

# CIVIL ENGINEERING

## Engineering Mechanics



Comprehensive Theory  
*with Solved Examples and Practice Questions*



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**Corporate Office:** 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016 | **Ph. :** 9021300500

**Email :** infomep@madeeasy.in | **Web :** www.madeeasypublications.org

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EDITIONS

First Edition : 2020

Second Edition : 2021

Third Edition : 2022

Fourth Edition : 2023

**Fifth Edition : 2024**

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# Basics of Vectors

## 1.1 VECTORS AND SCALARS

Many physical quantities are completely described by a numerical value alone and are added according to the ordinary rules of algebra. As an example the mass of a system is described by saying that it is 5 kg. If two bodies one having a mass of 5 kg and the other having a mass of 2 kg are added together to make a composite system, the total mass of the system becomes  $5 \text{ kg} + 2 \text{ kg} = 7 \text{ kg}$ . Such quantities are called scalars.

So, a scalar is any positive or negative physical quantity that can be completely specified by its magnitude. Other examples of scalar quantities are length, mass and time.

While the complete description of certain physical quantities requires a numerical value as well as a direction in space. Velocity of a particle is an example of this kind. The magnitude of velocity is represented by a number such as 5 m/s and tells us how fast a particle is moving. But the description of velocity becomes complete only when the direction of velocity is also specified. We can represent this velocity by drawing a line parallel to the velocity and putting an arrow showing the direction of velocity.

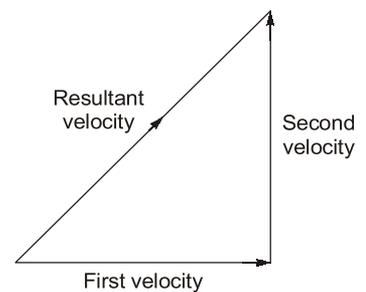


Fig. Triangle law to obtain resultant velocity

Further, if a particle is given two velocities simultaneously, its resultant velocity is different from the two velocities and is obtained by using a special rule known as triangle law.

The physical quantities which have magnitude and direction and which can be added according to the laws of vector addition are called vector quantities. Other examples of vector quantities are force, linear momentum, electric field, magnetic field etc.

## 1.2 EQUALITY OF VECTORS

Two vectors (representing two values of the same physical quantity) are called equal if their magnitudes and directions are same. Thus, a parallel translation of a vector does not bring about any change in it.

### 1.3 UNIT VECTORS

A vector whose magnitude is unity (i.e. 1 unit) is called a unit vector. Unit vector is a director vector. Unit

vector in the direction of any given vector  $\vec{a}$  is given as  $\frac{\vec{a}}{|\vec{a}|}$ .

#### EXAMPLE : 1.1

The unit vector in the direction of  $\vec{A} = 5\hat{i} + \hat{j} - 2\hat{k}$  is

**Solution:**

$$|\vec{A}| = \sqrt{5^2 + 1^2 + (-2)^2} = \sqrt{30}$$

$$\text{The required unit vector is } \frac{\vec{A}}{|\vec{A}|} = \frac{5}{\sqrt{30}}\hat{i} + \frac{1}{\sqrt{30}}\hat{j} - \frac{2}{\sqrt{30}}\hat{k}$$

### 1.4 ADDITION OF VECTORS

The triangle rule of vector addition is already described above. If  $\vec{a}$  and  $\vec{b}$  are two vectors to be added, a diagram is drawn in which the tail of  $\vec{b}$  coincides with the head of  $\vec{a}$ . The vector joining the tail of  $\vec{a}$  with the head of  $\vec{b}$  is the vector sum of  $\vec{a}$  and  $\vec{b}$ .

The same rule may be stated in a slightly different way by parallelogram law.

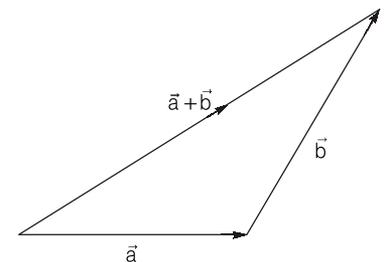


Fig. Vector addition as per triangle law

In parallelogram law we draw vectors  $\vec{a}$  and  $\vec{b}$  with both the tails coinciding as shown in figure. Taking these two as the adjacent sides we complete the parallelogram. The diagonal through the common tails gives the sum of the two vectors.

Suppose the magnitude of  $\vec{a} = a$  and that of  $\vec{b} = b$ . If the angle between  $\vec{a}$  and  $\vec{b}$  is  $\theta$ , then

$$\begin{aligned} AD^2 &= (AB + BE)^2 + (DE)^2 \\ &= (a + b\cos\theta)^2 + (b\sin\theta)^2 \\ &= a^2 + 2ab\cos\theta + b^2 \end{aligned}$$

Thus, the magnitude of  $\vec{a} + \vec{b}$  is

$$= \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

Its angle with  $\vec{a}$  is  $\alpha$  where,

$$\tan\alpha = \frac{DE}{AE} = \frac{b\sin\theta}{a + b\cos\theta}$$

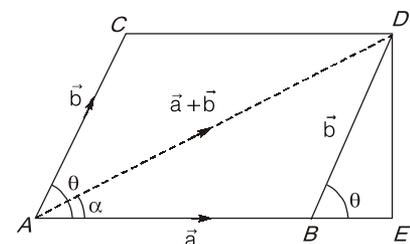


Fig. Vector addition as per parallelogram law

#### Special cases:

(a) When two vectors are acting in same direction, then,

$$\theta = 0^\circ$$

$\therefore$

$$|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab} = a + b$$

and 
$$\tan\alpha = \frac{b \times \sin 0^\circ}{a + b \cos 0^\circ} = 0$$

$\Rightarrow \alpha = 0^\circ$

Thus, the magnitude of sum of vectors  $\vec{a}$  and  $\vec{b}$  is equal to the sum of magnitudes of two vectors acting in same direction and their resultant acts in direction of  $\vec{a}$  and  $\vec{b}$ .

**(b)** When two vectors acts in opposite directions:

Then,

$$\theta = 180^\circ$$

$\therefore$

$$|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 - 2ab} = a - b$$

and

$$\tan\alpha = \frac{b \times \sin(180^\circ)}{a + b \times \cos(180^\circ)} = 0$$

$\Rightarrow$

$$\alpha = 0^\circ \text{ or } 180^\circ$$

Thus, the magnitude of sum of the vectors  $\vec{a}$  and  $(-\vec{b})$  is equal to the difference of magnitudes of two vectors and their resultant acts in direction of bigger vector.

## 1.5 MULTIPLICATION OF VECTOR BY A NUMBER

Suppose  $\vec{a}$  is a vector of magnitude  $a$  and  $k$  is a number. We define the vector  $\vec{b} = k\vec{a}$  as a vector of magnitude  $|ka|$ . If  $k$  is positive the direction of the vector  $\vec{b} = k\vec{a}$  is same as that of  $\vec{a}$ . If  $k$  is negative, the direction of  $\vec{b}$  is opposite to  $\vec{a}$ . In particular, multiplication by  $(-1)$  just inverts the direction of the vector. The vectors  $\vec{a}$  and  $-\vec{a}$  have equal magnitudes but opposite directions.

If  $\vec{a}$  is a vector of magnitude  $a$  and  $\hat{u}$  is a vector of unit magnitude in the direction of  $\vec{a}$ , we can write  $\vec{a} = a\hat{u}$ .

## 1.6 SUBTRACTION OF VECTORS

Let  $\vec{a}$  and  $\vec{b}$  be two vectors. We define  $\vec{a} - \vec{b}$  as the sum of the vector  $\vec{a}$  and the vector  $(-\vec{b})$ . To subtract  $\vec{b}$  from  $\vec{a}$ , invert the direction of  $\vec{b}$  and add to  $\vec{a}$ .

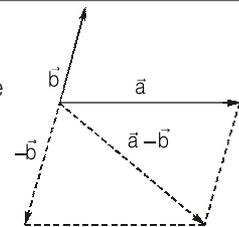


Fig. Vector diagram representing  $\vec{a} + (-\vec{b})$

## 1.7 RESOLUTION OF VECTORS

Consider a vector  $\vec{a} = \vec{OA}$  in the X-Y plane drawn from the origin O. The length OB is called the projection of  $\vec{OA}$  on X-axis. Similarly OC is the projection of  $\vec{OA}$  on Y-axis. According to the rules of vector addition

$$\vec{a} = \vec{OA} = \vec{OB} + \vec{OC}$$

We have resolved the vector  $\vec{a}$  into two parts, one along OX and the other along OY. The magnitude of the part along OX is  $OB = a \cos\alpha$  and the magnitude of the part along OY is  $OC = a \cos\beta$ . If  $\hat{i}$  and  $\hat{j}$  denote vectors of unit magnitude along OX and OY respectively, we get

$$\vec{OA} = a \cos\alpha \hat{i} \text{ and } \vec{OC} = a \cos\beta \hat{j}$$

So, that,

$$\vec{a} = a \cos\alpha \hat{i} + a \cos\beta \hat{j}$$

If the vector  $\vec{a}$  is not in the X-Y plane, it may have nonzero projections along X, Y, Z axes and we can resolve it into three parts i.e. along the X, Y and Z axes. If  $\alpha, \beta, \gamma$  be the angles made by the vector  $\vec{a}$  with the three axes respectively, we get

$$\vec{a} = a \cos\alpha \hat{i} + a \cos\beta \hat{j} + a \cos\gamma \hat{k}$$

where  $\hat{i}$ ,  $\hat{j}$  and  $\hat{k}$  are the unit vectors along X, Y and Z axes respectively. The component of vector  $\vec{a}$  along direction making angle  $\theta$  with it is  $a \cos\theta$  which is the projection of  $\vec{a}$  along the given direction.

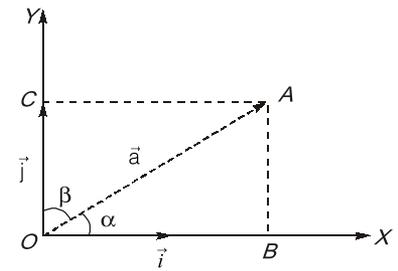


Fig. Resolution of vector OA in x and y directions

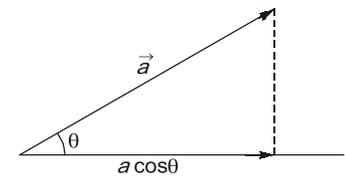


Fig. Horizontal projection of  $\vec{a}$

### EXAMPLE : 1.2

The magnitudes of vectors  $\vec{OA}$ ,  $\vec{OB}$  and  $\vec{OC}$  in the figure shown are equal. Find the direction of  $\vec{OA} + \vec{OB} - \vec{OC}$ .

#### Solution:

Let,

$$OA = OB = OC = F$$

$$x\text{-component of } \vec{OA} = F \cos 30^\circ = F \frac{\sqrt{3}}{2}$$

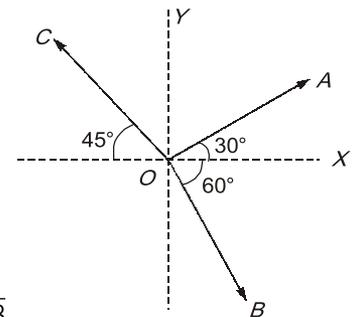
$$x\text{-component of } \vec{OB} = F \cos 60^\circ = \frac{F}{2}$$

$$x\text{-component of } \vec{OC} = F \cos 135^\circ = -\frac{F}{\sqrt{2}}$$

$$x\text{-component of } \vec{OA} + \vec{OB} - \vec{OC} = \left(\frac{F\sqrt{3}}{2}\right) + \left(\frac{F}{2}\right) - \left(-\frac{F}{\sqrt{2}}\right) = \frac{F}{2}(\sqrt{3} + 1 + \sqrt{2})$$

$$y\text{-component of } \vec{OA} = F \cos 60^\circ = \frac{F}{2}$$

$$y\text{-component of } \vec{OB} = F \cos 120^\circ = -\frac{F\sqrt{3}}{2}$$



$$y\text{-component of } \overline{OC} = F \cos 45^\circ = \frac{F}{\sqrt{2}}$$

$$y\text{-component of } \overline{OA} + \overline{OB} - \overline{OC} = \left(\frac{F}{2}\right) + \left(-\frac{F\sqrt{3}}{2}\right) - \left(\frac{F}{\sqrt{2}}\right) = \frac{F}{2}(1 - \sqrt{3} - \sqrt{2})$$

Angle of  $\overline{OA} + \overline{OB} - \overline{OC}$  with the  $x$ -axis

$$= \tan^{-1} \frac{\frac{F}{2}(1 - \sqrt{3} - \sqrt{2})}{\frac{F}{2}(1 + \sqrt{3} + \sqrt{2})} = \tan^{-1} \frac{(1 - \sqrt{3} - \sqrt{2})}{(1 + \sqrt{3} + \sqrt{2})}$$

## 1.8 DOT PRODUCT OR SCALAR PRODUCT OF TWO VECTORS

The dot product (also called scalar product) of two vectors  $\vec{a}$  and  $\vec{b}$  is defined as

$$\vec{a} \cdot \vec{b} = ab \cos \theta$$

The dot product is commutative and distributive.

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

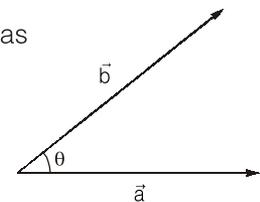


Fig. Vector  $\vec{a}$  and  $\vec{b}$  having angle  $\theta$  between them

### EXAMPLE : 1.3

The work done by a force  $\vec{F}$  during a displacement  $\vec{r}$  is given by  $\vec{F} \cdot \vec{r}$ . Suppose a force of 12 N acts on a particle in vertically upward direction and particle is displaced through 2 m in the vertically downward direction. Find the work done by the force during this displacement?

**Solution:**

The angle between the force  $\vec{F}$  and the displacement  $\vec{r}$  is  $180^\circ$ . Thus, the work done is

$$W = \vec{F} \cdot \vec{r} = Fr \cos \theta = 12 \times 2 \times \cos 180^\circ$$

$$W = -24 \text{ N-m} = -24 \text{ J}$$

## 1.9 DOT PRODUCT OF TWO VECTORS IN TERMS OF THE COMPONENTS ALONG THE COORDINATE AXES

Consider two vectors  $\vec{a}$  and  $\vec{b}$  represented in terms of the unit vectors  $\hat{i}, \hat{j}, \hat{k}$  along the coordinate axes as

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

and

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

Then,

$$\vec{a} \cdot \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k})$$

$$= \left[ \begin{array}{l} a_x b_x \hat{i} \cdot \hat{i} + a_x b_y \hat{i} \cdot \hat{j} + a_x b_z \hat{i} \cdot \hat{k} \\ + a_y b_x \hat{j} \cdot \hat{i} + a_y b_y \hat{j} \cdot \hat{j} + a_y b_z \hat{j} \cdot \hat{k} \\ + a_z b_x \hat{k} \cdot \hat{i} + a_z b_y \hat{k} \cdot \hat{j} + a_z b_z \hat{k} \cdot \hat{k} \end{array} \right] \quad \dots (1)$$

Since  $\hat{i}, \hat{j}$  and  $\hat{k}$  are mutually orthogonal.

We have  $\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{i} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = \hat{k} \cdot \hat{j} = 0$

Also,  $\hat{i} \cdot \hat{i} = 1 \times 1 \cos 0 = 1$

Similarly,  $\hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

Using these relations in equation (1), we get

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

## 1.10 CROSS PRODUCT OR VECTOR PRODUCT OF TWO VECTORS

The cross product or vector product of two vectors  $\vec{a}$  and  $\vec{b}$ , denoted by  $\vec{a} \times \vec{b}$  is itself a vector. The magnitude of this vector is

$$|\vec{a} \times \vec{b}| = ab \sin \theta$$

where  $a$  and  $b$  are the magnitude of  $\vec{a}$  and  $\vec{b}$  respectively and  $\theta$  is the smaller angle between the two. When two vectors are drawn with both the tails coinciding, two angles are formed between them. One of the angles is smaller than  $180^\circ$  and the other is greater than  $180^\circ$  unless both are equal to  $180^\circ$ . The angle  $\theta$  used in equation (2) is the smaller one. If both the angles are equal to

$180^\circ$ ,  $\sin \theta = \sin 180^\circ = 0$  and hence  $|\vec{a} \times \vec{b}| = 0$ .

Similarly if  $\theta = 0$ ,  $\sin \theta = 0$  and  $|\vec{a} \times \vec{b}| = 0$ .

The cross product of two parallel vectors is zero.

The direction  $\vec{a} \times \vec{b}$  is perpendicular to both

$\vec{a}$  and  $\vec{b}$ .

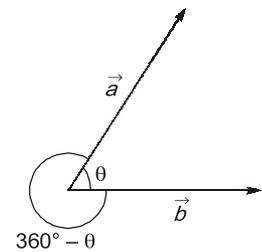


Fig. Vector  $\vec{a}$  and  $\vec{b}$  having angle  $\theta$  between them  
 $\vec{a} \times \vec{b}$

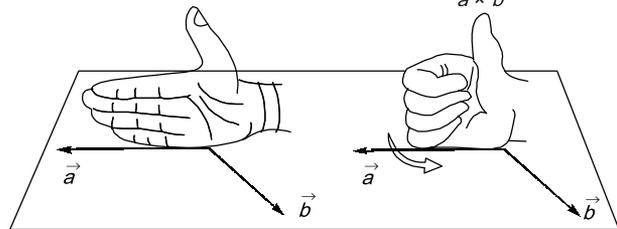


Fig. Right hand thumb rule displaying direction of  $\vec{a} \times \vec{b}$

Thus, it is perpendicular to the plane formed by  $\vec{a}$  and  $\vec{b}$ . To determine the direction of arrow on this perpendicular, we use **right hand thumb rule**.

Draw the two vectors  $\vec{a}$  and  $\vec{b}$  with both the tails coinciding. Now place your stretched right palm perpendicular to the plane of  $\vec{a}$  and  $\vec{b}$  in such a way that the fingers are along the vector  $\vec{a}$  and when the fingers are closed they go towards  $\vec{b}$ . The direction of the thumb gives the direction of arrow to be put on the vector  $\vec{a} \times \vec{b}$ .

Note that this rule makes the cross product non-commutative. In fact,

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

The cross product follows distributive law,

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

It does not follow the associative law

$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$$

When we choose a coordinate system any two perpendicular lines may be chosen as X and Y axes. However, once X and Y axes are chosen, there are two possible choices of Z-axis. The Z-axis must be perpendicular to the X-Y plane. But the positive direction of Z-axis may be defined in two ways. We choose the positive direction of Z-axis in such a way that

$$\hat{i} \times \hat{j} = \hat{k}$$

Such a coordinate system is called a **right handed system**. In such a system

$$\hat{j} \times \hat{k} = \hat{i} \text{ and } \hat{k} \times \hat{i} = \hat{j}$$

Of course,

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

**EXAMPLE : 1.4**

If  $\vec{A} = 2\hat{i} - 3\hat{j} + 7\hat{k}$ ,  $\vec{B} = \hat{i} + 2\hat{k}$  and  $\vec{C} = \hat{j} - \hat{k}$ , then the magnitude of the product

$\vec{A} \cdot (\vec{B} \times \vec{C})$  is

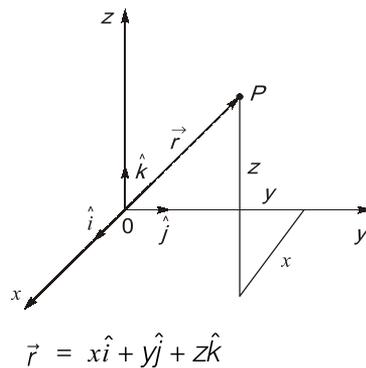
- |                |                |
|----------------|----------------|
| (a) 0          | (b) $\sqrt{2}$ |
| (c) $\sqrt{3}$ | (d) 2          |

**Solution: (a)**

$$\begin{aligned} \vec{B} \times \vec{C} &= (\hat{i} + 2\hat{k}) \times (\hat{j} - \hat{k}) \\ &= \hat{i} \times (\hat{j} - \hat{k}) + 2\hat{k} \times (\hat{j} - \hat{k}) \\ &= \hat{i} \times \hat{j} - \hat{i} \times \hat{k} + 2\hat{k} \times \hat{j} - 2\hat{k} \times \hat{k} \\ &= \hat{k} + \hat{j} - 2\hat{i} = -2\hat{i} + \hat{j} + \hat{k} \\ \vec{A} \cdot (\vec{B} \times \vec{C}) &= (2\hat{i} - 3\hat{j} + 7\hat{k}) \cdot (-2\hat{i} + \hat{j} + \hat{k}) \\ &= (2)(-2) + (-3)(1) + (7)(1) \\ &= 0 \end{aligned}$$

**1.11 POSITION VECTOR**

Directed line segment  $r$  from the origin of a coordinate system to a point  $P$  in space is called the position vector.



## 1.12 MOMENT OF A FORCE ABOUT A POINT

- (a) **Simple Case:** Moment of a force about a point is product of force magnitude times the perpendicular distance to line of action of the force from the given point.

$$M_A = F_P \times d$$

- (b) **Complex Case:**  $\vec{M} = \vec{r} \times \vec{F}$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad (\text{position vector})$$

$$\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k} \quad (\text{force vector})$$

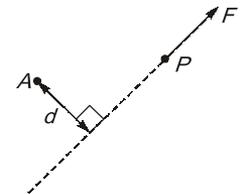


Fig. Moment of  $F$  about  $A$

## 1.13 MOMENT OF FORCE ABOUT AN AXIS

Consider a body rotating about a given axis of rotation  $AB$ . Let  $\vec{F}$  be acting at point  $P$  of the body.  $O$  is the origin.

$$\vec{M}_{AB} = \vec{r} \times \vec{F}$$

$$\vec{M}_{AB} = \text{Moment of force about } AB \text{ axis}$$

where,  $\vec{r}$  = Position vector of point  $P$  and  $\vec{F}$  = Force vector acting at point  $P$

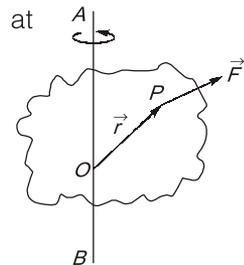


Fig. Moment of  $\vec{F}$  about axis  $AB$

## 1.14 VARIGNON'S THEOREM

For a concurrent force system, the vector summation of moments by the applied forces on the system about a point will be exactly the same as the moment given by the resultant force of all the applied forces the same point.

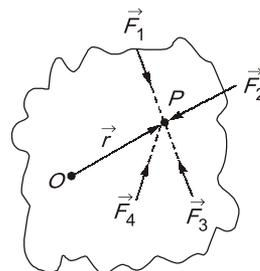


Fig. Representation of concurrent forces acting at point  $P$

Let  $O$  be the origin (reference point) and  $P$  be the point of action of all the forces

$$\begin{aligned} \Sigma M_P &= r \times \vec{F}_1 + r \times \vec{F}_2 + r \times \vec{F}_3 + r \times \vec{F}_4 \\ &= r \times [\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4] = r \times \Sigma \vec{F} \end{aligned}$$

$$\Sigma M_P = r \times \vec{F}_R \quad [\vec{F}_R = \Sigma \vec{F}]$$

$$\vec{F}_R = \text{Resultant of all the applied forces acting at point } P$$





**OBJECTIVE  
BRAIN TEASERS**

**Q.1** A particle whose speed is 25 m/sec moves along the line from A(2, 1) to B(9, 25). The velocity vector of the particle in the form  $a\hat{i} + b\hat{j}$  is

- (a)  $(7\hat{i} + 7\hat{j})$  m/s      (b)  $(7\hat{i} + 24\hat{j})$  m/s  
(c)  $(24\hat{i} + 12\hat{j})$  m/s      (d)  $(24\hat{i} + 7\hat{j})$  m/s

**Q.2** Two forces are acting on a body,  $\vec{F}_1 = 2\hat{i} + 3\hat{j}$  and it does 8J of work,  $\vec{F}_2 = 3\hat{i} + 5\hat{j}$  and it does -4J of work on body. The magnitude of displacement traversed by the body is

- (a) 57.27 m      (b) 30.53 m  
(c) 54.40 m      (d) 61.06 m

**Q.3** The angle between two vectors  $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$

and  $\vec{B} = \hat{i} - \hat{k}$  is

- (a) 30°      (b) 60°  
(c) 0°      (d) 90°

**Q.4** If two vectors are given as:

$$\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{B} = -\hat{i} - \hat{j} + \hat{k}$$

then, the vector perpendicular to  $(\vec{A} \times \vec{B})$  can be

- (a)  $2\hat{i} + 4\hat{j} + 2\hat{k}$       (b)  $\hat{i} + \hat{j} + \hat{k}$   
(c)  $25\hat{i} - 625\hat{j} - 25\hat{k}$       (d)  $3\hat{i} - 2\hat{j} + 3\hat{k}$

**Q.5** If  $(\vec{a} + \vec{b})$  is perpendicular to  $\vec{b}$  and  $\vec{a} + 2\vec{b}$  is perpendicular to  $\vec{a}$ . If  $|\vec{a}| = a$  and  $|\vec{b}| = b$ , then

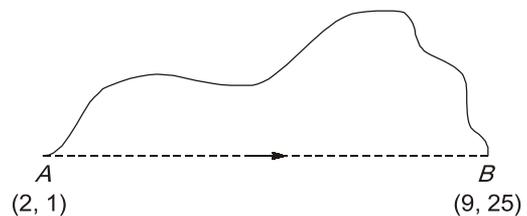
- (a)  $a = b$       (b)  $a = 2b$   
(c)  $b = 2a$       (d)  $a = b\sqrt{2}$

**ANSWER KEY**

1. (b)    2. (d)    3. (a)    4. (c)    5. (d)

**HINTS & EXPLANATIONS**

1. (b)  
Velocity vector is given by the product of magnitude of velocity (speed) multiplied by the unit vector along velocity.



Unit vector along velocity,

$$\widehat{AB} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{(9-2)\hat{i} + (25-1)\hat{j}}{\sqrt{7^2 + 24^2}}$$

$$\begin{aligned} \vec{v} &= |\vec{v}| \widehat{AB} = 25 \left( \frac{7\hat{i} + 24\hat{j}}{25} \right) \\ &= (7\hat{i} + 24\hat{j}) \text{ m/s} \end{aligned}$$

2. (d)  
Let the displacement of the body is given by

$$\vec{r} = x\hat{i} + y\hat{j}$$

work done by first force,

$$W_1 = \vec{F}_1 \cdot \vec{r} = 8$$

$$\therefore (2\hat{i} + 3\hat{j}) \cdot (x\hat{i} + y\hat{j}) = 8$$

$$2x + 3y = 8 \quad \dots (i)$$

Work done by second force,

$$W_2 = \vec{F}_2 \cdot \vec{r} = -4$$

$$(3\hat{i} + 5\hat{j}) \cdot (x\hat{i} + y\hat{j}) = -4$$

$$3x + 5y = -4 \quad \dots \text{(ii)}$$

Solving (i) and (ii) equation, we get

$$x = 52, y = -32$$

$$\therefore \vec{r} = 52\hat{i} - 32\hat{j}$$

$$|\vec{r}| = \sqrt{(52)^2 + (-32)^2} = \sqrt{3728}$$

$$= 61.06 \text{ m}$$

3. (a)

The dot product of two vectors is given by

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\theta = \cos^{-1} \left[ \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right]$$

$$\theta = \left[ \frac{(2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{k})}{\sqrt{2^2 + 1^2 + (-1)^2} \sqrt{1^2 + (-1)^2}} \right]$$

$$= \cos^{-1} \left[ \frac{2+1}{\sqrt{6}\sqrt{2}} \right] = \cos^{-1} \left[ \frac{3}{2\sqrt{3}} \right]$$

$$= \cos^{-1} \left[ \frac{\sqrt{3}}{2} \right] = 30^\circ$$

4. (c)

The cross product of  $(\vec{A} \times \vec{B})$  is given as

$$(\vec{A} \times \vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(1) - \hat{j}(0) + \hat{k}(-1+2) = \hat{i} + \hat{k}$$

$\vec{C}$  will be perpendicular to  $(\vec{A} \times \vec{B})$ , it  $\vec{C} \cdot (\vec{A} \times \vec{B})$

gives a result zero. From options,

$$(25\hat{i} - 625\hat{j} - 25\hat{k}) \cdot (\hat{i} + \hat{k})$$

$$\therefore 25 - 25 = 0$$

5. (d)

From the given conditions,  $(\vec{a} + \vec{b})$  is perpendicular

to  $\vec{b}$ , therefore their dot product is zero.

$$(\vec{a} + \vec{b}) \cdot \vec{b} = 0$$

$$\vec{a} \cdot \vec{b} + |\vec{b}|^2 = 0$$

$$\vec{a} \cdot \vec{b} + b^2 = 0 \quad \dots \text{(i)}$$

Similarly,

$$(\vec{a} + 2\vec{b}) \cdot \vec{a} = 0$$

$$|\vec{a}|^2 + 2\vec{a} \cdot \vec{b} = 0$$

$$a^2 + 2\vec{a} \cdot \vec{b} = 0 \quad \dots \text{(ii)}$$

From equations (i) and (ii), we get

$$-\frac{a^2}{2} = -b^2$$

$$\therefore a = \pm\sqrt{2}b$$

Neglecting negative sign as magnitude cannot be negative, therefore  $a = \sqrt{2}b$ .

