# CIVIL ENGINEERING

## Strength of Materials

Comprehensive Theory
*with* Solved Examples *and* Practice Questions

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# Strength of Materials

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CHAPTER

# 1

# Properties of Materials

## 1.1 INTRODUCTION

Strength of material is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading and internal forces developed due to these loading. A thorough understanding of mechanical behaviour is essential for the safe design of all structures, whether buildings, bridges, machines, motors, submarines or airplanes. Hence, strength of material is a basic subject in many engineering fields.

The objective of our analysis will be to determine the stresses, strains and deflections produced by the loads in different structures. Theoretical analysis and experimental results have equally important role in the study of strength of materials. So these quantities are found for all values of load upto the failure load, and then we will have a complete picture of the mechanical behaviour of the body.

The behaviour of a member subjected to forces depends not only on the fundamental law of Newtonian mechanics that govern the equilibrium of the forces but also on the mechanical characteristics of materials of which the member is fabricated. Sometimes, to predict the behaviour of material some necessary information regarding the characteristics of material comes from laboratory tests.

## 1.2 STRESS

The fundamental concept of stress can be understood by considering a prismatic bar that is loaded by axial force *P* at the ends as shown.

A prismatic bar is a straight structural member having constant cross-sectional area throughout its length. In the figure (a), axial force is acting away from the cross-section producing a uniform stretching of the bar, hence the bar is said to be in tension. Similarly in figure (c), axial force is acting towards the cross-section producing uniform compression of the bar, hence the bar is said to be in compression.

To investigate the internal stresses produced in the bar by axial forces, we make an imaginary cut at section *mn* as shown in figure (b) and (d). This section is taken perpendicular to the longitudinal axis of bar. Hence it is known as cross-section.

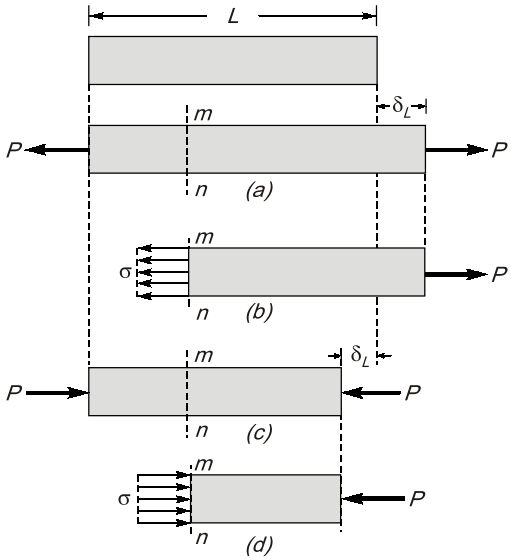



*Fig. Axial stress*

Now isolating the part of the bar to the right of the cut and considering the right of the cut as a free body. The force *P* has a tendency to move free body in the direction of load, so to restrict the motion of bar an internal force is induced which is uniformly distributed over cross-sectional area. The intensity of force developed, that is, internal force per unit area is called the **stress**.

Stress differs from pressure because pressure is defined as the externally applied force on unit area while stress is internal resistive force on unit area. To have better understanding of difference between externally applied force and internal resistance. Consider a bar suspended from a fixed end and a weight *W* is gradually applied at its free end as shown in figure.

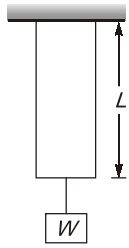



*Fig. Axial load on bar*

**Case-I: Weight, W is applied gradually**

Gradual loading means that value of load is zero at the starting time and gradually increases to value of *W*. Here, the bar gradually elongates with the increasing value of load. With increase in elongation, resistance forces say R will also increase gradually.

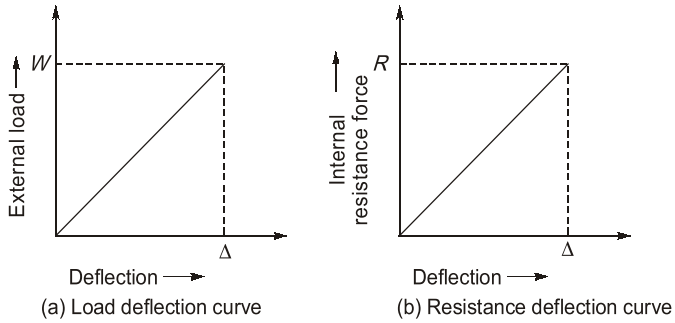



*Fig. External load is applied gradually*

**Case-II: Weight, W is applied suddenly**

Here, external load variation with elongation of bar is such as that its value instantly increases to *W*. This sudden load will result into elongation of bar say Δ. When external load is applied suddenly, resistance force will be set up in bar, but unlike external load which is sudden, resistance force has always linear variation with elongation of bar.

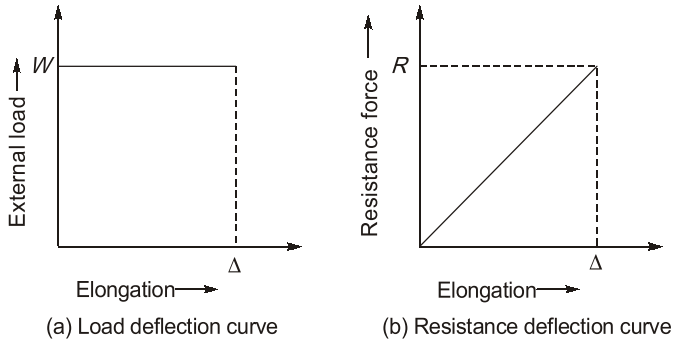



*Fig. External load is applied suddenly*

Now, as clear from figure (a) and (b), intensity of pressure is not equal to stress induced in bar.

Thus, stress can be defined as – **"Stress is the internal resistance of a material offered against deformation which is expressed in terms of force per unit area"**.

Stress induced in material depends upon the nature of force, point of application and cross-sectional area of material. Stress can be **tensile** or **compressive** in nature depending on the nature of load. Generally, stress is represented by the Greek letter σ. We can calculate stress mathematically as

$$\sigma = \frac{P}{A}$$

**General Sign Convention:**
Tensile stresses = +ve
Compressive stresses = –ve

**Unit:** (i) $N/m^2$ or Pa (SI unit)
(ii) $N/mm^2$ or MPa

- Stresses are induced only when motion of bar is restricted either by some force or reaction induced. If body or bar is free to move or free expansion is allowed, then no stresses will be induced.
- Pressure has same unit but pressure is different physical quantity than stress. Pressure is external normal force distributed over surface.

On the basis of cross-sectional area considered during calculation of stresses, direct stresses can be of following two types:

**(a) Engineering or nominal stress :** It is the stress where the original cross-sectional area of specimen is taken.

Mathematically, $\sigma = \frac{P}{A_0}$

where, $A_0$ = Original cross-sectional area of specimen taken

**(b) True or actual stress :** It is the stress where the actual cross-sectional area of specimen at any time of loading is considered.

Mathematically, $\sigma = \frac{P}{A_a}$

where, $A_a$ = Actual cross-sectional area of specimen at any time of loading i.e. changed area of cross-section due to loading

$A_a = A_0 \pm \Delta A$ as per our convention '+' for compression and '–' for tension is taken.

- In tension, true or actual stress is always greater than engineering or nominal stress.
- In compression, true or actual stress is always less than engineering or nominal stress.

## 1.3 STRAIN

An axially loaded bar undergoes a change in length, becoming longer when in tension and shorter when in compression. The elongation or shortening in axially loaded member per unit length is known as strain. Strain is represented by ∈.

Mathematically, strain can be calculated as

$$\in = \frac{\Delta L}{L}$$

Strain is dimensionless quantity and is always expressed in the form of number. If the member is in tension then the strain is called tensile strain. If the member is in compression, then the strain is called compressive strain.

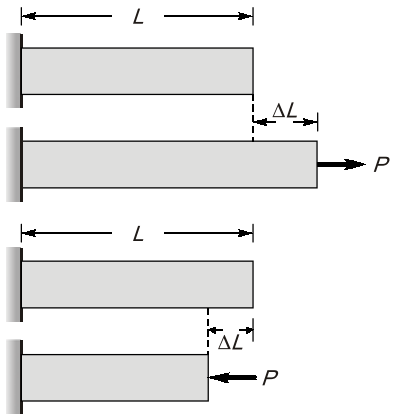



*Fig. Strain in bars*

On the basis of length of member used in calculation of strain, strain can be of following two types:

**(a) Engineering or Nominal Strain:** Engineering or nominal strain is strain calculated, when length of member is taken as original length

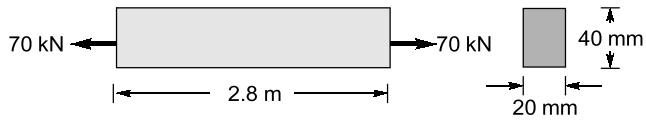
Mathematically, $\epsilon_0 = \dfrac{\Delta l}{l_0}$ where, $l_0$ = original length of member

**(b) True or Actual Strain :** True or actual strain is strain calculated, when length of member is taken as actual length of member at loading

Mathematically, $\epsilon_a = \dfrac{\Delta l}{l_a}$ where, $l_a$ = Actual length of member

$l_a = l_0 \pm \Delta l$ '+' sign for tension; '–' sign for compression

**Example 1.1** A prismatic bar with rectangular cross-section (20 mm × 40 mm), length $L$ = 2.8 m is subjected to an axial tensile force of 70 kN. The measured elongation of the bar is 1.2 mm. Calculate the tensile stress and strain in the bar.




**Solution:**

Assuming that force acts at *CG* of section. We know that,

$$\text{Stress, } \sigma = \frac{P}{A} = \frac{70 \times 10^3 \text{N}}{20 \times 40\ \text{mm}^2} = 87.5\ \text{N/mm}^2 = 87.5\ \text{MPa}$$

and

$$\text{Strain, } \epsilon = \frac{\Delta L}{L} = \frac{1.2\ \text{mm}}{2.8 \times 1000\ \text{mm}} = 4.286 \times 10^{-04}$$

## 1.4 TENSILE TEST FOR MILD STEEL

The mechanical properties of materials used in engineering are determined by experiments performed on small specimen. These experiments are conducted in laboratories equipped with testing machines that are capable of loading in tension or compression.

The American Society for Testing and Materials (ASTM) has published guidelines for conducting test. Tensile test is generally conducted on Universal Testing Machine (UTM).

### 1.4.1 General Specifications of Specimen

- Specimen is solid cylindrical rod
- Diameter of middle section 0.5" (inches)
- Gauge length 2" (inches)
- *L*/*D* ratio = 4.0

### 1.4.2 Stress Strain Curve for Tension

- ***A* is limit of proportionality:** Beyond this linear variation ceases. Hooke's law is valid in OA.
- ***B* is elastic limit:** The maximum stress upto which a specimen regains its original length on removal of applied load. For mild steel, *B* is very near to *A*. However, for other materials *B* may be greater than *A*.

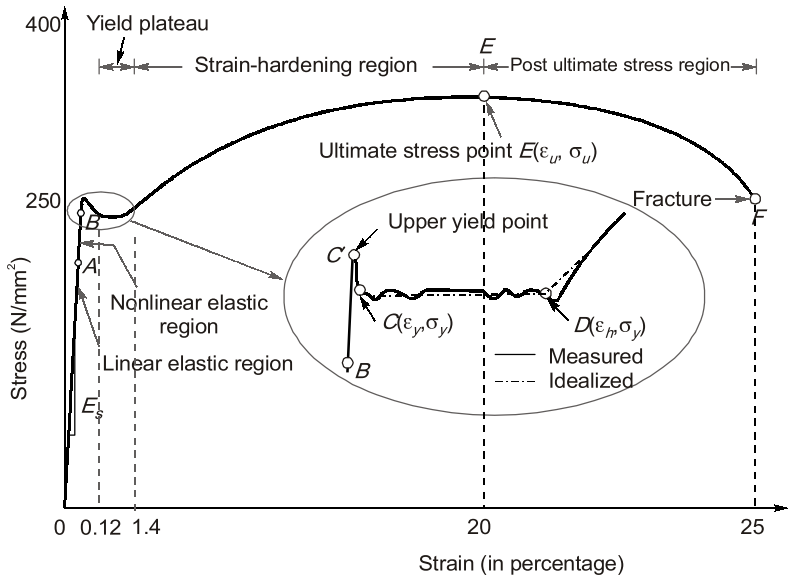



*Fig. Ideal Tensile stress-strain diagram for Mild Steel*

- ***C′* is upper yield point:** The magnitude of the stress corresponding to *C′* depends on the cross-sectional area, shape of the specimen and the type of the equipment used to perform the test. It has no practical significance.
- ***C* is lower yield point:** This is also called actual yield point. The stress at *C* is the yield stress ($\sigma_y$) with a typical value of $\sigma_y$ = 250 N/mm$^2$ (for mild steel). The yielding begins at this stress.
- ***CD* represents perfectly plastic region:** It is the strain which occurs after the yielding point *C*, without any increase in stress. The strain corresponding to point *D* is about 1.4% and corresponding to *C* is about 0.12% for mild steel. Hence, plastic strain is 10 to 15 times of elastic strain.
- ***DE* represents strain hardening region:** In this range further addition of stress gives additional strain. However, strain increases with faster rate in this region. The material in this range undergoes change in its crystalline structure, resulting in increased resistance to further deformation. This portion is not used for structural design.
- ***E* is ultimate point:** The stress corresponding to this point is ultimate stress ($\sigma_u$) and the corresponding strain is about 20% for mild steel.
- ***F* is fracture point:** Stress corresponding to this is called breaking stress and strain is called fracture strain. It is about 25% for mild steel.
- ***EF* post ultimate stress region:** In this range, necking occurs, i.e. area of cross-section is drastically decreased.

NOTE

1. Strain that occurs before the yield point is called elastic strain and that which occurs after yield point with no increase in stress is called plastic strain. For mild steel, plastic strain is 10 to 15 times of elastic strain.
2. Ideal curve for tension is shown in figure. However actual behaviour is different and indicates apparently reduced yield stress in compression after strain hardening in tension. The divergence between tension and compression results is explained by Bauschinger and is called **Bauschinger effect**.

### 1.4.3 Actual Curve v/s Engineering Curve in Tension and Compression for Mild Steel

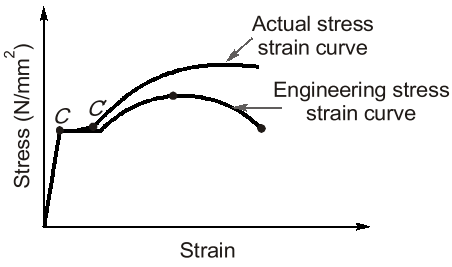



*Fig. Tension curve for mild steel*

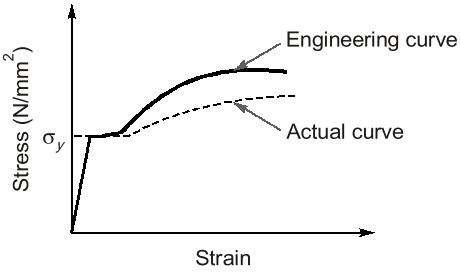



*Fig. Compression curve for Mild steel*

NOTE

- The fracture strain depends upon % carbon present in steel.
- With increase in percentage carbon, fracture strain reduces.
- With increase carbon content, steel has higher yield stress and higher ultimate stresses.

- In compression, engineering stress-strain curve lies above the actual stress-strain curve, while in tension actual stress-strain curve lies above the engineering stress-strain curve.
- In compression mild steel has yield stress $\sigma_y = 263\ N/mm^2$, slightly greater than tension.
- Mild steel has same Young's modulus of elasticity in compression and tension, $E = 2.1 \times 10^5\ N/mm^2$.

**Relation between engineering and actual stress**

$$\sigma_a = \sigma_0(1 \pm \epsilon_0)$$

where, $\sigma_a$ = Actual stress; $\sigma_0$ = Engineering stress; $\epsilon_0$ = Engineering strain

As per our convention, for tension, take positive (+ve) sign and take negative (–ve) sign for compression.'

**NOTE :** While deriving above equation volume changes is neglected which is true in plastic region (Non-elastic region).

### 1.4.4 Stress-strain Curve for other Grades of Steel in Tension

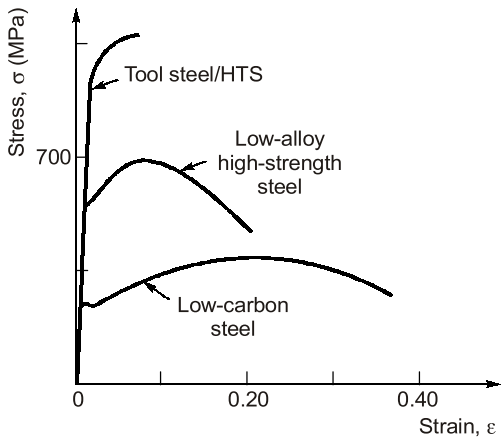



*Fig. Tensile stress-strain diagram for different grades of steel*

**REMEMBER**

- All the grades of steel have same Young's modulus of elasticity.
- Among all steel grades high tension steel (HTS) is more brittle and mild steel is more ductile.
- High tension steel has higher ultimate strength than other grades of steel.

### 1.4.5 Stress-strain Curve for Different Materials

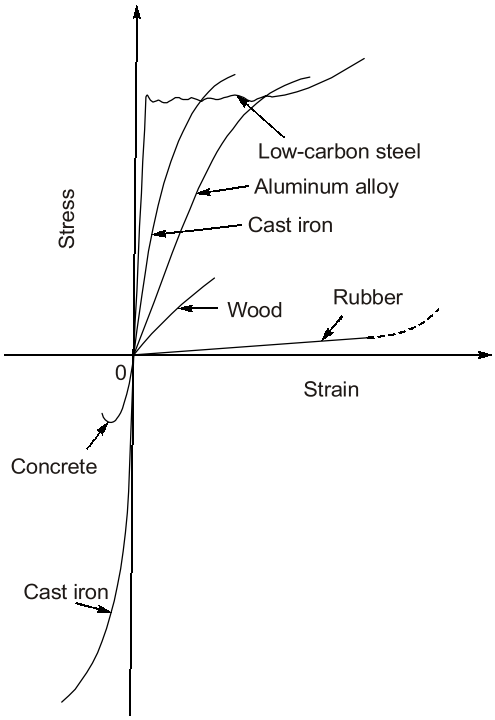



*Fig. Stress-strain diagram for different material*

## 1.5 PROPERTIES OF METALS

### 1.5.1 Ductility

Ductility is the property by which material can be stretched. Large deformations are thus possible in ductile materials before the absolute failure or rupture takes place. These materials have post-elastic strain (Plastic strain) greater than 5%. Some of the examples of ductile materials are mild steel, aluminium, copper, manganese, lead, nickel, brass, bronze etc.

### 1.5.2 Brittleness

Brittleness is the lack of ductility i.e. materials can not be stretched. In brittle materials, fracture takes place immediately after elastic limit with a relatively smaller deformation. For the brittle materials, fracture and ultimate points are same and after proportional limit very small strain is seen. Brittle materials have post elastic strain less than 5%. Some examples of brittle materials are cast iron, concrete and glass.

REMEMBER

To distinguish between these two type of materials, materials with post elastic strain less than 5% at fracture point are regarded as brittle and those having post elastic strain greater than 5% at fracture point are called ductile (this value for mild steel at fracture is about 25%).

### 1.5.3 Malleability

Malleability is the property of metal due to which a piece of metal can be converted into a thin sheet by pressing it. A malleable material possess a high degree of plasticity. This property is of great use in operations like forging, hot rolling, drop (stamping) etc.

### 1.5.4 Hardness

- Hardness is resistance to scratch or abrasion.
- There are two methods of hardness measurements:
  (a) Scratch hardness-commonly measured by Mohr's test
  (b) Indentation hardness (abrasion) measured by
    - Brinell's hardness method
    - Vickers hardness
    - Rockwell hardness
    - Knoop hardness

## 1.6 CREEP

Creep is permanent deformation which is recorded with passage of time at constant loading. Total creep deformation continues to increase with time asymptotically. Consider a prismatic bar of length L on which an external static load P is applied. Due to applied static load, goes a deformation of $D_e$, but after some time it is observed that bar has gone permanent and some stress developed in bar released. This effect is called creep.

where, $\Delta_e$ = Elastic deflection = $\frac{PL}{AE}$

$P$ = Static load

$\Delta_c$ = Deformation due to creep

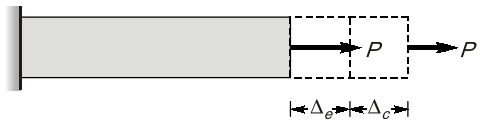



*Fig. Creep in bar*

Factors affecting creep are as follows:

1. Magnitude of load
2. Type of loading (static or dynamic)
3. Time or age of loading
4. Temperature
   - At higher temperature, due to greater mobility of atoms most of the materials loose their strength and elastic constants also get reduced. Hence, greater deformation at elevated temperature results, even under constant loading. Therefore, creep is more pronounced at higher temperature, and thus it must be considered for design of engines and furnaces.
   - Temperature at which the creep becomes very appreciable is half of the melting point temperature on absolute scale and is known as **homologous temperature**.

## OBJECTIVE BRAIN TEASERS

**Q.1** Match **List-I (Type of material)** with **List-II (Characteristics)** and select the correct answer using the codes given below the lists:

**List-I**

A. Elastic material
B. Rigid material
C. Plastic material
D. Resilient material

**List-II**

1. Does not store energy
2. Has no plastic region in stress strain curve
3. Behave as a spring
4. Offers resistance to deformation
5. Does not offer resistance to deformation

**Codes:**

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 4 | 1 | 5 | 3 |
| (b) | 4 | 1 | 5 | 2 |
| (c) | 2 | 3 | 4 | 5 |
| (d) | 1 | 3 | 4 | 5 |

**Q.2** A structure is said to be linearly elastic if

(a) Load $\propto$ displacement

(b) $\text{Load} \propto \frac{1}{\text{Displacement}}$

(c) Energy $\propto$ displacement

(d) Energy $\propto$ Load

**Q.3** Which one is not the characteristics of fatigue fracture?

(a) Rough fracture surface
(b) Rough and smooth areas on fracture surface
(c) Plastic deformation
(d) Conchoidal markings on fracture surface

**Q.4** A fatigue crack in a sound and smooth specimen takes

(a) longer time in initiation than propagation
(b) longer time in propagation than initiation
(c) equal time in initiation and propagation
(d) no time in propagation

**Q.5** Stress curve is always a straight line for

(a) elastic material
(b) materials obeying Hooke's law
(c) elasto plastic materials
(d) none of the above

**Q.6** The term nominal stress in stress-strain curve for mild steel implies

(a) average stress
(b) actual stress
(c) yield stress
(d) stress at necking

**Q.7** Consider the following statements:
The principle of superposition is applied to
1. Linear elastic bodies
2. Bodies subjected to small deformations
Which of these statements is/are correct?

(a) 1 alone (b) 1 and 2
(c) 2 alone (d) neither 1 nor 2

**Q.8** The strain at a point is a

(a) Scalar (b) Vector
(c) Tensor (d) None of these

**Q.9** If the value of Poisson's ratio is zero, then it means that

(a) the material is rigid
(b) the material is perfectly plastic
(c) there is no longitudinal strain in the material
(d) the longitudinal strain in the material is infinite

**Q.10** Consider the following statements
1. Strength of steel increases with carbon content
2. Young's modulus of steel increase with carbon content
3. Young's modulus of steel remain unchanged with variation of carbon content

Which of these statements is/are correct?

(a) 1 only (b) 2 only
(c) 1 and 2 (d) 1 and 3

**Q.11** True stress $\sigma$ is related with conventional stress $\sigma_0$ as

(a) $\frac{\sigma}{\sigma_0} = (1+\epsilon)^2$ (b) $\frac{\sigma}{\sigma_0} = \frac{1}{(1+\epsilon)^2}$

(c) $\frac{\sigma}{\sigma_0} = \frac{1}{(1+\epsilon)}$ (d) $\frac{\sigma}{\sigma_0} = 1+\epsilon$

**Q.12** Steel has its yield strength of 400 N/mm$^2$ and modulus of elasticity of $2 \times 10^5$ MPa. Assuming the material to obey Hooke's law up to yielding, what is its proof resilience?

(a) 0.8 N/mm$^2$ (b) 0.4 N/mm$^2$
(c) 0.6 N/mm$^2$ (d) 0.7 N/mm$^2$

**Q.13** What would be the shape of the failure surface of a standard cast iron specimen subjected to torque?

(a) Cup and cone shape at the center.
(b) Plane surface perpendicular to the axis of the specimen.
(c) Pyramid type wedge-shaped surface perpendicular to the axis of the specimen.
(d) Helicoidal surface at 45° to the axis of the specimen.

**Q.14** The greatest stress that a material can withstand for a specified length of time without excessive deformation is called

(a) Fatigue strength
(b) Endurance limit
(c) Creep rupture strength
(d) Creep strength

**Q.15** Consider the following statements:

1. Creep is usually more important at higher temperatures and higher stress.
2. Creep depends on temperature level, stress level, time and type of loading (static or dynamic).
3. The nature of creep is elastic because as soon as we remove the load, the material regains its original length.
4. Creep in concrete may relieve same tensile stress.

Which of these statements are correct?

(a) 1, 2 and 3 (b) 1, 3 and 4
(c) 1, 2 and 4 (d) 2 and 4 only

**Q.16** The following observations refer to two metal samples *A* and *B* of same size subjected to uni-axial tension test upto failure

1. Elastic strain energy of *A* is more than that of *B*.
2. Area under stress-strain curve of *A* is less than that of *B*.
3. The yield strength of *A* is more than that of *B*.
4. The percentage elongation of *A* and *B* at elastic limit are equal.

Which of the following statement is true in this regard?

(a) Specimen A is more ductile than specimen B
(b) Specimen B is more ductile than specimen A
(c) The ductility of the two specimen is equal
(d) The data is insufficient to compare the ductilities of the two specimens

**Q.17** Consider the following statements:

1. If only shear stress is acting then volume of the specimen does not change.
2. The ultimate shear strength observed from shear stress-strain curve is almost the same as ultimate normal strength observed from normal stress-strain curve.
3. For an isotropic material, number of independent and distinct elastic constants are 21.

Which of the statement are CORRECT?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1 only

**Q.18** What is the number of independent stress components in a body loaded under a general state of stress and a plane stress condition respectively in order to completely specify the state of stress at a point?

(a) 9 and 4 (b) 6 and 4
(c) 9 and 3 (d) 6 and 3

**Directions:** The following items consists of two statements; one labelled as **'Assertion (A)'** and the other as **'Reason (R)'**. You are to examine these two statements carefully and select the answers to these items using the codes given below:

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**Q.19 Assertion (A):** Many materials do not have well defined yield point.

**Reason (R):** 0.2% offset parallel to the initial tangent of the stress-strain curve intersects the curve at yield stress.

**Q.20 Assertion (A):** Strain is a fundamental behaviour of the material, while the stress is a derived concept.

**Reason (R):** Strain does not have a unit while the stress has a unit.

**Q.21 Assertion (A):** The amount of elastic deformation at a certain point, which an elastic body undergoes, under given stress is the same irrespective of the stresses being tensile or compressive.

**Reason (R):** The modulus of elasticity and Poisson's ratio are assumed to be the same in tension as well as compression.

**Q.22 Assertion (A):** A mild steel tension specimen has a cup and cone fracture at failure.

**Reason (R):** Mild steel is weak in shear and failure of the specimen in shear takes place at 45° to the direction of the applied tensile force.

**Q.23 Assertion (A):** In a tensile test on a specimen, true stress in the specimen is more than nominal stress.

**Reason (R):** Grip of universal testing machine introduces stress concentrations.

**Q.24 Assertion (A):** The failure surface of a mild steel specimen subject to a torque about its axis is along a surface perpendicular to its axis.

**Reason (R):** Mild steel is relatively weaker in shear than in tension and the plane of maximum shear is perpendicular to its axis.

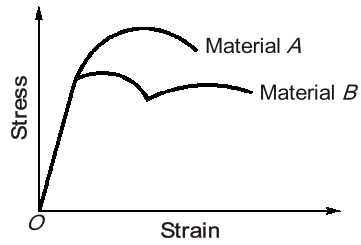
**Q.25 Assertion (A):** In a tension test on a cast iron specimen, the failure of the specimen is on a cross-section perpendicular to the axis of the specimen.

**Reason (R):** The failure of the specimen is on a plane subjected to maximum tensile-stress and cast iron is relatively weak in tension.

**Q.26 Assertion (A):** In a tension test on a mild steel specimen, the failure of the specimen is along a plane at 45° to cross-section.

**Reason (R):** The failure of the specimen is on a plane subjected to maximum shear stress and mild steel is relatively weak in shear.

**Q.27** The stress strain diagram for two materials *A* and *B* is shown below:




**Assertion (A):** Material *A* is more brittle than material *B*.

**Reason (R):** The ultimate strength of material *B* is more than that of *A*.

**Q.28 Assertion (A):** In strain hardening region, the material appears to loose some of its strength and hence offers more resistance, thus requiring increased tensile load for further deformation.

**Reason (R):** The material undergoes changes in its atomic and crystalline structure in this region.

**Q.29 Assertion (A):** There are two independent elastic constants for an isotropic material.

**Reason (R):** All metals at micro-level are isotropic.

## ANSWERS KEY

| | | | | |
|---|---|---|---|---|
| 1. (a) | 2. (a) | 3. (a) | 4. (a) | 5. (b) |
| 6. (a) | 7. (b) | 8. (c) | 9. (c) | 10. (d) |
| 11. (d) | 12. (b) | 13. (d) | 14. (d) | 15. (c) |
| 16. (b) | 17. (d) | 18. (d) | 19. (b) | 20. (b) |
| 21. (a) | 22. (a) | 23. (b) | 24. (a) | 25. (a) |
| 26. (a) | 27. (c) | 28. (d) | 29. (c) | |

## HINTS & EXPLANATIONS

**5. (b)**

Stress ∝ strain (Hooke's law)

Which is valid within proportional limit.

Within elastic limit stress – strain curve may be linear or nonlinear. For e.g. Rubber.

**6. (a)**

$$\text{Nominal stress} = \frac{\text{Load}}{\text{Original area}} = \frac{P}{A_0}$$

Nominal stress is also called engineering stress or average stress.

$$\text{Actual stress} = \frac{\text{Load}}{\text{Actual area}} = \frac{P}{A_a}$$

Actual area at instant of loading does not remain constant and decreases with increase in elongation. Actual stress is also called true stress.

**10. (d)**

Strength of steel increases with carbon content but Young's modulus remains constant.

**12. (b)**

$$\text{Proof resilience} = \frac{\sigma_y^2}{2E} = \frac{400^2}{2\times 2\times 10^5} = 0.4\text{N/mm}^2$$

**13. (d)**

Brittle materials fails in a plane at 45° from the axis when subjected to torque because they are weak in tension compare to shear. If ductile materials are subjected to torque, then the failure surface will be in a plane at 90° from the axis of shaft.

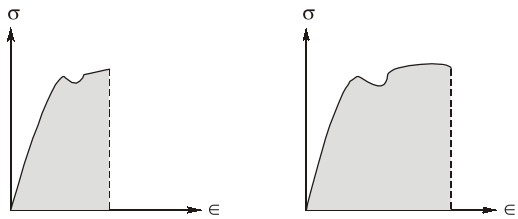
**15. (c)**

At higher temperature creep become more important because after temperature half of melting point, it becomes uncontrollable.

The nature of creep is elastic as well as plastic.

**16. (b)**

$u_A > u_B$




For material A For material B

Strain energy per unit volume of *A* is less than *B*.

$$\frac{u_A}{V_A} < \frac{u_B}{V_B}$$

Since materials are same size, $\frac{u_A}{A\cdot L_A} = \frac{u_B}{A\cdot L_B}$

$\therefore \quad L_A > L_B$

and given that $\sigma_{y_A} > \sigma_{y_B}$

$\therefore$ Material *B* is more ductile than that of *B*.

**17. (d)**

For isotropic material number of elastic constant are 2 i.e. (*E* and μ). If body are stressed by shear stress as in pure shear case, then the change in volume of stressed body is zero.

**20. (b)**

During experiment in laboratory, strain is measured that is why it is called fundamental quantity. While stress is derived from strain.

**22. (a)**

Mild steel is enough strong in tension and compression but it is weak in shear. Hence, the failure of the specimen takes place due to shear.

**23. (b)**

For true stress, the actual area at any time used is less than original area, due to elongation in specimen, therefore, true stress is more than nominal or engineering stress.

**29. (c)**

The two elastic constants for isotropic materials are usually expressed as Young's modulus and the Poisson's ratio. However the other elastic constants *K*, *G* can also be used. For isotropic material, *G* and *K* are found out from *E* and μ. All metals at micro level are anisotropic.

## CONVENTIONAL BRAIN TEASERS

**Q.1** For a brass alloy, the following engineering stresses produce the corresponding plastic engineering strains, prior to necking :

| Engineering Stress (MPa) | Engineering Strain |
|---|---|
| 235 | 0.194 |
| 250 | 0.296 |

On the basis of this information, compute the engineering stress necessary to produce an engineering strain of 0.25.

**Solution :**

(i) As per given information, we first need to convert engineering stresses and strains to true stresses and strains.

True stress, $\sigma_T = \sigma(1 + \varepsilon)$

$(\sigma_T)_1 = \sigma_1(1 + \varepsilon) = 235(1 + 0.194) = 280.59$ MPa

True strain, $\varepsilon_T = \ln(1 + \varepsilon)$

$(\varepsilon_T)_1 = \ln(1 + \varepsilon_1) = \ln(1 + 0.194) = 0.1773$

Similarly, true stress $(\sigma_{T2}) = \sigma_2(1 + \varepsilon_2) = 250(1 + 0.296) = 324$ MPa

True strain, $(\varepsilon_T)_2 = \ln(1 + \varepsilon_2) = \ln(1 + 0.296) = 0.259$

As we know, true stress and strain relationship given by :

$$\sigma_T = K\varepsilon_T^n$$

$$\ln \sigma_T = \ln (K) + n \ln (\varepsilon_T)$$

$$\ln (280.59) = \ln (K) + n \ln (0.1773) \quad \text{...(i)}$$

$$\ln (324) = \ln (K) + n \ln (0.259) \quad \text{...(ii)}$$

By solving equation (i) and (ii) respectively :

$$n = 0.379$$

$$\ln (K) = 6.2935$$

$$K = 541.0436 \text{ MPa}$$

General equation, $\sigma_T = K\varepsilon_T^n$

$$\sigma_T = 541.0436\,(\varepsilon_T)^{0.379}$$

For engineering strain, $\varepsilon = 0.25$

True strain, $\varepsilon_T = \ln(1 + 0.25) = 0.223$

$$\sigma_T = 541.0486(0.223)^{0.379}$$

$$\sigma_T = 306.3654 \text{ MPa}$$

$$\sigma_T = \sigma(1 + \varepsilon)$$

$$306.3654 = \sigma(1 + 0.25)$$

Required engineering stress,

$$\sigma = \frac{306.3654}{1.25}$$

$$\sigma = 245.092\ \text{MPa}$$

**Q.2** Prove that for maximum strain hardening ,true strain is equal to work hardening coefficient. For a material true stress-strain curve follows the relationship $s_T = s_0 + k\hat{I}_T^n$. If true stress, $s_T = 360$ MPa, flow stress at zero plastic strain, $s_0 = 250$ MPa, strain hardening exponent, $n = 0.28$ and true plastic strain $\hat{I}_T = 0.07$.

Determine the value of $\frac{d\sigma_T}{d\in_T}$.

**Solution :**

$$\text{True strain} = \int_0^{\in} d\in = \int_{l_0}^{l} \frac{dl}{l}$$

True strain, $$\in = [\ln l]_{l_0}^{l} = \ln\left(\frac{l}{l_0}\right)$$

$$\left(\frac{l}{l_0} - 1\right) + 1 = e + 1$$

$$\in = \ln(1 + e) \qquad \text{(where, } e \text{ is engineering strain)}$$

True stress: $$\sigma_f = \frac{P}{A} \times \frac{A_0}{A_0}$$

$$\sigma_f = \sigma_0 \times \frac{A_0}{A}$$

$$A_0 l_0 = Al = \text{Volume.}$$

$$\frac{A_0}{A} = \frac{l}{l_0}$$

Now, $$\frac{A_0}{A} = \left(\frac{l}{l_0} - 1\right) + 1$$

$$\frac{A_0}{A} = e + 1$$

$$\sigma_f = \sigma_0(1 + e)$$

$$\frac{dP}{d\in} = \frac{d}{d\in} \times \sigma_f A$$

$$\frac{dP}{d\in} = \sigma_f \times \frac{dA}{d\in} + A \times \frac{d\sigma_f}{d\in} \quad ...(1)$$

$$\frac{d}{d\in} V = \frac{d}{d\in} \times Al$$

$$0 = A\frac{dl}{d\in} + l\frac{dA}{d\in}$$

$$0 = \frac{A}{l}\frac{dl}{d\in} + \frac{dA}{d\in} \qquad \left(\because \frac{dl}{d\in} = l\right)$$

$$0 = A + \frac{dA}{d\in}$$

$$\frac{dA}{d\in} = -A \quad ...(2)$$

Now by eq. (1) and (2):

$$\frac{dP}{d\in} = -\sigma_f \times A + A \times \frac{d\sigma_f}{d\in}$$

At U.T.S.

$$\frac{dP}{d\in} = 0$$

Now,
$$\frac{d\sigma_f}{d\in} = \sigma_f \quad ...(3)$$

$$\frac{d}{d\in}\sigma_f = \frac{d}{d\in} k\in^n$$

$$\frac{d\sigma_f}{d\in} = kn\in^{n-1} \times \frac{\in}{\in}$$

$$\frac{d\sigma_f}{d\in} = \frac{n}{\in}(k\in^n)$$

$$\frac{d\sigma_f}{d\in} = \frac{n}{\in} \times \sigma_f \quad ...(4)$$

by eq. (3) and (4)

$$n = \in$$

For maximum strain harding true strain is equal to work hardening coefficient.

$$\sigma_T = \sigma_0 + k\in_T^n$$

$$k = \frac{\sigma_T - \sigma_0}{\in_T^n}$$

Differentiating equation (i) with respect of $\in_T$.

$$\frac{d\sigma_T}{d\in_T} = \frac{d\sigma_0}{d\in_T} + k\frac{d\in_T^n}{d\in_T}$$