CIVIL ENGINEERING

Fluid Mechanics and Hydraulic Machines

Comprehensive Theory
with Solved Examples *and* Practice Questions

MADE EASY
PUBLICATIONS

www.madeeasypublications.org

MADE EASY Publications Pvt. Ltd.

Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016 | **Ph. :** 9021300500

Email : infomep@madeeasy.in | **Web :** www.madeeasypublications.org

Fluid Mechanics and Hydraulic Machines

EDITIONS

First Edition : 2015
Second Edition : 2016
Third Edition : 2017
Fourth Edition : 2018
Fifth Edition : 2019
Sixth Edition : 2020
Seventh Edition : 2021
Eighth Edition : 2022
Ninth Edition : 2023

Tenth Edition : 2024

CONTENTS

Fluid Mechanics and Hydraulic Machines

CHAPTER 15

Impact of Jets 415-432

CHAPTER 16

Hydraulic Turbines 433-470

CHAPTER 17

Hydraulic Pumps 471-509

CHAPTER

1

Fluid Properties

1.1 INTRODUCTION

- A substance in the liquid or gas phase is referred as a fluid.
- Fluid is capable of flowing and conforms to the shape of the containing vessel.
- Fluid undergoes continuous deformation under the influence of shearing forces no matter how small the forces may be.
- This property of continuous deformation in technical terms is known as 'flow property', whereas this property is absent in solids.
- The distinction between a solid and a fluid is made on the basis of their ability to resist an applied shear stress. A solid can resist an applied shear stress by deforming itself by a fixed amount. On the other hand, a fluid shows its flow property under the application of shear stresses due to which it deforms continuously and does not come back to its previous position.
- In case of solids, total deformation is significant, whereas, in case of fluids, rate of deformation is significant in defining the properties.
- If a fluid is at rest, there can be no shearing forces acting and therefore, all forces in the fluid must be perpendicular to the planes upon which they act.
- Fluids may be classified as Ideal fluids or real fluids.

(i) Ideal Fluids: Ideal fluids are those fluids which have neither viscosity nor surface tension and they are incompressible. In nature, the ideal fluids do not exist and therefore, they are only imaginary fluids.

(ii) Real Fluids: Practical or real fluids are those fluids which possess viscosity, surface tension and compressibility.

1.2 FLUID MECHANICS

- Fluid mechanics is the study of fluids at rest (fluid statics) or in motion (fluid dynamics).
- The basic laws which are applicable to any fluid for analysis of any problem in fluid mechanics, are
  (i) The law of conservation of mass
  (ii) Newton's second law of motion
  (iii) The principle of angular momentum
  (iv) The first law of thermodynamics
  (v) The second law of thermodynamics

1.3 FLUID AS A CONTINUUM

- In a fluid system on macro scale, the intermolecular spacing between the fluid particles is treated as negligible and the entire fluid mass system is assumed as continuous distribution of mass, and such continuous mass of fluid is known as continuum.
- This assumption is valid only if the fluid system is very large as compared to the spacing between the particles. (Continuum is invalid at low pressure i.e. at high elevation)
- As a consequence of the continuum, each fluid property is assumed to have a definite value at every point in space. Thus, the fluid properties such as density, temperature and velocity etc., are considered as continuous functions of position and time.

For Example:

Velocity field, $\vec{V} = \vec{V}(x, y, z, t)$ or $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

where, each velocity component, *u, v* and *w* will be a function of x, y, z and t.

$\vec{V}(x, y, z, t)$ indicates the velocity of a fluid particle that is passing through the point x, y, z at time instant t.

Thus, the velocity is measured at the same location at different points of time.

In case of steady flow,

$$\frac{\partial \vec{V}}{\partial t} = 0$$

Therefore, $$\vec{V} = \vec{V}(x,y,z)$$

1.3.1 The No Slip Condition

- Consider the flow of a fluid over a stationary solid surface that is non-porous. As per the experimental observation, it has been found out that a fluid in motion comes to a complete stop at the surface of solid body and assumes zero relative velocity with solids surface. It represents that the fluid in direct contact with a solid, stick to the surface and there is no slip. This is known as “no slip condition”.
- The fluid property responsible for the no slip condition and development of the boundary layer is viscosity.
- The no slip condition is responsible for the development of velocity profile.
- Another consequence of no slip condition is the surface drag or skin friction drag.

1.4 FLUID PROPERTIES

- Any characteristic of a fluid system is called a fluid property.
- Fluid properties are of two types:
  - (i) **Intensive Properties:** Intensive properties are those that are independent of the size of the system or the amount of material in it. **Example:** Temperature, pressure, density etc.
  - (ii) **Extensive Properties:** Extensive properties are those whose values depend on the size or extent of the system. **Example:** Total mass, total volume, total momentum etc.
- Following are some of the intensive and extensive properties of a fluid system.

  (i) Viscosity (ii) Surface tension (iii) Vapour pressure (iv) Compressibility and elasticity

1.4.1 Some other Important Properties

1. **Mass Density:** Mass density or specific mass (ρ) of a fluid is the mass which it possesses per unit volume. Its SI unit is kg/m^3.

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

2. **Specific Weight:** Specific weight or weight density (γ) of a fluid is the weight it possesses per unit volume. Its SI unit is N/m^3. The mass density and specific weight γ has following relationship:
   $\gamma = \rho g$; $\rho = \gamma/g$. Both mass density and specific weight depend upon temperature and pressure.
3. **Relative Density (R.D.):** It is defined as the ratio of density of one substance to the density of other substance. Mathematically, $\rho_{1/2} = \frac{\rho_1}{\rho_2}$.
   where, $\rho_{1/2}$ = Relative density of substance '1' with respect to substance '2'.
4. **Specific Gravity:** Specific gravity (S) is the ratio of specific weight (or mass density) of a fluid to the specific weight (or mass density) of a standard fluid. The standard fluid chosen for comparison is pure water at 4°C for liquids and air or hydrogen for gases at some specified temperature and pressure.

$$S(\text{for liquid}) = \frac{\text{Specific weight of liquid}}{\text{Specific weight of water}} = \frac{\text{Specific weight of liquid}}{9810 \text{ N/m}^3}$$

$$S(\text{for gases}) = \frac{\text{Specific weight of gas}}{\text{Specific weight of air}}$$

   If specific gravity $< 1 \Rightarrow$ Fluid is lighter than standard fluid.
   If specific gravity $> 1 \Rightarrow$ Fluid is heavier than standard fluid.
   Specific gravity is unitless property.
5. **Specific Volume:** Specific volume of a fluid is the volume of fluid per unit mass. Thus it is the reciprocal of density. It is generally denoted by ν. Its SI unit is m^3/kg.

Example 1.1 Three litres of petrol weigh 23.7 N. Calculate the mass density, specific weight, specific volume and specific gravity of petrol.

Solution :

Mass density of petrol, $\rho_p = \frac{M}{V} = \frac{W/g}{V} = \frac{W}{gV} = \frac{23.7}{9.81 \times 3}$ = 0.805 kg/litre = 805 kg/m^3

Mass density of water, ρ_w = 1000 kg/m^3

$$\text{Specific gravity of petrol} = \frac{\rho_p}{\rho_w} = \frac{805}{1000} = 0.805$$

$$\text{Specific weight of petrol} = \frac{W}{V} = \frac{23.7}{3.0} = 7.9 \text{ N/litre} = 7.9 \text{ kN/m}^3$$

$$\text{Specific volume} = \frac{V}{M} = \frac{1}{\rho_p} = \frac{1}{805} = 1.242 \times 10^{-3} \text{ m}^3\text{/kg}$$

1.5 VISCOSITY

- Viscosity is the property of fluids by virtue of which they offer resistance to shear or angular deformation.
- It is primarily due to cohesion (in case of liquids) and molecular momentum exchange (in case of gases) between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers.

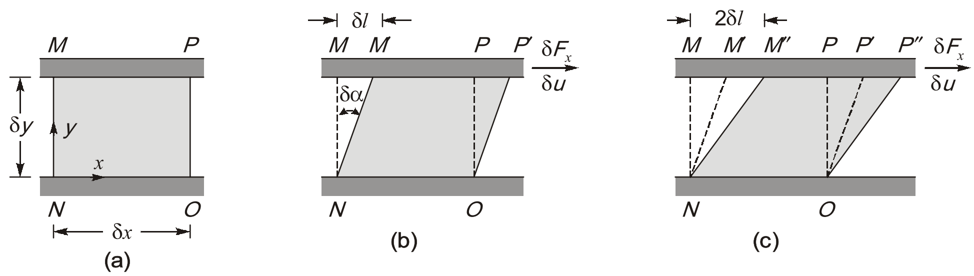


Fig: (a) Fluid element at time t, **(b)** Deformation of fluid element at time $t + \delta t$, and **(c)** Deformation of fluid element at time $t + 2\delta t$.

- Consider a fluid element between the two infinite plates. The rectangular fluid element is initially at rest at time t. Let us now suppose a constant rightward force δF_x is applied to the upper plate so that it is dragged across the fluid at constant velocity δu. The relative shearing action of the plates produces a shear stress, τ_{yx}, which acts on the fluid element and is given by

$$\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y},$$

where δA_y is the area of contact of the fluid element with the plate and δF_x is the force exerted by the plate on that element.

Various positions of the fluid element, illustrate the deformation of the fluid element from position $MNOP$ at time t, to $M'NOP'$ at time $t + \delta t$, to $M''NOP''$ at time $t + 2\delta t$, due to the imposed shear stress. The deformation of the fluid is given by

$$\text{Deformation rate} = \lim_{\delta t \to 0} \frac{\delta \alpha}{\delta t} = \frac{d\alpha}{dt}$$

Distance between the points M and M' is given by,

$$\delta l = \delta u \, \delta t \quad \text{...(1)}$$

Alternatively, for small angles, $\delta l = \delta y \delta \alpha$ ...(2)

Equating equations (1) and (2), $\delta u \delta t = \delta y \delta \alpha$

or

$$\frac{\delta \alpha}{\delta t} = \frac{\delta u}{\delta y}$$

Taking the limits of both sides

$$\lim_{\delta t \to 0} \frac{\delta \alpha}{\delta t} = \lim_{\delta t \to 0} \frac{\delta u}{\delta y}$$

$$\frac{d\alpha}{dt} = \frac{du}{dy}$$

Thus, the rate of angular deformation is equal to velocity gradient across the flow.

1.5.1 Newton's law of viscosity

- According to Newton's law of viscosity, shear stress is directly proportional to the rate of deformation or velocity gradient across the flow.

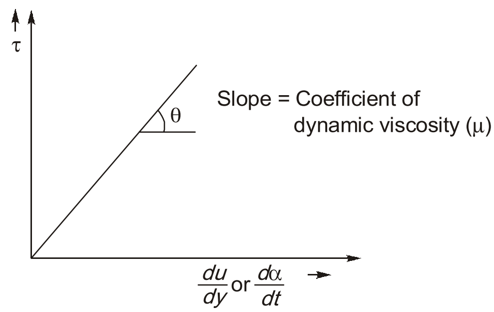


Fig. Newton's law of viscosity

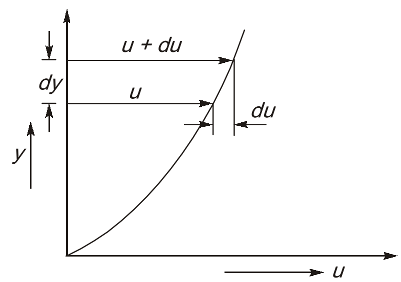


Fig. Velocity profile

Thus, $$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

where, μ = Coefficient of dynamic viscosity

Dynamic Viscosity (μ)

- Dimension of $\mu = [M L^{-1} T^{-1}]$
- Unit of $\mu = Ns/m^2$ or Pa.s
- In C. G. S. units, μ is expressed as 'poise', 1 poise = 0.1 N-s/m^2
- A 20°C and at standard atmospheric pressure, $(\mu)_{water} \approx 10^{-3}$ Ns/m^2; $(\mu)_{air} \approx 1.81 \times 10^{-5}$ Ns/m^2

NOTE

- Water is nearly 55 times viscous than air.
- **Linearization of Newton's law of viscosity:** If the flow is taking place between two parallel plates where the gap between the plates is very small then velocity gradient is assumed to be constant. If the gap is large then velocity gradient will be variable.

Kinematic Viscosity (ν)

- The kinematic viscosity (ν) is defined as the ratio of dynamic viscosity to mass density of the fluid. Therefore, $\nu = \mu/\rho$
- Dimension of $\nu = [L^2 T^{-1}]$
- Unit of $\nu = m^2/s$ or cm^2/s (stoke, in C.G.S. units)
- 1 stoke = 10^{-4} m^2/s
- At 20°C and standard atmospheric pressure, $\nu_{water} = 1 \times 10^{-6}$ m^2/s, $\nu_{air} = 15 \times 10^{-6}$ m^2/s

NOTE: Kinematic viscosity of air is about 15 times greater than the corresponding value of water.

1.5.2 Variation of viscosity with Temperature

1. **Dynamic viscosity:** Increase in temperature causes a decrease in the dynamic viscosity of a liquid, whereas viscosity of gases increases with temperature growth.

   The reason for the above phenomena is that; in liquids; viscosity is primarily due to molecular cohesion which decreases due to increase in volume due to temperature increment, while in gases, viscosity is due to molecular momentum transfer which increases due to increase in number of collision between gas molecules.

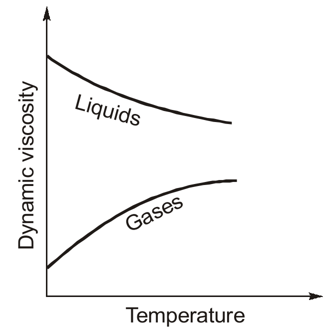


Fig: Variation of Dynamics Viscosity with Temperature

2. **Kinematic Viscosity:** Kinematic viscosity is ratio of dynamic viscosity to the density of fluid. In case of liquids with increase in temperature, the dynamic viscosity as well as density both decrease but decrease in dynamic viscosity is very high as compared to density. So, overall kinematic viscosity will decrease for liquids. On the other hand, in case of gases, with increase in temperature dynamic viscosity increases and density decreases. So overall kinematic viscosity increases for gases.

1.5.3 Variation of viscosity with pressure

1. **Dynamic viscosity:** In fluids, dynamic viscosity is practically independent of pressure except at extremely high pressure.
2. **Kinematic viscosity:** In liquids, kinematic viscosity is independent of pressure at low to moderate pressure.

   In case of gases, density increases with increase in pressure, therefore kinematic viscosity decreases.

1.5.4 Types of Fluids

The fluids are classified into following types based on shear stress variation with velocity gradient:

(i) Newtonian Fluids

- Fluids which obey newton's law of viscosity are known as Newtonian fluids.
- General relationship between shear stress and velocity gradient is given by,

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

- For Newtonian fluids, $n = 1$, $A = \mu$ and $B = 0$,

  Thus, $$\tau = \mu\frac{du}{dy}$$

  Examples: Air, water, Mercury, Petrol, Kerosene, etc.

(ii) Non-Newtonian Fluids

- Fluids for which shear stress is not directly proportional to deformation rate are Non-Newtonian fluids.

  Examples: Toothpaste and paint.
- Non-Newtonian fluids are commonly classified as having time-independent or time-dependent behavior.

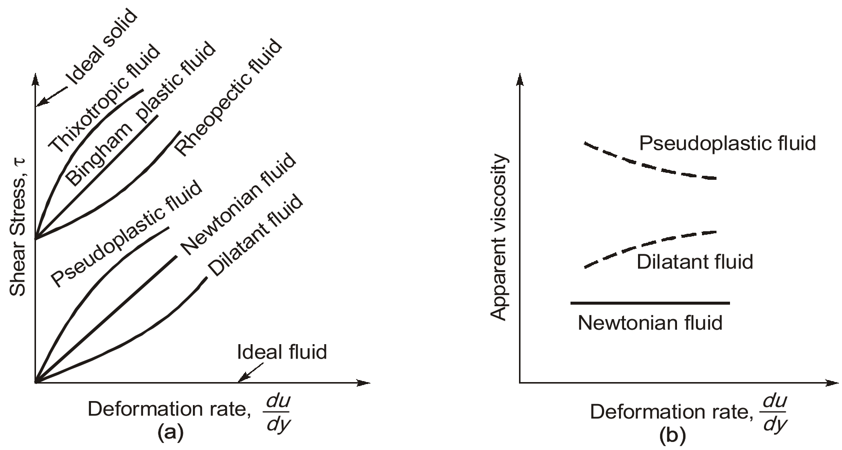


Fig: (a) Variation Shear stress rate with deformation **(b)** Variation of Apparent viscosity with deformation rate

- Relation between shear stress and rate of deformation for non-Newtonian fluid can be represented as:

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

where, n = flow behavior index; A = consistency index; B = Residual strength

Above equation can also be represented as:

$$\tau = A\left(\frac{du}{dy}\right)^{n-1}\left(\frac{du}{dy}\right) + B = \eta\frac{du}{dy} + B$$

where, $\eta = A\left(\frac{du}{dy}\right)^{n-1}$ is referred as the apparent viscosity

NOTE: Dynamic viscosity (μ) doesn't depend on the shear rate, while apparent viscosity (η) depends on the shear rate.

- Various types of non-Newtonian fluids are :
  1. **Pseudoplastic fluids**: Fluids in which the apparent viscosity decreases with increasing deformation rate ($n < 1$) are called pseudoplastic fluids or shear thinning fluid. Most Non-Newtonian fluids fall into this group. These are time independent fluids.
     Example: Polymer solutions, colloidal suspensions, milk, blood and paper pulp in water, etc.
  2. **Dilatant fluids:** If the apparent viscosity increases with increasing deformation rate ($n > 1$), then the fluid is termed as dilatant or shear thickening fluid. These are time independent fluids.
     Example: Suspensions of starch, saturated sugar solution, etc.
  3. **Bingham Plastic fluids:** Fluids that behave as a solid until a minimum yield stress, τ_y, is reached and flow after crossing this stress are known as Ideal plastic or Bingham plastic fluids. The corresponding shear stress model is $\tau = \tau_y + \mu\frac{du}{dy}$.
     Example: Clay suspensions, drilling muds, sewage sludge, creams, toothpaste, etc.
  4. **Thixotropic fluid:** Apparent viscosity (η) for thixotropic fluids decreases with time under a constant applied shear stress. These are time dependent fluids.
     Example: Paints, printer inks, etc.

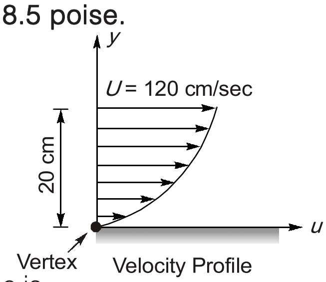
5. **Rheopectic fluid:** Apparent viscosity (η) for rheopectic fluids increases with time under constant shear stress. These are time dependent fluids.
   Example: Gypsum pastes.

NOTE

- **Viscoelastic fluids:** Fluids which after some deformation partially return to their original shape when the applied stress is released are called viscoelastic fluids. Example, polymerised fluid with drag reduction features.
- **Rheology:** It is the branch of science which deals with the studies of different types of fluid behaviours.

Example 1.2 If the velocity profile of a fluid over a plate is parabolic with free stream velocity of 120 cm/s occurring at 20 cm from the plate, calculate the velocity gradients and shear stress at a distance of 0, 10 and 20 cm from the plate. Take the viscosity of the fluid as 8.5 poise.

Solution:



Given:
Distance of surface from plate = 20 cm
Velocity at surface, $U = 120$ cm/s

Viscosity, $\mu = 8.5 \text{ poise} = \dfrac{8.5}{10}\,\text{Ns/m}^2 = 0.85\,\text{Ns/m}^2$

The velocity profile is given as parabolic. Hence equation of velocity profile is

$$u = ay^2 + by + c \qquad \text{...(i)}$$

where a, b and c are constants. Their values are determined from boundary conditions as:

(a) at $y = 0$, $u = 0$
(b) at $y = 20$ cm, $u = 120$ cm/s
(c) at $y = 20$ cm, $\dfrac{du}{dy} = 0$

Boundary condition (*a*) on substitution in equation (i), gives

$$c = 0$$

Boundary condition (*b*) on substitution in equation (i) gives

$$120 = a(20)^2 + b(20) = 400a + 20b \qquad \text{...(ii)}$$

Boundary condition (*c*) on substitution in equation (i) gives

$$\frac{du}{dy} = 2ay + b \qquad \text{...(iii)}$$

or $\qquad 0 = 2 \times a \times 20 + b = 40\,a + b$

Solving equations (ii) and (iii) for *a* and *b*

From equation (iii), $\qquad b = -40\,a$

Substituting this value in equation (ii), we get

$$120 = 400\,a + 20 \times (-40a)$$
$$= 400a - 800a = -\,400a$$

$$\therefore \qquad a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$$\therefore \qquad b = (-40) \times (-0.3) = 12.0$$

Substituting the values of *a*, *b* and *c* in equation (i)

$$u = -0.3\,y^2 + 12\,y$$

Velocity Gradient

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

at $y = 0$, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 = 12$ per sec

at $y = 10$ cm, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 = 6$ per sec

at $y = 20$ cm, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 = 0$

Shear Stresses

Shear stress is given by $\tau = \mu \frac{du}{dy}$

(i) Shear stress at $y = 0$, $\tau = \mu\left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.2 \text{ N/m}^2$

(ii) Shear stress at $y = 10$, $\tau = \mu\left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6.0 = 5.1 \text{ N/m}^2$

(iii) Shear stress at $y = 20$, $\tau = \mu\left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0 = 0$

Example 1.3 Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerin. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surfaces at a speed of 0.6 m/s, if:

(i) the thin plate is in the middle of the two plane surfaces, and
(ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces?
Take the dynamic viscosity of glycerin = 8.10×10^{-1} Ns/m^2. Assume linear velocity distribution in transverse direction.

Solution :

Given:

Distance between two large surfaces = 2.4 cm

Area of thin plate, $A = 0.5 \text{ m}^2$

Velocity of thin plate, $u = 0.6$ m/s

Viscosity of glycerin, $\mu = 8.10 \times 10^{-1}$ Ns/m^2

Case-I: When the thin plate is in the middle of the two plane surfaces.

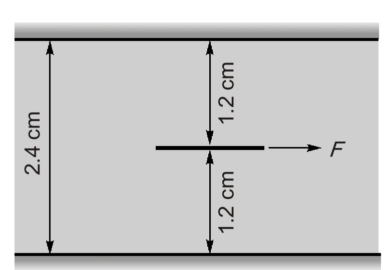


Fig: Case-I

Let, F_1 = Shear force on the upper side of the thin plate
F_2 = Shear force on the lower side of the thin plate
F = Total force required to drag the plate

Then, $F = F_1 + F_2$

The shear stress (τ_1) on the upper side of the thin plate is given by equation,

$$\tau_1 = \mu\left(\frac{du}{dy}\right)_1$$

where, du = Relative velocity between thin plate and upper large plane surface = 0.6 m/s.
dy = Distance between thin plate and upper large plane surface
= 1.2 cm = 0.012 m (plate is a thin one and hence thickness of plate is neglected)

Assuming linear velocity distribution between large plane surfaces and thin plate.

$$\therefore \quad \tau_1 = 8.10\times10^{-1}\times\left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

Now shear force, F_1 = Shear stress × Area
$= \tau_1 \times A = 40.5 \times 0.5 = 20.25$ N

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = \mu\left(\frac{du}{dy}\right)_2 = 8.10\times10^{-1}\times\left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

∴ Shear force, $F_2 = \tau_2 \times A = 40.5 \times 0.5 = 20.25$ N

∴ Total force, $F = F_1 + F_2 = 20.25 + 20.25 = 40.5$ N

Case II: When the thin plate is at a distance of 0.8 cm from one of the plane surfaces.

Let the thin plate is at a distance 0.8 cm from the lower plane surface.

Then distance of the plate from the upper plane surface

= 2.4 – 0.8 = 1.6 cm = 0.016 m (Neglecting thickness of the plate)

The shear force on the upper side of the thin plate,

$$F_1 = \text{Shear stress} \times \text{Area} = \tau_1 \times A$$

$$= \mu\left(\frac{du}{dy}\right)_1 \times A$$

$$= 8.10\times10^{-1}\times\left(\frac{0.6}{0.016}\right)\times 0.5 = 15.19 \text{ N}$$

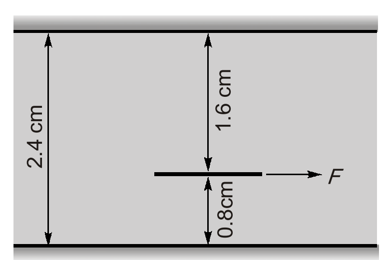


Fig: Case-II

The shear force on the lower side of the thin plate,

$$F_2 = \tau_2 \times A = \mu\left(\frac{du}{dy}\right)_2 \times A$$

$$= 8.10\times10^{-1}\times\left(\frac{0.6}{0.8/100}\right)\times 0.5 = 30.38 \text{ N}$$

∴ Total force required $= F_1 + F_2 = 15.19 + 30.38 = 45.57$ N **Ans.**

1.6 SURFACE TENSION

- It has been seen that a drop of blood forms a hump on horizontal glass. Similarly a drop of mercury forms a near perfect sphere and can be rolled just like a steel ball over a smooth surface.

- In these observations, liquid droplets behave like small balloons filled with the liquid and surface of the liquid acts like a stretched elastic membrane under tension. These pulling forces that causes this tension acts parallel to the surface and is due to attractive forces between the molecules of the liquid. The magnitude of this force per unit length is called surface tension (σ) and is expressed in unit N/m.
- To visualize that how surface tension arise at interface, consider the microscopic view of molecule on surface and inside the liquid filled in a container. The attractive force applied on the interior molecule by the surrounding molecules balance each other because of symmetry. But for the molecule on the surface or at interface between two different mediums, the attractive forces are not symmetric. The attractive forces applied by the air/gas molecule above are usually small. Therefore, there is a net attractive force acting on the molecule at the surface of the liquid. These are balanced by repulsive forces from the molecules below the surface that are trying to be compressed. Due to this, liquid minimise its surface area.

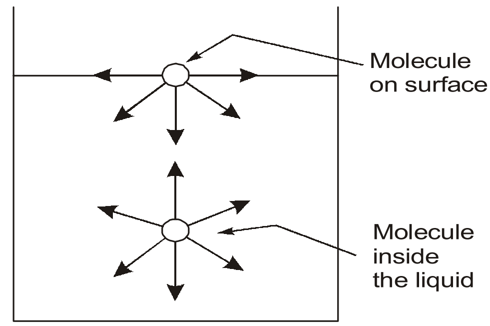


Fig: Surface tension

  This is the reason for the tendency of liquid droplets to attain a spherical shape which has the minimum surface area for a given volume.
- Surface tension is due to "cohesion" between the liquid particles.
- Whenever a liquid is in contact with other liquids or gases, or solid surface, an interface develops that acts like a stretched elastic membrane, creating surface tension.
- There are two features to this stretched elastic membrane : the contact angle θ, and the magnitude of the surface tension, σ (N/m). Both of these, depend on both the type of liquid and the type of solid surface (or other liquid or gas) with which it shares an interface.

  For example, the car's surface will get wetted when water is applied to the surface. If before applying water, waxing is done to the car's surface and then water is applied, the car's surface will not get wet. This is because of the change of the contact angle from being smaller than 90°, to larger than 90°. The waxing has changed the nature of the solid surface.

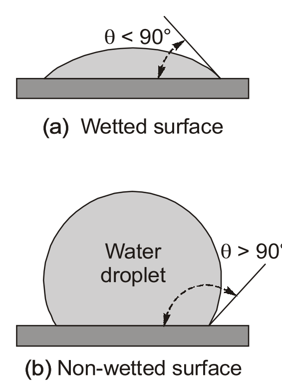


Fig: Surface tension effect on water droplets

- For liquids, surface tension decreases with increase in temperature.
- Due to surface tension, pressure change occurs across a curved interface.
- Surface tension is also defined as work done per unit increase in surface area. This work done is stored in the form of surface energy.

$$\sigma = \frac{\text{Work done}}{\text{Area}}$$

- A liquid droplet takes spherical shape because surface area is minimum in spherical condition. Therefore, the surface energy is minimum. Minimum surface energy leads to most stable state.

NOTE

- For air-water interface, $\sigma = 0.073$ N/m.
- For water glass interface, contact angle, $\theta \approx 0$.
- For air-mercury interface, $\sigma = 0.44$ N/m.
- For mercury glass interface, contact angle, $\theta = 130°$.

1.7 APPLICATIONS OF SURFACE TENSION

1.7.1 Droplet and Jet

- When a droplet is separated initially from the surface of the main body of liquid, then due to surface tension there is a net inward force exerted over the entire surface of the droplet which causes the surface of the droplet to contract from all the sides and results in increasing the internal pressure within the droplet.
- The contraction of the droplet continues till the inward force due to surface tension is in balance with the internal pressure and the droplet forms into sphere which is the shape for minimum surface area.
- The internal pressure within a jet of liquid is also increased due to surface tension.
- The internal pressure intensity within a droplet and a jet of liquid in excess of the outside pressure intensity may be determined by the expressions derived below.

(i) Pressure intensity inside a water droplet : Consider a spherical water droplet of radius r having internal pressure intensity p in excess of the outside pressure intensity. If the droplet is cut into two halves, then the forces acting on one half will be those due to :

(a) Pressure intensity (p) on the projected area (πr^2).

(b) Tensile force due to surface tension (σ) acting around the circumference ($2\pi r$).

These two forces will be equal and opposite for equilibrium and hence, we have,

$$p(\pi r^2) = \sigma(2\pi r)$$

or

$$p = \frac{2\sigma}{r} = \frac{4\sigma}{d}$$

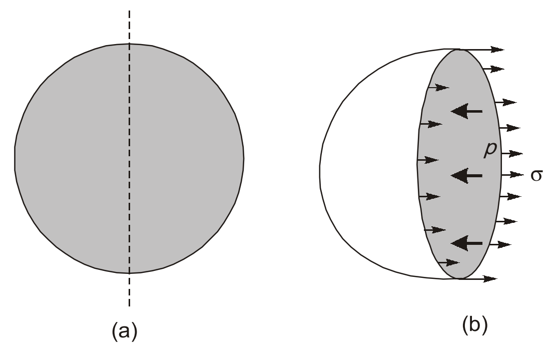


Fig: Surface Tension (σ) and Internal Pressure (p) in a droplet

NOTE

- Above equation indicates that the internal pressure intensity increase with the decrease in the size of droplet.
- In case of an air bubble in water, an interface is formed where air is inside and water is outside. Therefore it is same as water droplet.

$$\Delta p = \frac{2\sigma}{r} = \frac{4\sigma}{d}$$

(ii) Pressure intensity inside a soap bubble: A spherical soap bubble has two surfaces in contact with air, one inside and the other outside, each one of which contributes the same amount of tensile force due to surface tension. Therefore, consider a hemispherical section of a soap bubble of radius r, the tensile force due to surface tension is equal to $2\sigma(2\pi r)$. However, the

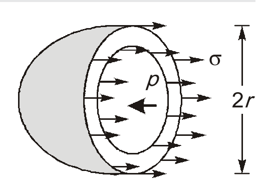


Fig. Soap Bubble

pressure force acting on the hemispherical section of the soap bubble is same as in the case of a droplet and it is equal to $p(\pi r^2)$. Thus, equating these two forces for equilibrium, we have

$$p(\pi r^2) = 2\sigma(2\pi r)$$

or
$$p = \frac{4\sigma}{r} = \frac{8\sigma}{d}$$

(iii) Pressure intensity inside a liquid jet: Consider a jet of liquid of radius r, length l and having an internal pressure intensity p in excess of outside pressure intensity. If the jet is cut into two halves, then the forces acting on one half will be those due to pressure intensity p on the projected area ($2rl$) and the tensile force due to surface tension (σ) acting along the two sides ($2l$). These two forces will be equal and opposite for equilibrium and hence we have,

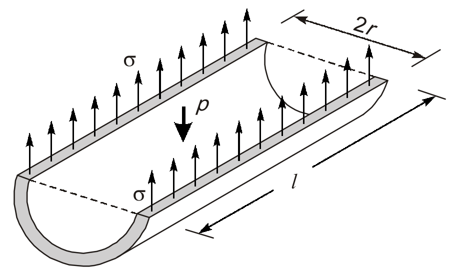


Fig. Liquid Jet

$$p(2rl) = \sigma(2l)$$

or
$$p = \frac{\sigma}{r} = \frac{2\sigma}{d}$$

NOTE: Some insects can land on water or even walk on water and small steel needle can float on water. All these phenomenons are possible due to surface tension which balances the weight of these objects.

Example 1.4 If the surface tension at the air-water interface is 0.073 N/m, estimate the pressure difference between inside and outside of an air bubble of diameter 0.01 mm.

Solution:

An air bubble has only one surface.

Hence
$$\Delta p = \frac{2\sigma}{R} = \frac{2\times 0.073}{\left(\frac{0.01}{2}\right)\times 10^{-3}} = 29200 \text{ N/m}^2 = 29.2 \text{ kPa}$$

1.7.2 Capillarity

- The phenomenon of rise or fall of a liquid in a small diameter tube inserted into a liquid is known as capillary effect.
- The capillary effect depends upon both the cohesive forces (the forces between like molecules) and adhesive forces (the forces between unlike molecules). Relative magnitude of these forces will determine whether a liquid rises or falls in the tube.
- If the adhesive forces are predominant, then the fluid tends to rise along the glass surface. For example, the water molecules are more strongly attached to the glass molecules than they are to other water molecules. Thus, water rise in the glass tube.
- If the cohesive forces are predominant, then fluid tends to fall down along the glass surface. For example, in case of mercury, due to predominant cohesive forces between its molecules, than they are to glass molecules. Thus, mercury falls in the glass tube.

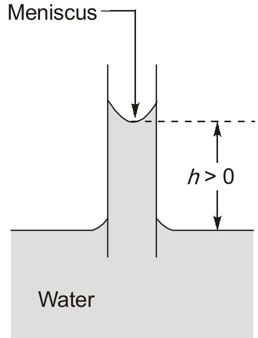


Fig. Capillary rise of water

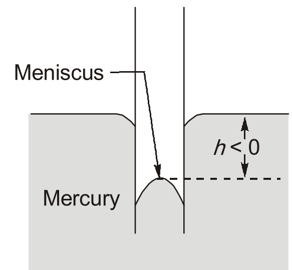
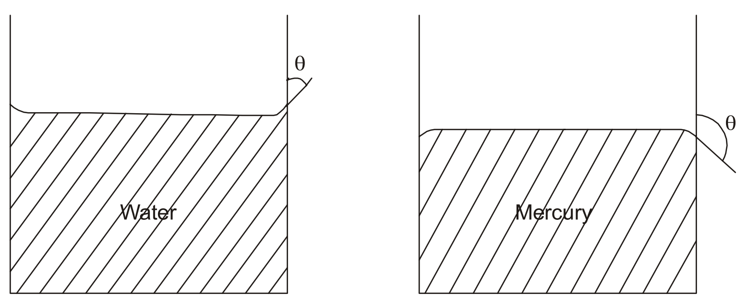


Fig. Capillary fall of Mercury

- This effect can also be expressed by saying that water wets the glass (by sticking to it) while mercury does not. The strength of capillary effect is quantified by the contact angle which is defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact. The surface tension force acts along the tangent line towards the solid surface.
- A liquid is said to wet the surface when contact angle is less than 90° ($\theta < 90°$) and not to wet the surface when contact angle is more than 90° ($\theta > 90°$).
- Surface tension force acts upward on a water in a glass tube, tending to pull the water up because the contact angle of water with glass is nearly zero. As a result, water will rise in the tube until the weight of the liquid in the tube is balanced by surface tension force. The contact angle of mercury is 130° at glass interface. Therefore, surface tension will act downwards tending to fall the mercury level in the glass tube.



(a) Wetting fluid

(b) Non-Wetting fluid

Fig. The contact angle for wetting and non-wetting fluids

- Contact angle for Kerosene glass interface is 26° in air.

Expression for capillary rise or fall

- Let the level of liquid rises (or fall) by height, h above (or below) the general liquid surface when a tube of radius r is inserted in a liquid having specific gravity 'S'. By equilibrium condition, the weight of liquid column of height h(or the total internal pressure in the case of capillary depression) must be balanced by the force, at surface of the liquid, due to surface tension σ.

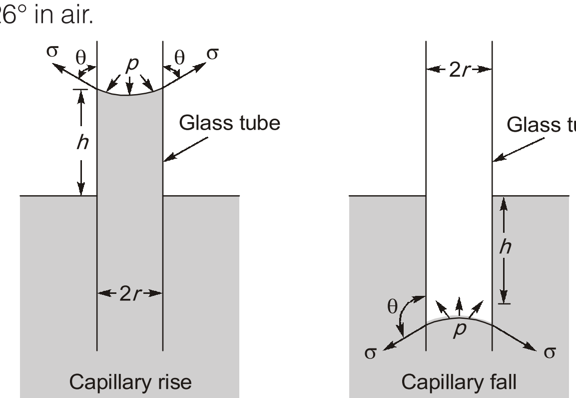


Fig. Capillarity in circular glass tubes

Thus, $S\gamma_w \pi r^2 h = 2\pi r \sigma \cos\theta$

where, S = specific gravity of liquid, γ_w = specific weight of water, θ = contact angle

$$h = \frac{4\sigma\cos\theta}{S\gamma_w d}$$

Since, the contact angle θ for water and glass is equal to zero, $h = \dfrac{2\sigma}{\gamma_w r}$

Assumptions in deriving above equations

(a) The meniscus of the curved liquid surface is a section of sphere.

(b) The liquid and tube surface are extremely clean.

REMEMBER

- At 20°C and for water and glass, $h = \dfrac{0.30}{d}$ m

  where, d = diameter of tube in cm
- With increase in diameter of the tube, capillary rise decreases. For tube of diameter more than 6 mm (radius > 3 mm), the capillary rise is negligible.

- If an annular tube, is immersed in a liquid, with outer radius r_o and inner radius r_i, then capillary rise is given by,

$$h = \frac{2\sigma\cos\theta}{(r_0 - r_i)S\gamma_w}$$

- If a tube of radius 'r' is inserted in mercury (Specific gravity, S_1) above which a liquid of specific gravity, S_2 lies, then the capillary fall or depression h is given by,

$$h = \frac{2\sigma\cos\theta}{r\gamma_w(S_1 - S_2)}$$

- If two vertical plates 't' distance apart are held partially immersed in a liquid of surface tension σ and specific gravity, S, then capillary rise or depression h is given by,

$$h = \frac{2\sigma\cos\theta}{S\gamma_w t}$$

NOTE

On reducing the height of capillary tube than the required height, the curvature of the liquid surface inside the capillary tube rearranges itself. The contact angle increases for a wetting interface and it decreases for a non-wetting interface.

Example 1.5 Derive an expression for the capillary rise between two vertical parallel plates of width b partially immersed in a liquid of specific gravity S in terms of the distance t between the plates, surface tension σ and the contact angle θ between the liquid and the plates.