantity of interest in the design and evaluation of heat transfer equipment.
- Heat transfer deals with modes of heat transfer and temperature profile within the object.

1.1.2 Temperature

Temperature is measure of amount of energy caused by the molecules. It tells about the hotness and coldness of the object. Temperature difference is driving force for heat transfer.

1.2 MODES OF HEAT TRANSFER

The process of heat transfer taken as place by three distinct modes: Conduction, Convection and Radiation.

1.2.1 Conduction

The mechanism of heat transfer due to a temperature gradient in a stationary medium is called conduction. The medium may be a solid or a fluid. In liquids and gases, conduction is due to the collisions of molecules in course of their random motions. In solids, the conduction of heat is attributed to two effects:

(i) the flow of free electrons and

(ii) the lattice vibrational waves caused by the vibrational motions of the molecules at relatively fixed positions called a lattice.

The law which describes the rate of heat transfer in conduction is known as Fourier's law.

According to Fourier's law,

$$q_x = -k\frac{dT}{dx} \quad ...(1.1)$$

Assumptions of Fourier's Law

(i) Steady state heat conduction.

(ii) Linear temperature profile.

(iii) No heat generation within object.

(iv) Object faces are isothermal (means no change in temperature with time).

(v) One direction flow of heat.

(vi) Isotropic material (thermal conductivity must be constant)

- Where q_x is the rate of heat flow per m^2 of heat area normal to the direction of heat flow.
- The minus sign in Equation (1.1) indicates that heat flows in the direction of decreasing temperature.
- The constant k is known as thermal conductivity.

When the temperature becomes a function of three space coordinates, say, x, y, z in a rectangular Cartesian frame, heat flows along the three coordinate directions. Equation (1.1) under the situation, is written in vector form as

$$q = -k\nabla T \quad ...(1.2)$$

where, $$q = iq_x + jq_y + kq_z$$

and, $$\nabla T = i\frac{\partial T}{\partial x} + j\frac{\partial T}{\partial y} + k\frac{\partial T}{\partial z}$$

Example 1.1 The rate of heat transfer from a hot surface to a cold surface is directly proportional to the difference in temperature between the two surfaces and the surface area normal to the direction of heat flow. This is

(a) Newton's law of cooling
(b) Kirchhoff's law
(c) Fourier's law
(d) Wien's law

Answer : (c)

Example 1.2 Heat transfer takes place according to

(a) Fick's law
(b) Zeroth law of thermodynamics
(c) First law of thermodynamics
(d) Second law of thermodynamics

Answer : (d)

NOTE

- Thermal conductivity is a transport property of the medium through which heat is conducted.
- For an isotropic medium, the thermal conductivity *k* is a scalar quantity which depends upon temperature only.

1.2.2 Convection

The mode by which heat is transferred between a solid surface and the adjacent fluid in motion when there is a temperature difference between the two is known as convection heat transfer.

- The mode of convective heat transfer comprises of two mechanisms:
  (i) Conduction at the solid surface and
  (ii) Advection by the bulk or macroscopic motion of the fluid a little away from the solid surface.
- The convection is of two types: **Forced convection** and **Free convection**.
- In **Forced convection**, the fluid is forced to flow over a solid surface by external means such as fan, pump or atmospheric wind.
- When the fluid motion is caused by the buoyancy forces that are induced by density differences due to the variation in temperature in the fluid, the convection is called **Natural** (or **Free**) **convection**.

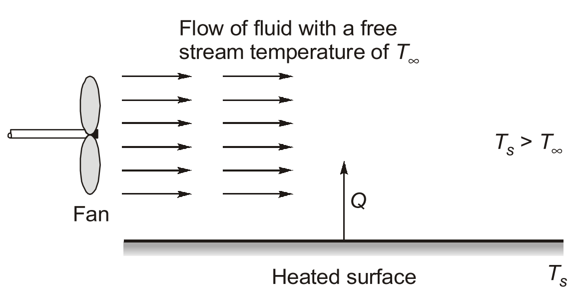


Figure 1.1 Forced convective heat transfer from a horizontal surface

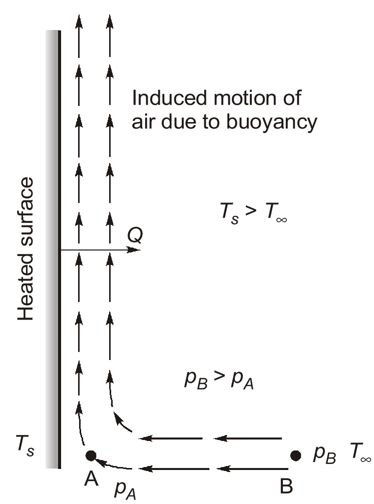


Figure 1.2 Free convective heat transfer from a heated vertical surface

- Irrespective of the details of the mechanism, the rate of heat transfer by convection (both forced and free) between a solid surface and a fluid is calculated from the relation

$$Q = \bar{h} A \Delta T \qquad \text{...(1.3)}$$

This equation is known as Newton's law of cooling.

where, Q = Rate of heat transfer by convection

A = Heat transfer area

$\Delta T = (T_s - T_f)$, is the difference between the surface temperature T_s and the temperature of the fluid T_f at some reference location.

$\bar{h}$ = Average convective heat transfer coefficient over the area A.

Solution :

Applying equation (1.5), we have

$$\frac{E}{A} = \in \sigma T^4 = 0.9 \times 5.67 \times 10^{-8} \times (50 + 273)^4 = 555.44 \text{ W/m}^2$$

1.3 THERMAL CONDUCTIVITY

Thermal conductivity of a material can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference. The thermal conductivity of a material is a measure of the ability of the material to conduct heat. A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or insulator. The thermal conductivities of some common materials at room temperature are given in Table 1.1.

Table 1.1 Thermal conductivity of some materials at room temperature (300 K)

Material	k(W/(m°C))
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminium	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.613
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Refrigerant-12	0.072
Glass fibre	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

1.3.1 Solids

In solids, heat conduction is due to two effects - flow of free electrons and propagation of lattice vibrational waves. The thermal conductivity is therefore determined In the addition of these two components. In a pure metal, the electronic component is more prominent than the component of lattice vibration and gives rise to a very high value of thermal conductivity. The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. Highly ordered crystalline non-metallic solids like diamond, silicon, quartz exhibit very high thermal conductivities (more than that of pure metals) due to lattice vibration only, but are poor conductors of electricity.

1.4 THERMAL CONDUCTIVITY OF LIQUIDS AND GASES

The thermal conductivity for liquids and gases is attributed to the transfer of kinetic energy between the randomly moving molecules due to their collisions. The kinetic theory of gases predicts and the experiments confirm that the thermal conductivity of gases is proportional to the square root of the thermodynamic temperature T, and inversely proportional to the square root of the molar mass M. Therefore, the thermal conductivity of a gas increases with increasing temperature and decreasing molar mass. So it is not surprising that the thermal conductivity of helium (M = 4) is much higher than those of air (M = 29) and argon (M = 40).

Unlike gases, the thermal conductivities of most liquids decrease with increasing temperature, with water being a notable exception. Like gases, the conductivity of liquids decreases with increasing molar mass.

NOTE

- The thermal conductivity of gases is independent of pressure in a wide range of pressures encountered in practice.
- Because of large intermolecular spaces and hence a smaller number of molecular collisions, the thermal conductivities exhibited by gases are lower than those of the liquids.

Example 1.5 In general, the thermal conductivity of a substance is

(a) independent of temperature
(b) a strong function of pressure
(c) strongly temperature dependent
(d) independent of pressure

Answer : (c)

Example 1.6 With increase in temperature, the thermal conductivity of gases

(a) decreases
(b) increases
(c) remain constant
(d) first increase and then decreases

Answer : (b)

Example 1.7 Which liquid metal can be taken as the best conductor?

(a) Tin
(b) Mercury
(c) Bismuth
(d) Sodium

Answer : (d)

Example 1.8 Choose the correct statement.

(a) The thermal conductivity of insulating solids increases with temperature
(b) The thermal conductivity of good electrical conductors is generally low
(c) The thermal conductivity of gases decreases with temperature
(d) The thermal conductivity of liquids is a strong function of temperature

Answer : (a)

Objective Brain Teasers

1. Eggs with a mass of 0.15 kg per egg and a specific heat of 3.32 kJ/kg°C are cooled from 32°C to 10°C at a rate of 300 eggs per minute. The rate of heat removal from the eggs is
(a) 11 kW (b) 80 kW
(c) 25 kW (d) 55 kW

2. Which equation below is used to determine the heat flux for conduction?
(a) $-kA\dfrac{dT}{dx}$ (b) $-k$ grad T
(c) $h(T_2 - T_1)$ (d) $\varepsilon\sigma T^4$

3. A 2 kW electric resistance heater submerged in 30 kg water is turned on and kept on for 10 min. During the process, 500 kJ of heat is lost from the water. The temperature rise of water is
(a) 5.6°C (b) 9.6°C
(c) 13.6°C (d) 23.3°C

4. A 1 kW electric resistance heater in a room is turned on and kept on for 50 minutes. The amount of energy transferred to the room by the heater is
(a) 1 kJ (b) 50 kJ
(c) 3000 kJ (d) 3600 kJ

5. Which equation below is used to determine the heat flux for convection?
(a) $-kA\dfrac{dT}{dx}$ (b) $-k$ grad T
(c) $h(T_1 - T_2)$ (d) $\varepsilon\sigma T^4$

6. A hot 16 cm × 16 cm × 16 cm cubical iron block is cooled at an average rate of 80 W. The heat flux is
(a) 195 W/m^2 (b) 521 W/m^2
(c) 3125 W/m^2 (d) 7100 W/m^2

7. Which equation below is used to determine the heat flux emitted by thermal radiation from a surface?
(a) $-kA\dfrac{dT}{dx}$ (b) $-k$ grad T
(c) $h(T_2 - T_1)$ (d) $\varepsilon\sigma T^4$

ANSWERS

1.	(d)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(c)
6.	(b)	**7.**	(d)						

Hints & Explanation

1. (d)

$$m = 0.15 \text{ kg/egg}$$
$$c = 3.32 \text{ kJ/kg°C}$$
$$T_{\text{initial}} = 32°C$$
$$T_{\text{final}} = 10°C$$

Number of eggs cooled = 300 per minute

The rate of heat removal

= Mass of 1 egg × Number of eggs cooled per minute × specific heat × $[T_{\text{initial}} - T_{\text{final}}]$

= 0.15 × 300 × 3.32 × [32 – 10] = 3286.8 kJ/min

= 54.78 kW ≈ 55 kW

2. (b)

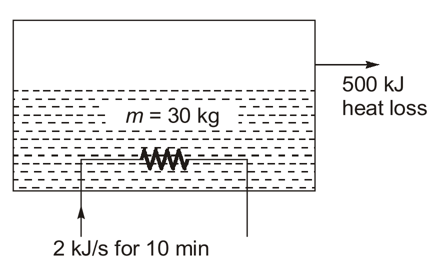
$$\text{Heat flux} = \frac{Q}{A}$$

From Fourier's law

$$\frac{Q}{A} = \text{Heat flux} = -k\frac{dt}{dx}$$
$$\frac{dT}{dx} = \text{grad or slope}$$

∴ Heat flux = $-k$ grad.

3. (a)



$$Q_{\text{input}} = \frac{2\text{ kJ}}{\text{s}} \times (10 \times 60)s = 1200 \text{ kJ}$$
$$Q_{\text{out}} = 500 \text{ kJ}$$

$$Q_{stored} = 1200 - 500 = 700 \text{ kJ}$$

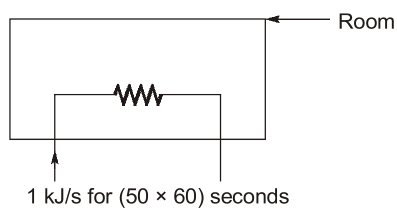
Heat stored is utilized in rise of temperature of water.

$$\text{Heat stored} = mcdT$$

$$700 = 30 \times 4.18 \times dT$$

$$dT = \frac{700}{30 \times 4.18} = 5.58°\text{C} \approx 5.6°\text{C}$$

4. (c)



Amount of energy transferred to the room by the heater

= Rate of energy × Time input

= 1 kJ/s × (50 × 60) second = 3000 kJ

5. (c)

$$Q = hA\Delta T \quad \text{(Connection heat transfer)}$$

$$Q/A = \text{heat flux for convection}$$

$$\text{Heat flux} = h\,\Delta T = h[T_1 - T_2]$$

6. (b)

Dimension of cube = 16 × 16 × 16 cm^3

Area of cube = $6a^2$

Heat flux is Q/A

$$= \frac{80}{6 \times 16 \times 16} = \frac{0.3125}{6} \text{ W/cm}^2$$

$$= \frac{0.3125}{(10^{-2})^2} = \frac{3125}{6} \text{ W/m}^2$$

$$= 520.83 = 521 \text{ W/m}^2$$

7. (d)

$$Q = \sigma \in AT^4$$

Heat flux:

$$Q/A = \sigma \in T^4$$

Student's Assignments

1. An insulated pipe of 50 mm outside diameter (ε = 0.8) is laid in a room at 30°C. It the surface temperature is 250°C and the convective heat transfer coefficient is 10 W/m^2 K, calculate the heat loss per unit length of pipe.

Ans. Q/L = 2232.4 W/m

2. An immersion water heater of surface area 0.1 m^2 and rating 1 kW is designed to operate fully submerged in water. Estimate the surface temperature of the heater when the water is at 40°C and the heat transfer coefficient is 300 W/m^2K.

Ans. T_s = 73.3°C

■■■■

CHAPTER

2

Steady State Heat Conduction

LEARNING OBJECTIVES

The reading of this chapter will enable the students:

- To understand the difference between steady- and unsteady-state heat transfer,
- To identify the different types of one-dimensional heat conduction problems,
- To analyze mathematically the different types of one-dimensional heat conduction problems with and without convection at boundary surfaces.

2.1 INTRODUCTION

In the case of steady-state heat conduction, the temperature ceases to be a function of time, it becomes a function of space coordinates only. Hence, we can express steady-state conduction

$$T = f(x, y, z) \quad ...(2.1)$$

Under many practical situations, the conduction of heat is significant in only one direction of the coordinate axes, and is negligible in the other two directions. The heat conduction is then termed one-dimensional heat conduction and temperature becomes $T = f(x, t)$

In the case of a steady one-dimensional heat conduction, the temperature is a function of one space coordinate only.

2.2 GENERALIZED HEAT CONDUCTION EQUATION

2.2.1 Cartesian Coordinate System

A rectangular parallelepiped is considered (Figure 2.1) as the control volume in a rectangular Cartesian frame of coordinate axes. Let us also consider that thermal energy is being generated along with the conduction of heat within the control volume.

The principle of conservation of energy for the control volume can be described as

$$\dot{q}_{in} + \dot{q}_g = \dot{q}_{out} + \dot{q}_{internal\ energy} \quad ...(2.2)$$

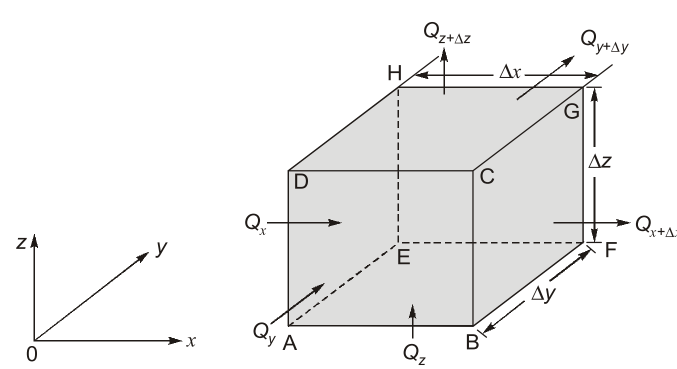


Figure 2.1 A control volume appropriate to a rectangular Cartesian coordinate system

Under steady state, we can write
$$Q = \frac{T_1 - T_2}{\frac{L_A}{k_A A}} = \frac{T_2 - T_3}{\frac{L_B}{k_B A}} = \frac{T_3 - T_4}{\frac{L_C}{k_C A}} \quad ...(2.15)$$

Equation (2.15) can also be written in a different fashion in terms of an overall temperature difference $(T_1 - T_4)$ as

$$Q = \frac{T_1 - T_4}{R_A + R_B + R_C} = \frac{T_1 - T_4}{\frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{L_C}{k_C A}} \quad ...(2.16)$$

Example 2.1 A plane wall of a refrigerated van is made of 1.5 mm steel sheet (k_s = 25 W/mK) at the outer surface, 10 mm plywood (k_P = 0.05 W/mK) at the inner surface and 20 mm glass wool (k_G = 0.01 W/mK) in between the outer and inner surfaces. The temperature of the cold environment inside the van is –15°C, while the outside surface is exposed to a surrounding ambient at 24°C.

The average values of convective heat transfer coefficients at the inner and outer surfaces of the wall are 12 W/m^{2}K and 20 W/m^{2}K respectively. The surface area of the wall is 0.75 m^2. Determine:

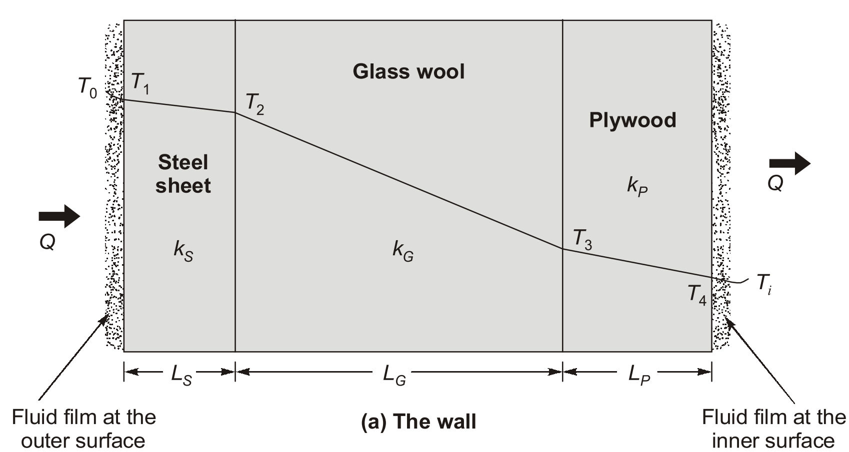
(a) The individual components of the thermal resistance to heat flow.

(b) The rate of heat flow through the wall.

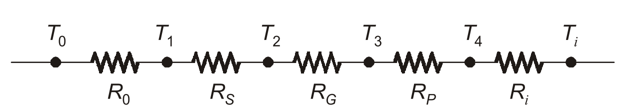
(c) The temperatures at (i) the outer surface of the wall, (ii) the interface between steel sheet and glass wool, (iii) the interface between glass wool and plywood, and (iv) the inner surface of the wall.

Solution :

The composite wall and the electrical analogous thermal circuit are shown in figure.



(a) The wall



(b) The analogous electrical circuit

The composite wall and the analogous electrical circuit

(a) R_o (convective resistance at the outer surface) $= \dfrac{1}{20 \times 0.75}$ = **0.067 KW^{-1}**

R_S(conduction resistance of steel sheet) $= \dfrac{1.5 \times 10^{-3}}{25 \times 0.75}$ = **8 × 10^{-5} KW^{-1}**

R_G (conduction resistance of glass wool) $= \dfrac{20 \times 10^{-3}}{0.01 \times 0.75}$ = **2.67 KW^{-1}**

R_P (conduction resistance of plywood) $= \dfrac{10 \times 10^{-3}}{0.05 \times 0.75}$ = **0.267 KW^{-1}**

R_i(convective resistance at the inner surface) $= \dfrac{1}{12 \times 0.75}$ = **0.111 KW^{-1}**

(b) The rate of heat flow,
$$Q = \frac{24-(-15)}{0.067 + 8 \times 10^{-5} + 2.67 + 0.267 + 0.111} = \textbf{12.52 W}$$

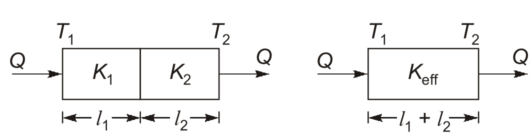
(c) The temperature at the interfaces are marked in above figure

$$T_1 = 24 - (12.52 \times 0.067) = 23.16°\text{C}$$
$$T_2 = 23.16 - (12.52 \times 8 \times 10^{-5}) = 23.16°\text{C}$$
$$T_3 = 23.16 - (12.52 \times 2.67) = -10.27°\text{C}$$
$$T_4 = -10.27 - (12.52 \times 0.267) = -13.61°\text{C}$$

Check for T_i
$$T_i = -13.61 - (12.52 \times 0.111)$$
$$= -15°\text{C} \qquad \text{(The value given in the problem)}$$

Effective Thermal Conductivity

Wall with multiple layers of different material having different thermal conductivity can be replace by a single material which will cause same rate of heat transfer having same temperature difference.



$$Q = \frac{T_1 - T_2}{\Sigma R}$$

$$Q = \frac{T_1 - T_2}{\left(\dfrac{l_1}{k_1 A} + \dfrac{l_2}{k_2 A}\right)} = \frac{T_1 - T_2}{\left(\dfrac{l_1 + l_2}{k_{eff} A}\right)}$$

Here K_{eff} is known as effective thermal conductivity.

$$\frac{l_1}{k_1 A} + \frac{l_2}{k_2 A} = \frac{l_1 + l_2}{k_{eff}}$$

For $l_1 = l_2$,
$$k_{eff} = \frac{2k_1 k_2}{(k_1 + k_2)}$$

Example 2.9 For a current carrying wire of 20 mm diameter exposed to air (h = 20 W/m^2 K), maximum heat dissipation occurs when thickness of insulation (0.5 W/mK) is

(a) 30 mm (b) 25 mm (c) 20 mm (d) 15 mm

Solution: (d)

For maximum heat dissipation, $r_c = \frac{k}{h} = \frac{k}{h} = \frac{0.5}{20} = 25$ mm

Thickness of insulation, $r_c - r = 25 - 10 = 15$ mm

Example 2.10 Consider a tube wall of inner and outer radii r_i and r_0 whose temperatures are maintained at T_i and T_0, respectively. The thermal conductivity of the cylinder is temperature dependent and may be represented by an expression of the form $k = k_o(1 + at)$ where k_0 and a are constants. Obtain an expression for the heat transfer per unit length of the tube.

Solution:

We start with equation (2.25), i.e.

$$\int_{r_1}^{r_2} \frac{Q}{2\pi rL} dr = -\int_{T_1}^{T_2} kdT$$

Here $k = k_o(1 + aT)$

Therefore, $$\int_{r_1}^{r_2} \frac{Q}{2\pi rL} dr = -k_o \int_{T_1}^{T_2} (1+aT)dT$$

or, $$\frac{Q}{2\pi L}\ln\left(\frac{r_2}{r_1}\right) = k_o(T_1 - T_2)\left[1 + a\left(\frac{T_1+T_2}{2}\right)\right]$$

or, $$Q = 2\pi L k_m \frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)}$$

where $k_m = k_0\left(1 + a\frac{T_1+T_2}{2}\right)$ is the thermal conductivity at mean temperature of $\frac{(T_1+T_2)}{2}$.

Example 2.11 A thin-walled copper tube of outside metal radius r = 0.01 m carries steam at 400 K. It is inside a room where the surrounding air temperature is 300 K. The tube is insulated with magnesia insulation of an approximate thermal conductivity of 0.07 W/(mK).

(a) What is the critical thickness of insulation for an external convective coefficient h = 4.0 W/(m^2 K) (Assume negligible conduction resistance due to the wall of the copper tube.)

(b) Under these conditions, determine the rate of heat transfer per metre of tube length for (i) a 0.002 m thick layer of insulation (ii) the critical thickness of insulation (iii) a 0.05 m thick layer of insulation.

Solution :

(a) $$r_c = \frac{k}{h} = \frac{0.07}{4.0} = 0.0175 \text{ m} = 17.5 \text{ mm}$$

Objective Brain Teasers

1. A composite plane wall is made of two different materials of same thickness with thermal conductivities k_1 and k_2 respectively. The equivalent thermal conductivity of the slab is
(a) $k_1 + k_2$ (b) k_1k_2
(c) $(k_1 + k_2) / (k_1k_2)$ (d) $(2k_1k_2) / k_1 + k_2$

2. A composite wall consists of three different materials having thermal conductivities, k, $2k$ and $4k$ respectively. The temperature drop across different materials will be in the ratio
(a) 1 : 1 : 1 (b) 1 : 2 : 4
(c) 4 : 2 : 1 (d) 2 : 4 : 1

3. The variation of temperature in a plane wall is determined to be $T(x) = 110 - 48\,x$ where x is in m and T is in °C. If the thickness of the wall is 0.75 m, the temperature difference between the inner and outer surfaces of the wall is
(a) 110°C (b) 74°C
(c) 55°C (d) 36°C

4. For pipes, the rate of heat transfer by conduction at the critical radius is
(a) equal to the rate of heat transfer by convections and is maximum
(b) equal to the rate of heat transfer by convection and is maximum
(c) greater than the rate of heat transfer by convection
(d) less than the rate of heat transfer by convection

5. The critical radius of insulation for a sphere is equal to
(a) $2kh$ (b) $2k/h$
(c) $h/2k$ (d) $\sqrt{2kh}$

6. A composite wall of three layers with thicknesses 0.3, 0.2 and 0.15 m and having thermal conductivities 0.3, 0.2 and 0.15 (W/mK) respectively will have a heat rate of (when inner and outer temperature are 1000°C and 40°C)
(a) 300 W/m^2 (b) 200 W/m^2
(c) 320 W/m^2 (d) 500 W/m^2

7. The temperature at the inner and outer surfaces of a 15 cm thick plane wall are measured to be 40°C and 28°C, respectively. The expression for steady, one-dimensional variation of temperature in the wall is (x is in m).
(a) $T(x) = 28\,x + 40$ (b) $T(x) = -40\,x + 28$
(c) $T(x) = 40\,x + 28$ (d) $T(x) = -80\,x + 40$

8. The ratio of heat flows from two walls of same area and thickness ratio 1 : 2 and thermal conductivity ratio 3 : 1 for the same temperature difference on the two sides is
(a) 5 : 1 (b) 6 : 1
(c) 2 : 3 (d) 3 : 2

9. The heat transfer rate by conduction for a hollow sphere with areas A_1 and A_2 varies as
(a) $\sqrt{A_1A_2}$ (b) $(A_1\,A_2)$
(c) $\left(\dfrac{1}{A_1A_2}\right)$ (d) $\dfrac{1}{\sqrt{A_1A_2}}$

10. A 30 mm OD pipe is to be insulated with asbestos having a thermal conductivity of 0.1 W/mK. The convective heat transfer coefficient is 5 W/m^{2}K. The critical radius of insulation for this pipe would be
(a) 10 mm (b) 20 mm
(c) 40 mm (d) 60 mm

11. Consider steady one-dimensional heat conduction through a plane wall, a cylindrical shell and a spherical shell of uniform thickness with constant thermophysical properties and no thermal energy generation. The geometry in which the variation of temperature in the direction of heat transfer will be linear is
(a) plane wall (b) cylindrical shell
(c) spherical shell (d) all of them

12. For conduction heat transfer the geometric mean areas is defined only in the case of a
(a) plane slab (b) hollow cylinder
(c) hollow sphere (d) truncated cone

13. The reduction of temperature drop in a heat generating solid can be most effectively achieved by reducing
    (a) the heat generation rate
    (b) the convection coefficient on the surface
    (c) the thermal conductivity
    (d) the linear dimension

14. The variation of temperature in a plane wall is determined to be $T(x) = 65\,x + 25$ where x is in m and T is in °C the thickness of the wall for 13°C temperature drop.
    (a) 2 m (b) 0.4 m
    (c) 0.2 m (d) 0.1 m

15. The temperature gradient in a sphere under steady state conduction at half the radius location will be
    (a) 2 times that at the surface
    (b) half of that at the surface
    (c) four times that at the surface
    (d) eight times that at the surface

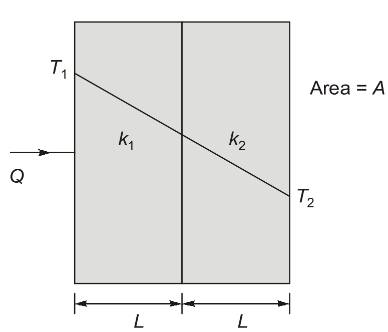
16. Up to the critical radius of insulation
    (a) added insulation will increase heat loss
    (b) added insulation will decrease heat loss
    (c) convection heat loss will be less than conduction heat loss
    (d) heat flux will decrease

■ ANSWERS

1. (d)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (b)
6. (c)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (b)
11. (a)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (c)
16. (a)				

■ Hints & Explanation

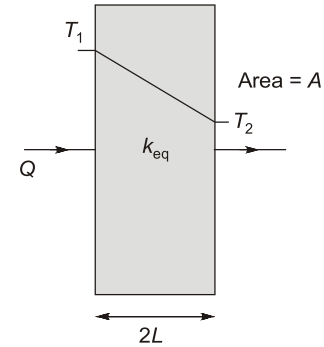
1. (d)



$$Q = \frac{T_1 - T_2}{\Sigma R}$$

$$= \frac{(T_1 - T_2)k_1k_2A}{(k_1 + k_2)L} \quad \text{...(1)}$$

$$R = \frac{L}{k_1A} + \frac{L}{k_2A} = \frac{L}{A}\left[\frac{k_1 + k_2}{k_1k_2}\right]$$



$$Q = \frac{T_1 - T_2}{\Sigma R}$$

$$= \frac{(T_1 - T_2)k_{eq}.A}{2L} \quad \text{...(ii)}$$

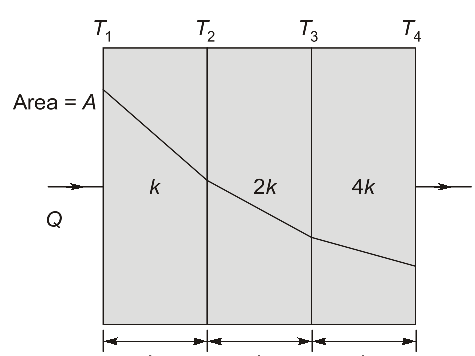
$$R = \frac{2L}{k_{eq}A}$$

Equating equation (1) and equation (ii)

$$\frac{(T_1 - T_2)k_1k_2A}{(k_1 + k_2)L} = \frac{(T_1 - T_2)\cdot k_{eq}A}{2L}$$

$$k_{eq} = \frac{2k_1k_2}{k_1 + k_2}$$

2. (c)



At steady state heat transfer through all layers will be same.

Temperature drop across Ist layer, is $(T_1 - T_2)$

$$\therefore \quad Q = \frac{(T_1 - T_2)}{\Sigma R} = \frac{(T_1 - T_2)}{L/kA}$$

$$\therefore \quad T_1 - T_2 = \frac{QL}{kA}$$

Similarly for other layers.

$$T_2 - T_3 = \frac{QL}{2kA}$$

$$T_3 - T_4 = \frac{QL}{4kA}$$

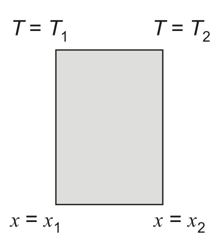
$$\therefore \quad (T_1 - T_2) : (T_2 - T_3) : (T_3 - T_4) = ?$$

$$\frac{T_1 - T_2}{T_2 - T_3} = \frac{QL/kA}{QL/2kA} = \frac{2}{1} = \frac{4}{2}$$

$$\frac{T_2 - T_3}{T_3 - T_4} = \frac{QL/2kA}{QL/4kA} = \frac{2}{1}$$

$$(T_1 - T_2) : (T_2 - T_3) : (T_3 - T_4)$$
$$= 2 : 1 :: 2 : 1$$
$$= 4 : 2 :: 2 : 1$$
$$= 4 : 2 : 1$$

3. (d)



$$T(x) = 110 - 48x$$

Differentiating both sides.

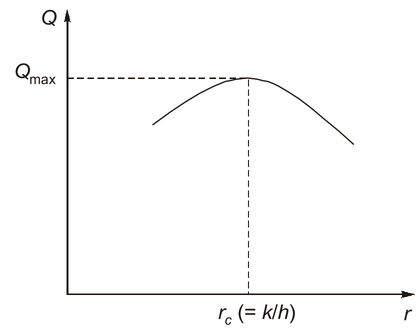
$$\frac{dT}{dx} = 0 - 48$$

Integrating with proper limit.

$$\int_{T_1}^{T_2} dT = -48 \int_{x_1}^{x_2} dx$$

$$T_2 - T_1 = -48 \times (x_2 - x_1)$$
$$= -48 \times 0.75$$
$$= -36°C$$
$$T_1 - T_2 = 36°C$$

4. (a)



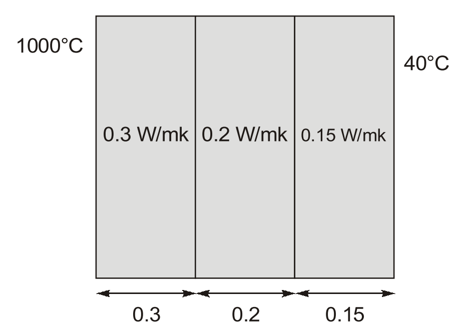
At steady state heat conducted is equal to heat convected and rate of heat transfer is maximum at critical radius.

5. (b)

$$(R_{critical})_{sphere} = \frac{2k}{h}$$

$$(R_{critical})_{cylinder} = \frac{k}{h}$$

6. (c)

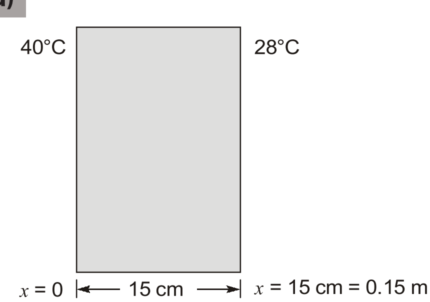


$$\text{Heat rate} = \frac{1000 - 40}{\Sigma R}$$

$$R = \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} = \frac{0.3}{0.3} + \frac{0.2}{0.2} + \frac{0.15}{0.15}$$
$$= 1 + 1 + 1 = 3 \text{ m}^2\text{K/W}$$

$$Q_{\text{Heat rate}} = \frac{960}{3} \text{ K/m}^2 \text{ K/W} = 320 \text{ W/m}^2$$

7. (d)



Let one dimensional variation of temperature in the wall be

$$T = a + bx$$

where x is in 'm' and T is in °C.

At $\quad x = 0 \quad T = 40°C$

$$40 = a + b \cdot 0$$

$\therefore \quad a = 40$

At $\quad x = 0.15, \quad T = 28$

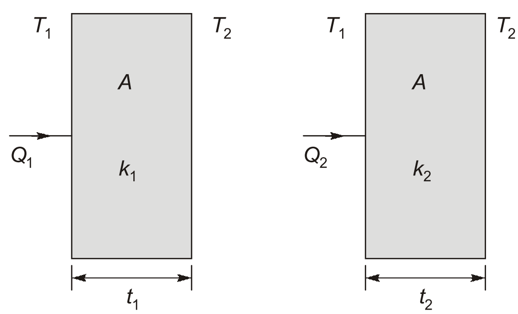
$$28 = 40 + 0.15\, b$$

$$-12 = 0.15\, b$$

$$b = \frac{-12}{0.15} = -80$$

$$T = 40 - 80x = -80x + 40$$

8. (b)



Given: $\quad \dfrac{t_1}{t_2} = \dfrac{1}{2} = 0.5$

$$\frac{k_1}{k_2} = 3$$

$$Q_1 = \frac{k_1 A(T_1 - T_2)}{t_1}$$

$$Q_2 = \frac{k_2 A(T_1 - T_2)}{t_2}$$

$$\frac{Q_1}{Q_2} = \frac{k_1}{t_1} \times \frac{t_2}{k_2}$$

$$= \frac{k_1}{k_2} \times \frac{1}{\frac{t_1}{t_2}} = 3 \times \frac{1}{0.5} = 6$$

$\therefore \quad Q_1 : Q_2 = 6 : 1$

9. (a)

$$Q = \frac{4\pi k\, r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)}$$

$$A_1 = 4\pi r_1^2$$

Taking square root

$$\sqrt{A_1} = r_1\sqrt{4\pi}$$

Similarly $\quad \sqrt{A_2} = r_2\sqrt{47}$

$$Q = \frac{k \cdot \sqrt{4\pi} \cdot \sqrt{4\pi}\, r_1 r_2 (T_1 - T_2)}{r_2 - r_1}$$

$$= \frac{k r_1\sqrt{4\pi} \cdot r_2\sqrt{4\pi}(T_1 - T_2)}{r_2 - r_1}$$

$$Q = \frac{k \cdot \sqrt{A_1} \cdot \sqrt{A_2} \cdot (T_1 - T_2)}{r_2 - r_1}$$

$$Q \propto \sqrt{A_1 A_2}$$

10. (b)

$$k = 0.1 \text{ W/mK}$$

$$h = 5 \text{ W/m}^2\text{ K}$$

$$(r_c)_{pipe} = (r_c)_{cylinder}$$

$$= \frac{k}{h} = \frac{0.1}{5}\text{ m}$$

$$= 0.02 \text{ m} = 20 \text{ mm}$$

11. (a)

$$\frac{d^2T}{dx^2} = 0 \qquad \text{[for plane wall]}$$

Integrating

$$\frac{dT}{dx} = c_1$$

$$T = c_1 x + c_2 \quad \rightarrow \text{linear}$$

For cylinder

$$\frac{1}{2} \cdot \left\{\frac{d}{dr}\left(r \cdot \frac{dT}{dr}\right)\right\} = 0$$

$$\Rightarrow \frac{1}{r} \cdot \left[r \cdot \frac{d^2T}{dr^2} + \frac{dT}{dr}\right] = 0$$

$$\Rightarrow \quad \frac{d^2T}{dr^2} + \frac{1}{r} \cdot \frac{dT}{dr} = 0$$

(Integrating will be not linear)