CHEMICAL ENGINEERING

Fluid Mechanics

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Fluid Mechanics

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CONTENTS

Fluid Mechanics

CHAPTER 7

CHAPTER 8

CHAPTER 9

CHAPTER 10

CHAPTER 11

CHAPTER 12

CHAPTER

1

Fluid Properties

1.1 INTRODUCTION

- A fluid is a substance which deforms continuously under the influence of shearing forces no matter how small the forces may be.
- Fluids are substance capable of flowing and they conforms to the shape of the containing vessel.
- This property of continuous deformation in technical terms is known as 'flow property', whereas this property is absent in solids.
- If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.
- Fluids are classified as ideal fluids and practical or real fluids.
- Ideal fluids are those fluids which have neither viscosity nor surface tension and they are incompressible. In nature, the ideal fluids do not exist and therefore, they are only imaginary fluids.
- Practical or real fluids are those fluids which possess viscosity, surface tension and compressibility.
- Fluids are considered to be continuum i.e., a continuous distribution of matter with no voids or empty spaces.
- Difference between fluid and solid is that solid can resist a shear stress by static deflection but fluid cannot resist it.

1.2 FLUID MECHANICS

- Fluid mechanics is study of fluids either at rest or in motion.
- Total fluid mechanics can be dealt with two different approaches, empirical hydraulics and classical hydrodynamics.
- Hydraulics is mainly concerned with motion of water. It is based on the physical principles and has close correlation with experimental studies which both complement and substantiate the fundamental analysis.
- Hydrodynamics is essentially mathematical science dealing with flow analysis based on concept of an ideal fluid, a fictitious fluid in which both fluid viscosity and fluid compressibility are assumed absent.

1.3 FLUID AS CONTINUUM

- Since fluids are aggregations of molecules widely spread for gas and closely spaced for a liquid. The distance between molecules is very large compared to molecular diameter.
- The molecules are not fixed in lattice but move about freely. Thus fluid density or mass per unit volume has no practical meaning because the numbers of molecule occupying a given volume continuously changes.
- But if chosen unit volume is too large there could be noticeable variation in the bulk aggregation of particle. So density can be written as

$$\rho = \lim_{\delta v \to \delta v'} \frac{\delta m}{\delta v}$$

- Since most engineering problems are connected with larger sample volume, so density being a point function and other fluid properties can be thought of as varying continually in space. Such a fluid is called a continuum, which simply means that its variation in properties is so smooth that differential calculus can be used to analyse the substance.

1.4 FLUID PROPERTIES

- Any characteristic of a fluid system is called a fluid property.
- Fluid properties are of two types:
  (i) **Intensive Properties:** Intensive properties are those that are independent of the mass of the fluid system.
  Example: Temperature, pressure, density etc.
  (ii) **Extensive Properties:** Extensive properties are those whose values depend on the size or extent of the system.
  Example: Total mass, total volume, total momentum etc.
- Following are some of the intensive and extensive properties of a fluid system.
  (i) Viscosity (ii) Surface tension (iii) Vapour pressure (iv) Compressibility and elasticity

1.4.1 Some other Important Properties

1. **Mass Density :** Mass density (or specific mass) of a fluid is the mass which it possesses per unit volume. It is denoted by the Greek symbol ρ. In SI system, the unit of ρ is kg/m^3.
2. **Specific Gravity :** Specific gravity (S) is the ratio of specific weight (or mass density) of a fluid to the specific weight (or mass density) of a standard fluid. The standard fluid chosen for comparison is pure water at 4°C.

$$\text{Specific gravity of liquid} = \frac{\text{Specific weight of liquid}}{\text{Specific weight of water}} = \frac{\text{Specific weight of liquid}}{9810\ \text{N/m}^3}.$$

3. **Relative Density (R.D.) :** It is defined as ratio of density of one substance with respect to other substance.

$$\rho_{1/2} = \frac{\rho_1}{\rho_2}$$

where, $\rho_{1/2}$ = Relative density of substance ‘1’ with respect to substance ‘2’.

4. **Specific Weight :** Specific weight (also called weight density) of a fluid is the weight it possesses per unit volume. It is denoted by the Greek symbol γ. For water, it is denoted by γ_w. In SI system, the unit of specific weight is N/m^3. The mass density and specific weight γ has following relationship $\gamma = \rho g$; $\rho = \gamma / g$. Both mass density and specific weight depend upon temperature and pressure.
5. **Specific Volume :** Specific volume of a fluid is the volume of the fluid per unit mass. Thus it is the reciprocal of density. It is generally denoted by v. In SI unit specific volume is expressed in cubic meter per kilogram, i.e., m^3/kg.

Example 1.1 Three litres of petrol weigh 23.7 N. Calculate the mass density, specific weight, specific volume and specific gravity of petrol.

Solution:

Mass density of petrol, $\rho_p = \dfrac{M}{V} = \dfrac{W/g}{V} = \dfrac{W}{gV} = \dfrac{23.7}{9.8 \times 3} = 0.805$ kg/litre = 805 kg/m^3

Mass density of water, $\rho_w = 1000$ kg/m^3

Specific gravity of petrol $= \dfrac{805}{1000} = 0.805$

Specific weight of petrol = Weight per unit volume

$= \dfrac{23.7}{3.0} = 7.9$ N/litre = 7.9 kN/m^3

Specific volume = volume per unit mass

$= \dfrac{1}{\rho_p} = \dfrac{1}{805} = 1.242 \times 10^{-3}$ m^3/kg

1.4.2 Viscosity

- Viscosity is a property of the fluids by virtue of which they offer resistance to shear or angular deformation.
- It is primarily due to cohesion and molecular momentum exchange between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers.

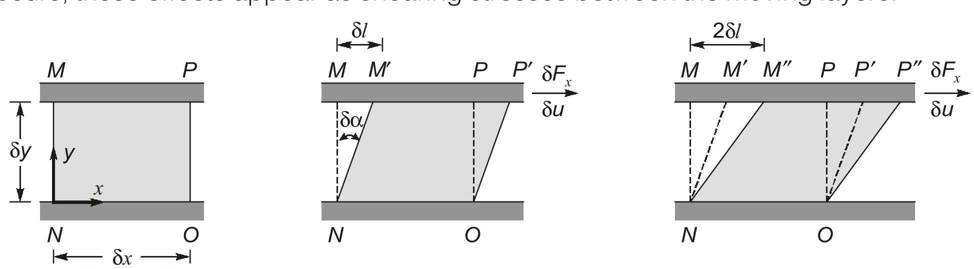


Fig. *(a) Fluid element at time t, (b) deformation of fluid element at time t + δt, and (c) deformation of fluid element at time t + 2δt.*

- Consider the behavior of a fluid element between the two infinite plates as shown in Fig. (a).
  The rectangular fluid element is initially at rest at time t. Let us now suppose a constant rightward force δF_x is applied to the upper plate so that it is dragged across the fluid at constant velocity δu. The relative shearing action of the plates produces a shear stress, τ_{yx}, which acts on the fluid element and

$$\tau = k\left(\frac{du}{dy}\right)^{n-1}\left(\frac{du}{dy}\right) = \eta\frac{du}{dy}$$

where, $\eta = k\left(\frac{du}{dy}\right)^{n-1}$ is referred as the apparent viscosity

NOTE: Dynamic viscosity (μ) is constant (except for temperature effects) while apparent viscosity (η) depends on the shear rate.

- Various types of non-Newtonian fluids are :
  1. **Pseudoplastic** : Fluids in which the apparent viscosity decreases with increasing deformation rate ($n < 1$) are called pseudoplastic fluids (or shear thinning). Most non-Newtonian fluids fall into this group.
     Example: Polymer solutions, colloidal suspensions, milk, blood and paper pulp in water.
  2. **Dilatant :** If the apparent viscosity increases with increasing deformation rate ($n > 1$), the fluid is termed as dilatant (or shear thickening).
     Example: Suspensions of starch, saturated sugar solution.
  3. **Bingham Plastic :** Fluids that behave as a solid until a minimum yield stress, τ_y, and flow after crossing this limit are known as ideal plastic or Bingham plastic. The corresponding shear stress model is $\tau = \tau_y + \mu\frac{du}{dy}$.
     Example: Clay suspensions, drilling muds, creams and toothpaste.
  4. **Thixotropic :** Apparent viscosity (η) for thixotropic fluids decreases with time under a constant applied shear stress.
     Example: Paints, printer inks
  5. **Rheopectic :** Apparent viscosity (η) for rheopectic fluids increases with time under constant shear stress.
     Example: Gypsum pastes.

NOTE

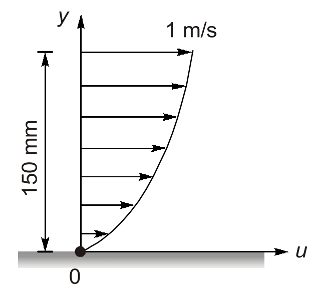
(i) There is no relative movement between fluid attached to the solid boundary and solid boundary i.e. the fluid layer just adjacent to the solid surface will have same velocity as of the solid surface.
(ii) Viscoelastic : Fluids which after some deformation partially return to their original shape when the applied stress is released such fluids are called viscoelastic.
(iii) Rheology : Branch of science which deals with the studies of different types of fluid behaviours.

Example 1.2 Calculate the velocity gradient at distance 0, 100, 150 mm from the boundary if the velocity is a parabola with vortex 150 mm from boundary, where velocity is 1 m/s. Also calculate the shear stress at these points if the fluids has a viscosity of 0.804 Ns/m^2.

Solution:

Let the equation of velocity profile

$$u = Ay^2 + By + C$$



Now apply boundary condition

(i) $u = 0$ at $y = 0 \Rightarrow c = 0$

(ii) $u = 1$ m/s at $y = 0.15$ m

$$1 = 0.15^2 \times A + 0.15\,B \qquad \text{...(ii)}$$

(iii) at $y = 0.15$ m at $\frac{du}{dy} = 0$

$$\frac{du}{dy} = 2Ay + B$$

$$2A \times 0.15 + B = 0 \qquad \text{...(iii)}$$

From Eq. (ii) and (iii), we get,

$A = -44.4$; $B = 13.33$

So velocity profile will be given as

$$u = -44.4\,y^2 + 13.33\,y$$

(a) at $y = 0$ mm $\quad \frac{du}{dy} = -2 \times 44.4 \times 0 + 13.33 = 13.33\ \text{sec}^{-1}$

Shear stress, $\quad \tau = \mu\frac{du}{dy} = 0.804 \times 13.33 = 10.8\ \text{N/m}^2$

(b) at $y = 100$ mm $\quad \frac{du}{dy} = -2 \times 44.4 \times 0.1 + 13.33 = 4.45\ \text{sec}^{-1}$

$$\tau = \mu\frac{du}{dy} = 0.804 \times 4.45 = 3.575\ \text{N/m}^2$$

(c) at $y = 150$ mm $\quad \frac{du}{dy} = -2 \times 44.4 \times 0.15 + 13.33 = 0$

$$\tau = 0$$

1.4.3 Surface Tension

- It is a force which exists on the surface of a liquid when it is in contact with another fluid or a solid boundary. Its magnitude depends upon the relative magnitude of cohesive and adhesive forces.
- Surface tension is a force in the liquid surface and acts normal to a line of unit length drawn imaginarily on the surface. Thus it is a line force.
- It represents surface energy per unit area. It has dimension MT^{-2} and SI unit is N/m.
- Whenever a liquid is in contact with other liquids or gases the interface develops that acts like a stretched elastic membrane, creating surface tension.

Example 1.3 A circular disc of diameter d is slowly rotated in a liquid of large viscosity 'μ' at a small distance 'h' from fixed surface. Derive expression for torque 'T' necessary to maintain an angular velocity 'ω'.

Solution:

Consider an element of disc at radius r and having a width dr

Linear velocity at this radius

$$V = r\omega$$

Example 1.7 The volume of a liquid is reduced by 1.1% by increasing the pressure from 0.5 MPa to 12.5 MPa. Estimate modulus of elasticity of liquid.

Solution:

Modulus of elasticity,
$$K = \frac{-dp}{\frac{dV}{V}} = -\left(\frac{12.5 - 0.5}{-\frac{1.1}{100}}\right) = 1090.91 \text{ MPa}$$

Example 1.8 For determining the depth of sea at a place, a charge was exploded at 100 m below the sea water surface. The first reflected wave was recorded after 3 sec at surface. Calculate the depth assuming the sea has a flat bottom. The average value of bulk modulus of elasticity of sea water is 1.96×10^9 N/m^2 and its specific weight = 10×10^3 N/m^3.

Solution:

$$K = 1.96 \times 10^9 \text{ N/m}^2$$

$$\rho = \frac{10^4}{9.81} = 1020 \text{ kg/m}^3$$

$$\text{Velocity of sound} = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{1.96 \times 10^9}{1020}} \simeq 1386 \text{ m/s}$$

Let the depth of sea below water surface be d meter.

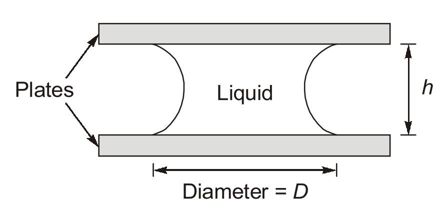
The distance travelled by reflected sound wave

$$= (d - 100) + d = (2d - 100)$$

Time taken by first reflected wave to reach surface in

$$3 = \frac{2d - 100}{1386} \quad \Rightarrow \quad d = 2129 \text{ m}$$

Example 1.9 A very small quantity of liquid having a surface tension σ forms a circular spot of diameter D and between two glasses plates separated by a small distance h, obtain expression for force required to pull the plate apart.



Solution:

Let p_0 and p_1 be the ambient pressure and pressure inside liquid and

$$\Delta p = p_1 - p_0 \text{ (where } p_0 > p_1)$$

The circular spot is stable if difference in pressure = Total surface tension force

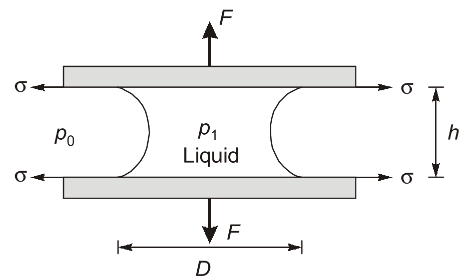
$$\pi Dh \times (\Delta p) = 2\sigma \times \pi D$$

$$\Delta p = \frac{2\sigma}{h}$$

So force required to pull the plate apart

$$F = \left(\frac{\pi D^2}{4}\right)(\Delta p)$$

$$= \frac{2\pi D^2 \sigma}{4h} = \frac{\pi}{2}\left(\frac{D}{h}\right)\sigma D$$



Summary

- A fluid is a substance that deforms continuously when subjected to even an infinitesimal shear stress.
- Fluid mechanics is the study of fluids at rest or in motion.
- The concept of a continuum assumes a continuous distribution of mass within the matter or system with no empty space.
- Viscosity is the property of a fluid by virtue of which it offers resistance to flow.
- The rate of deformation of any fluid element in a fluid flow is equal to the velocity gradient across the flow.
- Fluids which obey Newton's law of viscosity are called Newtonian fluids and which do not obey are called non-Newtonian fluids.
- It is due to surface tension that a curved liquid interface, in equilibrium, results in a greater pressure at concave side than that at its convex side.
- A liquid wets a solid surface and results in a capillary rise when the forces of cohesion between the liquid molecules are lower than the forces of adhesion between the molecules of the liquid and the solid in contact.
- Cavitation occurs when the local pressure reduces below vapour pressure of the liquid.
- Compressibility of a substance is the measure of its change in volume or density under the action of external forces.

Important Expressions

- Deformation rate $\left(\frac{d\alpha}{dt}\right)$ = velocity gradient $\left(\frac{du}{dy}\right)$
- Shear stress for Newtonian fluids : $\tau = \mu\left(\frac{du}{dy}\right)$
- Shear stress for non-Newtonian fluids: $\tau = k\left(\frac{du}{dy}\right)^n = k\left(\frac{du}{dy}\right)^{n-1}\frac{du}{dy} = \eta\frac{du}{dy}$
- Shear stress for Bingham plastics : $\tau = \tau_y + \mu\frac{du}{dy}$
- Pressure intensity p in excess of the outside pressure intensity in a droplet : $p = \frac{2\sigma}{r}$

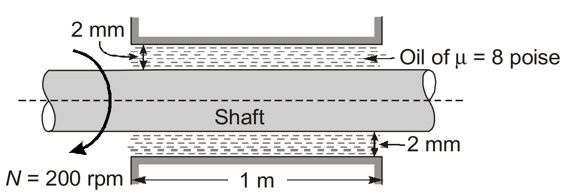
- Pressure intensity p in excess of the outside pressure intensity in a soap bubble: $p = \frac{4\sigma}{r}$
- Pressure intensity p in excess of the outside pressure intensity in a liquid jet : $p = \frac{\sigma}{r}$
- Capillary rise or fall : $h = \frac{2\sigma\cos\theta}{S\gamma_w r}$
- Compressibility : $\beta = \frac{1}{K} = \frac{-(\Delta V/V)}{dp} = \frac{(d\rho/\rho)}{dp}$

Objective Brain Teasers

Q.1 If the dynamic viscosity of a liquid is 0.012 poise and its R.D. is 0.79, then its kinematic viscosity in stoke is
(a) 0.0152 (b) 0.152
(c) 1.52 (d) 15.20

Q.2 The velocity distribution, in m/s near the solid wall at a section in a laminar flow is given by $u = 5\sin(5\pi y)$. If $\mu = 5$ poise, the shear stress at $y = 0.05$m, in N/m^2 is
(a) 39.27 (b) 27.77
(c) 38.9 (d) 26.66

Q.3 A solid shaft diameter of 350 mm, rotates at 200 rpm inside a fixed sleeve bearing as shown in figure. The dynamic viscosity of oil is 8 poise.



The power lost (in kW) due to viscosity in bearing is
(a) 4.9 (b) 5.9
(c) 11.8 (d) 2.95

Q.4 A fluid indicated the following shear stress and deformation rates :

$\frac{du}{dy}$ (units)	0	1	2	4
τ (units)	10	15	20	30

This fluid is classified as
(a) Newtonian (b) Bingham Plastic
(c) Dilatant (d) Pseudoplastic

Q.5 Kerosene is known to have a bulk modulus of elasticity $K = 1.43 \times 10^9$ N/m^2 and a relative density of 0.806. The speed of sound in kerosene, (in m/s) is
(a) 1332 (b) 1075
(c) 1197 (d) 184

Q.6 If 5.66 m^3 of oil weighs 4765 kg, then its mass density, specific weight and specific gravity respectively are
(a) 841.87 kg/m^3, 8.26 kN/m^3 and 0.842
(b) 8.26 kg/m^3, 841 kN/m^3 and 8.42
(c) 841.87 kg/m^3, 841 kN/m^3 and 8.42
(d) None of these

Q.7 A reservoir of capacity 0.01 m^3 is completely filled with a fluid of coefficient of compressibility 0.75×10^{-9} m^2/N. The amount of fluid that spill over (in m^3), if pressure in the reservoir is reduced by 2×10^7 N/m^2 is
(a) 0.15×10^{-4} (b) 1×10^{-4}
(c) 1.5×10^{-4} (d) None of these

Q.8 Assuming that sap in trees has the same characteristic as water and that it rises purely due to capillary phenomenon, what will be the average diameter of capillary tubes in a tree if the sap is carried to a height of 10 m? (Take surface tension of water = 0.0735 N/m and $\theta = 0°$)

(a) 0.003 mm (b) 0.03 mm
(c) 0.3 mm (d) 0.006 mm

Q.9 A small circular jet of mercury 0.1 mm in diameter issue from an opening. What is the pressure difference between the inside and outside of the jet when at 20°C? (Surface tension of mercury at 20°C is 0.514 N/m)

(a) 41 kPa (b) 21.5 kPa
(c) 10.28 kPa (d) 5.14 kPa

Q.10 An apparatus produces water droplets of diameter 70 μm. If the coefficient of surface tension of water in air is 0.07 N/m, the excess pressure in these droplets, in kPa, is

(a) 5.6 (b) 4.0
(c) 8.0 (d) 13.2

Q.11 If the surface tension of water air interface is 0.073 N/m, the gauge pressure inside a rain drop of 1 mm diameter is

(a) 146.0 N/m^2 (b) 0.146 N/m^2
(c) 73.0 N/m^2 (d) 292.0 N/m^2

Q.12 The capillary rise in a 3 mm tube immersed in a liquid is 15 mm. If another tube of diameter 4 mm is immersed in the same liquid, the capillary rise would be

(a) 11.25 mm (b) 20.00 mm
(c) 8.44 mm (d) 26.67 mm

Q.13 Which of the following is the correct expression for the bulk modulus of elasticity of a fluid?

(a) $\rho\dfrac{d\rho}{dp}$ (b) $\rho\dfrac{dp}{d\rho}$
(c) $\dfrac{d\rho}{\rho\, dp}$ (d) $\dfrac{dp}{\rho\, d\rho}$

Q.14 A Newtonian fluid fills the clearance between a shaft and a sleeve. When a force of 800 N is applied to the shaft, parallel to the sleeve, the shaft attains a speed of 1.5 cm/s. If a force of 2.4 kN is applied instead, the shaft would move with a speed of

(a) 1.5 cm/s (b) 13.5 cm/s
(c) 0.5 cm/s (d) 4.5 cm/s

Q.15 If the shear stress τ and shear rate $\left(\dfrac{du}{dy}\right)$ relationship of a material is plotted with τ on the Y-axis and $\dfrac{du}{dy}$ on the X-axis, the behaviour of an ideal fluid is exhibited by

(a) a straight line passing through the origin and inclined to the X-axis
(b) the positive X-axis
(c) the positive Y-axis
(d) a curved line passing through the origin

■ ANSWERS

1. (a)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (a)
6. (a)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (b)
11. (d)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (b)

Student's Assignments

Q.1 A glass tube 0.25 mm in diameter contains a mercury column with water above the mercury. The temperature is 20°C at which the surface tension of mercury in contact with water is 0.037 kg(f)/m. What will be the capillary depression of the mercury? Take angle of contact θ = 130°.

Ans. 3.02 cm

Q.2 The velocity distribution in the flow of a thin film of oil down an inclined channel is given by $u = \dfrac{\gamma}{2\mu}(d^2 - y^2)\sin\alpha$, where, d = depth of flow, α = angle of inclination of the channel to the horizontal, u = velocity at a depth y below the free surface, γ = unit weight of oil and μ = dynamic viscosity of oil. Calculate the shear stress : (i) on the bottom of the channel, (ii) at mid - depth, and (iii) at the free surface.

CHAPTER

2

Fluid Pressure and its Measurement

2.1 INTRODUCTION

- Intensity of pressure at any point is defined as force exerted per unit area. However if force is not uniformly distributed, the expression will give the average value only.
- When a certain mass of fluid is confined within solid boundary. The force exerted always acts in normal to surface.
- A fluid at rest is characterized by absence of relative motion between adjacent fluid layers. Viscosity of fluid has no effect on fluid at rest and therefore the ideal and real fluid behave exactly same.
- There are no tangential force as shear stress is zero because of no relative motion.
- In a fluid at rest the normal stress is called pressure.
- Fluid at rest cannot support shear stress. Normal stress on any plane through fluid element at rest is a point property called the fluid pressure, taken positive for compression by common convention.

2.2 PASCAL'S LAW FOR PRESSURE AT A POINT

- According to Pascal's law, pressure at a point in a fluid system is equally distributed in all directions (Fig.).
- It means that the pressure at a point in a fluid at rest, or in motion, is independent of direction as there are no shearing stresses present.
- Pressure in a fluid system has magnitude but not a specific direction and thus, it is a scalar quantity.

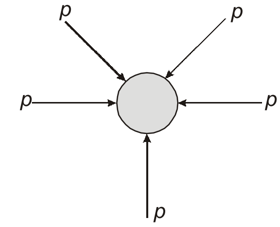


Fig. *A point in a fluid system*

- It applies to a fluid at rest.
- In case of flowing fluid, shear stresses will be set up as a result of relative motion between particles of the fluid.

  The pressure at a point is then considered to be the mean of the normal forces per unit area (stresses) on three mutually perpendicular planes. Since, these normal stresses are usually large compared to shear stresses, it is generally assumed that Pascal's law still applies.

Validation of the Law: Consider a small wedge-shaped fluid element of unit length in equilibrium as shown in Fig. The mean pressure at the three surfaces are p_1, p_2 and p_3 and the force acting on a surface is the product of mean pressure and the surface area. From Newton's second law, a force balance in the x-direction and z-direction gives

$$\Sigma F_x = ma_x = 0; \quad p_1 \Delta y \Delta z - p_3 \Delta y\, l \sin\theta = 0 \quad \text{...(i)}$$

$$\Sigma F_z = ma_z = 0; \quad p_2 \Delta y \Delta x - p_3 \Delta y\, l \cos\theta - \frac{1}{2}\rho g \Delta x\, \Delta y\, \Delta z = 0 \quad \text{...(ii)}$$

Geometric relations are: $\Delta x = l\cos\theta$, $\Delta z = l\sin\theta$

Apply geometric relations in equation (i) and (ii), we get

$$p_1 - p_3 = 0$$

$$\Rightarrow \quad p_1 = p_3$$

$$p_2 - p_3 - \frac{1}{2}\rho g\, \Delta z = 0$$

For infinitesimal element, $\Delta z \to 0$

then, $\quad p_2 = p_3$

$$\therefore \quad p_1 = p_2 = p_3 = p \quad \text{...(iii)}$$

Thus, we conclude that the pressure at a point in a fluid has the same magnitude in all directions.

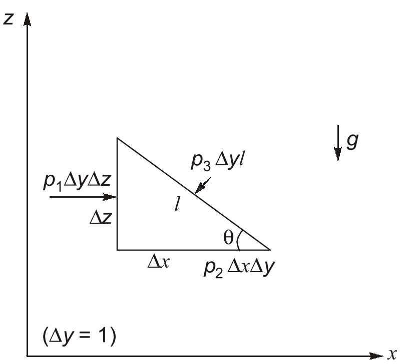


Fig. *Fluid Element*

2.3 ABSOLUTE AND GAUGE PRESSURE

- Pressures can be expressed in two different systems. The difference between two system is that their assumed datum is different.
- In absolute pressure system, the pressure is measured above absolute zero or complete vacuum.
- In gauge pressure system, the pressure is measured and expressed as difference between absolute value and local atmospheric pressure.
- If the pressure is below local atmospheric pressure, it is known as negative or vacuum or suction pressure.

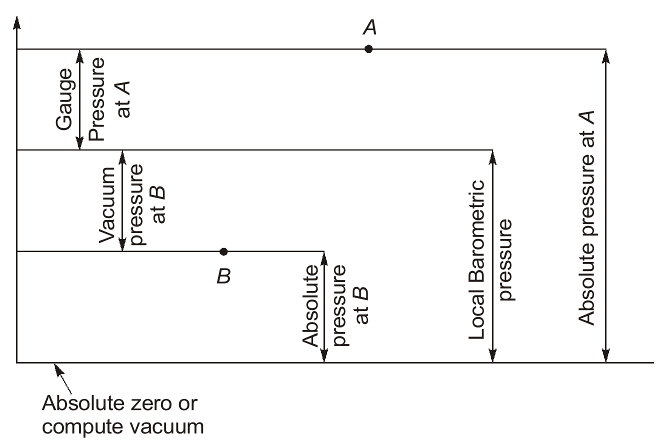


Fig. *Relation between gauge and Absolute pressure*

2.4 VARIATION OF PRESSURE IN A FLUID

- Consider a small fluid element of size $\delta x \times \delta y \times \delta z$ at any point in a static mass of fluid as shown in Fig. The forces acting on the element are the pressure forces on its faces and the self-weight of the element. Since, the element is in equilibrium under these forces, the algebraic sum of the forces acting on it in any direction must be zero.

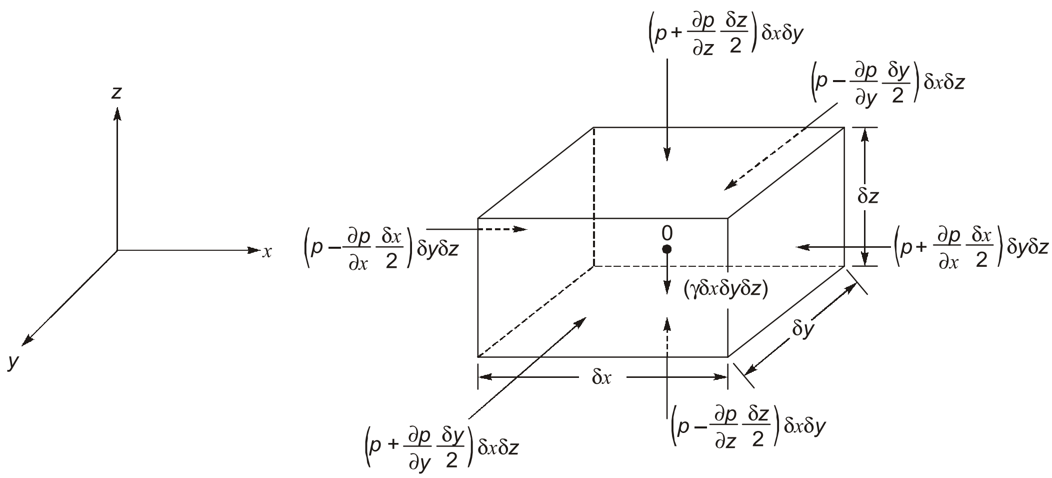


Fig. *Fluid Element*

i.e. $$\Sigma F_x = 0$$

or $$\left(p-\frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\cdot\delta y\delta z - \left(p+\frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\cdot\delta y\delta z = 0$$

or $$\frac{\partial p}{\partial x} = 0 \qquad \text{...(i)}$$

Also, $$\Sigma F_y = 0$$

or $$\left(p-\frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\cdot\delta x\delta z - \left(p+\frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\cdot\delta x\delta z = 0$$

or $$\frac{\partial p}{\partial y} = 0 \qquad \text{...(ii)}$$

Again, $$\Sigma F_z = 0$$

or $$\left(p-\frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\cdot\delta x\delta y - \left(p+\frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\cdot\delta x\delta y-\gamma(\delta x\delta y\delta z) = 0$$

or $$\frac{\partial p}{\partial z} = -\gamma \qquad \text{...(iii)}$$

Thus, Equations (i), (ii) and (iii) indicate that the pressure intensity 'p' at any point in a static fluid does not vary in x and y-directions and it varies only in z - direction. Partial derivative of Eq. (iii) can be reduced to total (or exact) derivative as follows

$$\frac{dp}{dz} = -\gamma = -\rho g \qquad \text{...(iv)}$$

- The minus sign(–) indicates that the pressure decreases in the direction in which z increases, i.e. in the upward direction.
- The above equation (iv) holds for both compressible and incompressible fluids and indicates that within a body of fluid at rest the pressure increases in the downward direction at the rate equivalent to the specific weight 'γ' of the liquid.

- A U-tube manometer can also be used to measure negative or vacuum pressure. For measurement of small negative pressure, a U-tube manometer without any manometric fluid may be used.

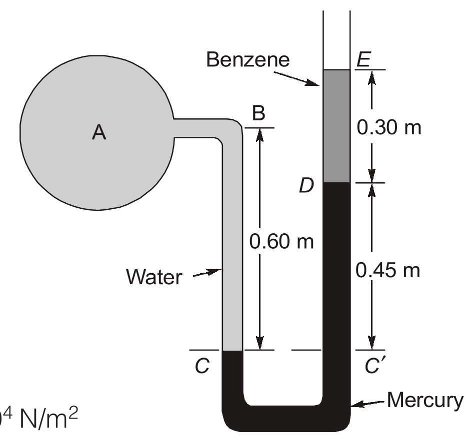
Limitations

- This method requires reading of fluid level at two or more points, since a change in pressure causes a rise of liquid in one limb of the manometer and a drop in the other.

Example 2.1 The left leg of U-tube mercury manometer is connected to a pipe-line conveying water. The level of mercury in the leg is 0.6 m below the center of pipe-line and the right leg is open to atmosphere. The level of mercury in the right leg is 0.45 m above that in the left leg and the space above mercury in the right leg contains Benzene (specific gravity 0.88) to a height of 0.3 m. Find the pressure in the pipe.

Solution:

In the accompanying figure, the pressures at C and C' are equal. Thus computing the pressure heads at C and C' from either side and equating the same, we get



$$\underbrace{\frac{p_A}{\gamma_w}+0.6}_{\text{(Left Leg)}} = \underbrace{0.45\times 13.6+0.3\times 0.88}_{\text{(Right Leg)}}$$

or $$\frac{p_A}{\gamma_w} = 5.784 \text{ m of water}$$

$\therefore$ $$p_A = (5.784\times 9810) = 5.674\times 10^4 \text{ N/m}^2$$

(iii) Single Column Manometer

- The limitation of U-tube manometer is removed in single column manometer.
- It is a modified form of U-tube manometer in which a shallow reservoir having a large cross-sectional area (about 100 times) as compared to the area of the tube is introduced into one limb of the manometer.
- For any change in pressure, the change in the liquid level in the reservoir will be so small that it may be neglected, and the pressure is indicated approximately by the height of the liquid in the other limb.
- Only one reading in the narrow limb of the manometer need to be taken for pressure measurement.

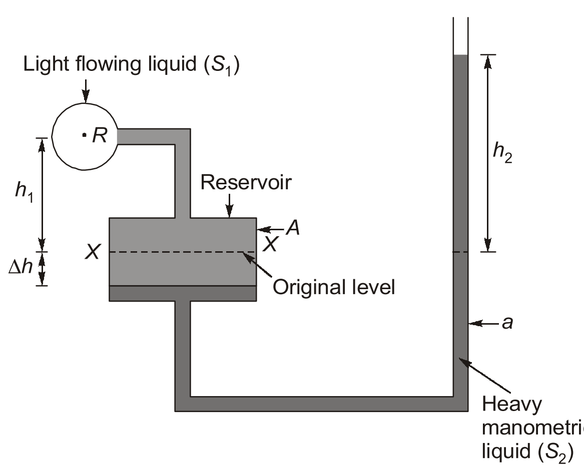


Fig. *Single column manometer*

- Narrow limb may be straight or inclined.
- The inclined type is useful for the measurement of small pressures as they are more sensitive than the vertical type.

$$A\Delta h = ah_2$$

or $$\Delta h = \frac{ah_2}{A}$$

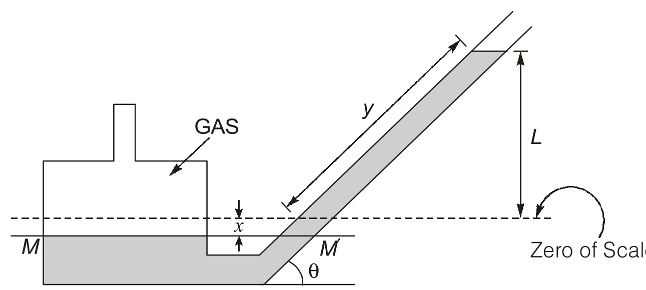
Now, $$p_R + \rho_1 g(h_1 + \Delta h) = \rho_2 g(h_2 + \Delta h)$$

$$p_R = \rho_2 gh_2 - \rho_1 gh_1 + \Delta hg(\rho_2 - \rho_1)$$

If Δh is very small, $$p_R = \rho_2 gh_2 - \rho_1 gh_1$$

Example 2.2 A manometer consists of an inclined glass tube which is connected to a metal cylinder standing upright. A manometric liquid fills the apparatus to a fixed zero mark on the tube when both cylinder and the tube are open to atmosphere. The upper end of the cylinder is then connected to a gas supply at a pressure p and the liquid rises in the tube.

Find an expression for the pressure p in cm of water when the liquid reads y cm in the tube, in terms of the inclination θ of the tube, the specific gravity of the liquid S, and the ratio a of the diameter of the cylinder to the diameter of the tube. Hence, determine the value of a so that the error due to disregarding the change in level in the cylinder will not exceed 0.1%, when $\theta = 30°$.



Solution:

Diameter ratio $= a$

So, Area ratio $= a^2$

Now, $$x\left(\frac{\pi D^2}{4}\right) = y\left(\frac{\pi d^2}{4}\right)$$

$$xa^2 = y$$

$$x = \frac{y}{a^2}$$

$$p_{(\text{incorrect})} = \rho_M g(y\sin 30°)$$

$$p_{(\text{correct})} = \rho_M g(y\sin 30° + x)$$

$$= \rho_M g\left(y\sin 30° + \frac{y}{a^2}\right)$$

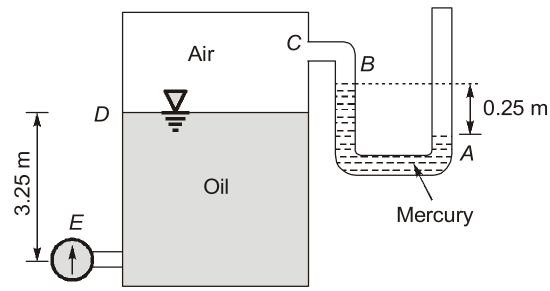
Now, $$\%\text{ Error} = \frac{p_{(\text{correct})} - p_{(\text{incorrect})}}{p_{(\text{correct})}} \times 100$$

$$0.1 = \frac{\frac{y}{a^2}}{y\sin 30° + \frac{y}{a^2}} \times 100$$

$\Rightarrow$ $$a^2 = 2000$$

$\Rightarrow$ $$a = 44.72$$

Example 2.10 The tank in the accompanying figure contains oil of specific gravity 0.750. Determine the reading of pressure gauge *A*.



Solution:

$$\text{Pressure at } B = (\text{Pressure at } A - 13.6 \times 0.25 \times 9.81)\ \text{kN/m}^2$$

Assuming every pressure in gauge format,

$$p_B = -13.6 \times 0.25 \times 9.81 = -33.354\ \text{kN/m}^2$$

$$\text{Pressure at } E = p_B + 3.25 \times 0.75 \times 9.81$$

$$p_A = -33.354 + 3.25 \times 0.75 \times 9.81 = -9.442125\ \text{kN/m}^2$$

(–ve) sign indicate that at point *A* there is a vacuum of 9.442125 kN/m^2.

Summary

- Normal stresses at any point in a fluid at rest, being directed towards the point from all directions, are of equal magnitude.
- Gauge pressure can be negative and positive.
- Pressure at all points on the plane parallel to the free liquid surface are same.
- Barometers are used for the measurement of atmospheric pressure.
- Manometers are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere.
- Piezometer tube measures the gauge pressure of a flowing liquid in terms of the height of liquid column.
- A simple U-tube manometer can be modified as an inclined tube manometer, an inverted tube manometer and a micromanometer to measure a small difference in pressure through a relatively large deflection of liquid column.

Important Expressions

- Fluid pressure at any point : $p = \dfrac{dF}{dA}$
- Variation in fluid pressure with depth : $\dfrac{dp}{dz} = -\gamma = -\rho g$
- Pressure head : $h = \dfrac{p}{\gamma}$

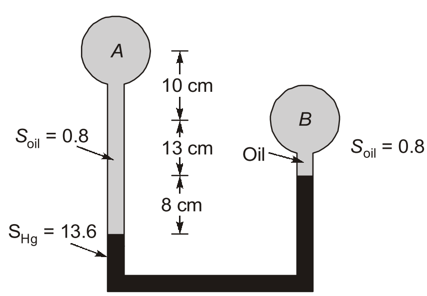
Objective Brain Teasers

Q.1 If a Mohr circle is drawn for a fluid element inside a fluid body at rest, it would be:
(a) a circle not touching the origin
(b) a circle touching the origin
(c) a point on the normal stress axis
(d) a point on the shear stress axis

Q.2 The pressure in meters of oil of specific gravity 0.8 equivalent to 80 m of water is
(a) 64 m (b) 88 m
(c) 80 m (d) 100 m

Q.3 The mass density of a liquid with variable density is given by $\rho = 1000 + 0.008\ y^{3/2}$, where ρ is in kg/m^3 ; y is measured in meters. The depth at which the pressure intensity will be 900 kPa, is
(a) 91.5 m (b) 101.5 m
(c) 112.5 m (d) 114.5 m

Q.4 The pressure difference between point *A* and *B* for the set up shown in figure in kPa is

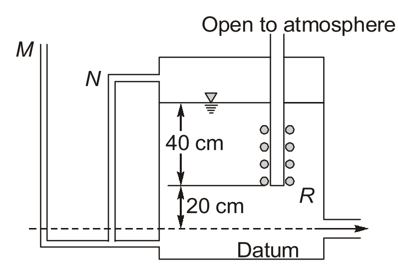


(a) 9.26 (b) 10.54
(c) 10.65 (d) 11.66

Q.5 In a mercury column-type barometer, the correct local atmospheric pressure is obtained by considering correction due to vapour pressure of mercury as follows; H_a =
(a) $H - h_v$ (b) $H_0 + h_v$
(c) H_0 / h_v (d) $h_v - H_0$
[where, H_a = correct local pressure in mm of mercury, H_0 = observed barometer reading in mm of mercury and h_v = vapour pressure of mercury in mm.]

Q.6 The standard atmospheric pressure is 101.32 kPa. The local atmospheric pressure at a location was 91.52 kPa. If a pressure is recorded as 22.48 kPa (gauge), it is equivalent to
(a) 123.80 kPa (abs) (b) 88.84 kPa(*abs*)
(c) 114.00 kPa(*abs*) (d) 69.04 kPa(*abs*)

Q.7 The tank shown in figure discharge water at constant rate for all water levels above the air inlet *R*. The height above datum to which water would rise in manometer tubes *M* and *N* respectively, are

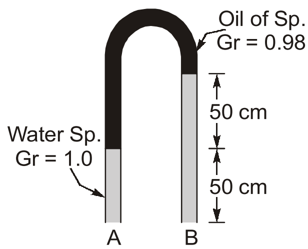


(a) (60 cm, 20 cm) (b) (40 cm, 40 cm)
(c) (20 cm, 20 cm) (d) (20 cm, 60 cm)

Q.8 Normal stresses are of the same magnitude in all directions at a point in a fluid
(a) only when the fluid is frictionless
(b) only when the fluid is at rest
(c) only when there is no shear stress
(d) in all cases of fluid motion

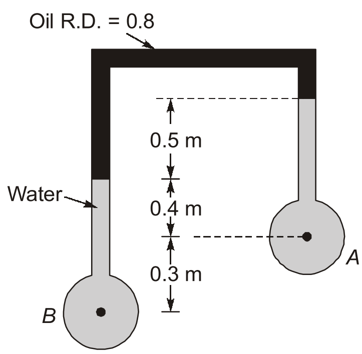
Q.9 Identify the CORRECT statement:
(a) Local atmospheric pressure is always less than the standard atmospheric pressure
(b) Local atmospheric pressure depends only on the elevation of the place
(c) A barometer reads the difference between the local and standard atmospheric pressure
(d) Standard atmospheric pressure is 760 mm of mercury

Q.10 In the setup shown in given figure assuming the specific weight of water as 10 kN/m^3, the pressure difference between the two points *A* and *B* will be



(a) 100 N/m^2 (b) – 100 N/m^2
(c) 200 N/m^2 (d) – 200 N/m^2

Q.11 An inverted differential manometer is shown in given figure. The differential pressure ($p_B - p_A$) in terms of column height of oil of relative density 0.8 is



(a) 0.25 m (b) 0.5 m
(c) 0.85 m (d) None of these

■ ANSWERS

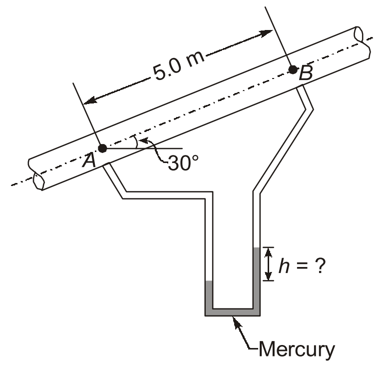
1. (c)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (b)
6. (c)	**7.** (d)	**8.** (b)	**9.** (d)	**10.** (a)
11. (a)				

Student's Assignments

Q.1 A certain fluid of specific gravity 0.8 flows upwards through a vertical pipe. *A* and *B* are two points on the pipe, *B* being 0.3 m higher than *A*. A U-tube mercury manometer is connected at points *A* and *B*. If the difference in pressure between *A* and *B* is 5 kPa, find the difference in the heights of the mercury columns in the manometer.

Ans. h = 21.4 mm

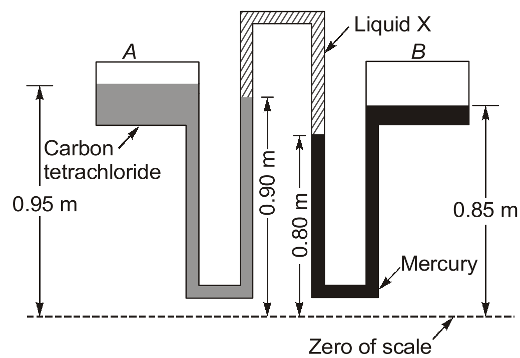
Q.2 If the pipe in the given figure contains water and there is no flow, calculate the value of the manometer reading *h*.



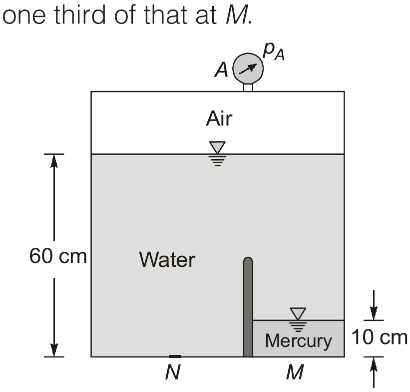
Ans. h = 0

Q.3 In the manometer shown in the given figure, the liquid on the left side is carbon - tetrachloride of specific gravity 1.60 and liquid on the right side is mercury. If ($p_A - p_B$) is 525 kg(f)/m^2 (5150.25 N/m^2), find the specific gravity of the liquid *X*.

Ans. 0.75



Q.4 For the system shown in given figure calculate the air pressure p_A to make the pressure at *N* one third of that at *M*.



Ans. p_A = 0.294 kPa

■■■■