CHEMICAL ENGINEERING

Mechanical Operations

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Mechanical Operations

CHAPTER

1

Properties and Handling of Particulate Solids

LEARNING OBJECTIVES

The reading of this chapter will enable the students

- To understand the characterization of solid particles.
- To understand the mixed and average size analysis.
- To understand the concept of screening and screening equipment.
- To understand the concept of screen effectiveness and capacity.

1.1 Introduction

Mechanical Unit Operations are the operations which are purely based on physical or mechanical forces such as

- Gravitational force
- Centrifugal force
- Mechanical and kinetic forces arising from flow

So, the unit operations of which are purely based on mechanical or physical forces such as gravitational force, centrifugal force, mechanical or kinetic forces arising from flow, etc. they are known as the kind of mechanical unit operations.

Mechanical Unit Operations can be classified based on phases interacting as

- **Solid-Solid operations:** Crushing, grinding, sieving, compaction, cutting, storage and transport of bulk solids, etc.
- **Solid-Fluid operations:** Filtration, sedimentation, centrifugation, floatation, cyclone separators, etc.

Unit operations involving particulate solids:

- Separation of solids from a suspension by filtration.
- Fractionation of solids of wide size distribution based on size by gravity settling or differential settling methods.
- Separation of immiscible liquids by centrifugation (or decanting) and separation of solids from liquids by centrifugation.

1.2 Characterization of Solid Particles

Individual solid particles are characterized by their size, shape, and density. Particles of homogeneous solids have the same density as the bulk material. Particles obtained by breaking up a composite solid, such as a metal-bearing ore, have various densities, usually different from the density of the bulk material. Size and shape are easily specified for regular particles, such as spheres and cubes, but for irregular particles (such as sand grains or mica flakes) the terms size and shape are not so clear and must be arbitrarily defined.

1.2.1 Particle Shape

The shape of an individual particle is conveniently expressed in terms of the sphericity ϕ_s, which is independent of particle size. For a spherical particle of diameter D_p, $\phi_s = 1$; for a non-spherical particle, the sphericity is defined by the relation

$$\phi_s = \frac{6v_p}{D_p s_p} \qquad \text{...(1.1)}$$

where, D_p = Equivalent diameter or nominal diameter of particle

s_p = Surface area of one particle, v_p = Volume of one particle

The equivalent diameter is sometimes defined as the diameter of a sphere of equal volume. For fine granular materials, however, it is difficult to determine the exact volume and surface area of a particle, and D_p is usually taken to be the nominal size based on screen analyses or microscopic examination. For many crushed materials ϕ_s is between 0.6 and 0.8, as shown in table given below, but for particles rounded by abrasion ϕ_s may be as high as 0.95.

For cubes and cylinders for which the length L equals the diameter, the equivalent diameter is greater than L and ϕ_s found from the equivalent diameter would be 0.81 for cubes and 0.87 for cylinders. It is more convenient to use the nominal diameter L for these shapes since the surface-to-volume ratio is $\frac{6}{D_p}$, the same as for a sphere, and this makes ϕ_s equal to 1.0. For column packings such as rings and saddles the nominal size is also used in defining ϕ_s.

1.2.2 Particle Size

In general, "diameters" may be specified for any equidimensional particle. Particles that are not equidimensional, i.e., that are longer in one direction than in others, are often characterized by the second longest major dimension. For needlelike particles, for example, D_p would refer to the thickness of the particles, not their length.

Material	Sphericity	Material	Sphericity
Sphres, cubes, short cylinders ($L = D_p$)	1.0	Ottawa sand	0.95
		Rounded sand	0.83
Raschig rings ($L = D_p$)		Coal dust	0.73
$L = D_o$, $D_I = 0.5D_o$	0.58	Flint sand	0.65
$L = D_o$, $D_i = 0.75D_o$	0.33	Crushed glass	0.65
Berl saddles	0.3	Mica flakes	0.28

***Table:** Sphericity of miscellaneous materials*

By convention, particle sizes are expressed in different units depending on the size range involved. Coarse particles are measured in inches or millimeters; fine particles in terms of screens size; very fine particles in micrometers or nanometers. Ultrafine particles are sometimes described in terms of their surface area per unit mass, usually in square meters per gram.

Sphericity of some regular particle:

Particle Shape	Sphericity
Cylinder (L = D)	0.87
Cylinder (L = 2D)	0.82
Cylinder (L = 3D)	0.78
Cube	0.80
Cuboid (1 : 2 : 3)	0.725

1.3 Mixed and Average Size Analysis

In a sample of uniform particles of diameter D_p, the total volume of the particles is $\frac{m}{\rho_p}$, where m and ρ_p are the total mass of the sample and the density of the particles, respectively. Since the volume of one particle is v_p, the number of particles in the sampel N is

$$N = \frac{m}{\rho_p v_p} \quad \text{...(1.2)}$$

The total surface area of the particles is, from eqs. (1.1) and (1.2),

$$A = Ns_p = \frac{6m}{\phi_s \rho_p D_p} \quad \text{...(1.3)}$$

To apply Eqs. (1.2) and (1.3) to mixtures of particles having various sizes and densities, the mixture is sorted into fractions, each of constant density and approximately constant size. Each fraction can then be weighed, or the individual particles in it can be counted or measured by any one of a number of methods. Equations (1.2) and (1.3) can then be applied to each fraction and the results added.

Information from such a particle-size analysis is tabulated to show the mass or number fraction in each size increment as a function of the average particle size (or size range) in the increment. An analysis tabulated in this way is called a **differential analysis**. The results are often presented as a histogram, as shown in fig. (a), with a continuous curve like the dashed line used to approximate the distribution. A second way to present the information is through a cumulative analysis obtained by adding, consecutively, the individual increments, starting with that containing the smallest particles, and tabulating or plotting the cumulative sums against the maximum particle diameter in the increment. Fig. (b) is a cumulative-analysis plot of the distribution shown in fig. (a). In a cumulative analysis the data may appropriately be represented by a continuous curve.

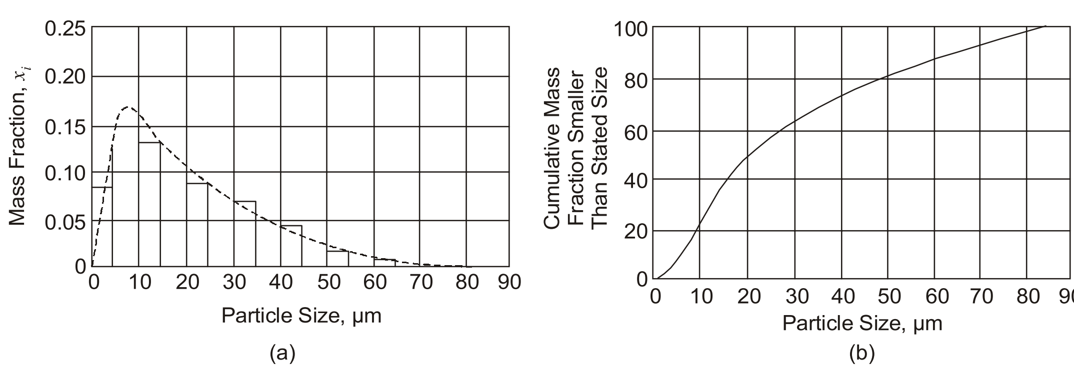

Fig. *Particle-size distribution for powder: (a) differential analysis; (b) cumulative analysis*

Calculations of average particle size, specific surface area, or particle population of a mixture may be based on either a differential or a cumulative analysis. In principle, methods based on the cumulative analysis are more precise than those based on the differential analysis, since when the cumulative analysis is used, the assumption that all particles in a single fraction are equal in size is not needed. The accuracy of particle-size measurements, however, is rarely great enough to warrant the use of the cumulative analysis, and calculations are nearly always based on the differential analysis.

Specific surface of mixture: If the particle density ρ_p and sphericity ϕ_s are known, the surface area of the particles in each fraction may be calculated from eq. (1.3) and the results for all fractions added to give A_w, the specific surface (the total surface area of a unit mass of particles). If ρ_p and ϕ_s are constant, A_w is given by

$$A_w = \frac{6x_1}{\phi_s\rho_d\bar{D}_{p1}} + \frac{6x_2}{\phi_s\rho_p\bar{D}_{p2}} + \ldots.. + \frac{6x_n}{\phi_s\rho_p\bar{D}_{pn}}$$

$$= \frac{6}{\phi_s\rho_p}\sum_{i=1}^{n}\frac{x_i}{\bar{D}_{pi}} \qquad \text{...(1.4)}$$

where, subscripts = Individual increments

x_i = Mass fraction in a given increment

n = Number of increments

$\bar{D}_{pi}$ = Average particle diameter, taken as arithmetic average of smallest and largest particle diameter in increment

1.3.1 Average Particle Size

The average particle size for a mixture of particles is defined in several different ways. Probably the most used is the volume-surface mean diameter $\bar{D}_s$, which is related to the specific surface area A_w. It is defined by the equation

$$\bar{D}_s \equiv \frac{6}{\phi_s A_w \rho_p} \qquad \text{...(1.5)}$$

Substitution from eq. (1.4) in eq. (1.5) gives

$$\bar{D}_s = \frac{1}{\sum_{i=1}^{n}\left(\frac{x_i}{\bar{D}_{pi}}\right)} \qquad \text{...(1.6)}$$

Other averages are sometimes useful. The arithmetic mean diameter $\bar{D}_N$ is

$$\bar{D}_N = \frac{\sum_{i=1}^{n}(N_i\bar{D}_{pi})}{\sum_{i=1}^{n}N_i} = \frac{\sum_{i=1}^{n}(N_i\bar{D}_{pi})}{N_T} \qquad \text{...(1.7)}$$

where N_T is the number of particles in the entire sample.

The mass mean diameter $\bar{D}_W$ is found from the equation

$$\bar{D}_W = \sum_{i=1}^{n} x_i\bar{D}_{pi} \qquad \text{...(1.8)}$$

Dividing the total volume of the sample by the number of particles in the mixture (see below) gives the average volume of a particle. The diameter of such a particle is the volume mean diameter $\bar{D}_V$, which is found from the relation

$$\bar{D}_V = \left[\frac{1}{\sum_{i=1}^{n}\left(\frac{x_i}{\bar{D}_{pi}^3}\right)}\right]^{1/3}$$

For samples consisting of uniform particles these average diameters are, of course, all the same. For mixtures containing particles of various sizes, however, the several average diameters may differ widely from one another.

Number of particles in mixture: To calculate, from the differential analysis, the number of particles in a mixture, Eq. (1.2) is used to compute the number of particles in each fraction, and N_w, the total population in one mass unit of sample, is obtained by summation over all the fractions. For a given particle shape, the volume of any particle is proportional to its "diameter" cubed, or

$$v_p = aD_p^3 \quad \text{...(1.9)}$$

where a is the volume shape factor. From eq. (1.2), then, assuming that a is independent of size,

$$N_w = \frac{1}{a\rho_p}\sum_{i=1}^{n}\frac{x_i}{\bar{D}_{pi}^3} = \frac{1}{a\rho_p \bar{D}_V^3} \quad \text{...(1.10)}$$

Screen analysis; standard screen series: Standard screens are used to measure the size (and size distribution) of particles in the size range between about 3 and 0.0015 in. (76 mm and 378 μm). Testing sieves are made of woven wire screens, the mesh and dimensions of which are carefully standardized. The openings are square. Each screen is identified in meshes per inch. The actual openings are smaller than those corresponding to the mesh numbers, however, because of the thickness of the wires. This set of screens is based on the opening of the 200-mesh screen, which is established at 0.074 mm. The area of the openings in any one screen in the series is exactly twice that of the openings in the next smaller screen. The ratio of the actual mesh dimension of any screen to that of the next smaller screen is, then, $\sqrt{2}$ = 1.41. For closer sizing, intermediate screens are available, each of which has a mesh dimension $\sqrt[4]{2}$ or 1.189, times that of the next smaller standard screen.

In making an analysis a set of standard screens is arranged serially in a stack, with the smallest mesh at the bottom and the largest at the top. The sample is placed on the top screen and the stack shaken mechanically for a definite time, perhaps 20 min. The particles retained on each screen are removed and weighed, and the masses of the individual screen increments are converted to mass fractions or mass percentages of the total sample. Any particles that pass the finest screen are caught in a pan at the bottom of the stack.

Since the particles on any one screen are passed by the screen immediately ahead of it, two numbers are needed to specify the size range of an increment, one for the screen through which the fraction passes and the other on which it is retained. Thus, the notation 14/20 means "through 14 mesh and on 20 mesh."

Average particle size for a mixture of particles is defined in several different ways:

(i) Volume Surface Mean Diameter ($\bar{D}_s$):

$$(\bar{D}_s) = \frac{6}{\phi_s \rho_p A_w} = \frac{1}{\sum_{i=1}^{n}\left(\frac{x_i}{\bar{D}_{p_i}}\right)}$$

It is defined as the diameter of sphere that has same volume/surface area ratio as a particle of interest.

(ii) Mass mean diameter $(\bar{D}_w)$:

$$\bar{D}_w = \sum_{i=1}^{n} x_i \bar{D}_{P_i}$$

(iii) Average volume of a particle:

$$= \frac{\text{Total volume of the sample}}{\text{Number of particles in the mixture}}$$

(iv) Volume mean diameter:

$$\bar{D}_V = \left[\frac{1}{\sum_{i=1}^{n}\left(\frac{x_i}{\bar{D}_{p_i}^3}\right)}\right]^{1/3}$$

(v) Arithmetic mean diameter $(\bar{D}_N)$:

$$\bar{D}_N = \frac{\sum_{i=1}^{n} N_i D_{Pi}}{N_T}$$

where, N_T is the number of particles in entire sample.

(vi) Number of particle in the mixture:

$$N_w = \frac{1}{a\rho_P}\sum_{i=1}^{n}\frac{x_i}{\bar{D}_{P_i}^3} = \frac{1}{a\rho_P \bar{D}_V^3}$$

where, a = Volume shape factor

N_w is the total population in one unit mass of sample, obtained by summation over all fractions.

Also, $V_P = aD_P^3$

(vii) Sauter mean diameter (SMD):

- It is a common measure in fluid dynamic as a way to estimate the average particle size.
- It is defined as the diameter of a sphere that has the same volume/surface area ratio as the particle of interest.
- Sauter mean diameter is typically defined in terms of

  Surface dia, $d_s = \sqrt{\frac{A_P}{\pi}}$, and

  Volume diameter, $d_V = \left(\frac{6V_P}{\pi}\right)^{\frac{1}{3}}$

  where, A_P and V_P are the surface area and volume of particle.
- S.M.D. for a given particle is

$$\text{S.D.} = D[3, 2] = d_{32} = \frac{d_V^3}{d_s^2}$$

Student's Assignments

Q.1 What is the sphericity of a cuboid whose length, breadth and depth are in the ratio 5 : 4 : 1?
(a) 0.726 (b) 0.614
(c) 0.81 (d) 0.563

Q.2 Find the shape factor of a cylindrical particle of 3 mm diameter and 3 mm length?
(a) 0.873 (b) 1.145
(c) 1 (d) 1.375

Q.3 What is the sphericity of a cylindrical particle whose length is equal to its diameter?
(a) 0.873 (b) 0.673
(c) 0.573 (d) 0.81

Q.4 For a cylindrical particle of height equal to twice the diameter, the sphericity value is
(a) 0.655 (b) 0.728
(c) 0.832 (d) 0.915

Q.5 The shape factor for a hemisphere is
(a) equal to 1 (b) greater than 1
(c) less than 1 (d) None of the above

Q.6 Find the sphericity of a cube of dimension $a \times a \times a$.

Q.7 Finely divided clay is used as a catalyst in the petroleum industry. It has a density of 1.2 g/cc and a sphericity of 0.5. The size analysis is as follows :

Average diameter, $D_{pi,avg}$(cm)	0.0252	0.0178	0.0126	0.0089	0.0038
Mass fraction, x_i (g/g)	0.088	0.178	0.293	0.194	0.247

Find the specific surface area and the sauter mean diameter of the clay material.

Q.8 Calculate the volume-surface mean diameter for the following particulate material.

Size range, μm	Mass of particles in the range, g
$-710+300$	30
$-300+180$	35
$-180+90$	65
$-90+38$	70
Pan	55

Q.9 Which of the following particle has the lowest value of sphericity?
(a) Rounded sand
(b) Pulverized coat
(c) Tungsten powder
(d) Mica flakes

Q.10 The following table gives the size distribution of a dust measured by micro scope. Convert these figures to obtain distribution on mass basis and calculate the specific surface in μm. Assuming spherical particle of specific gravity 2.65.

Size range (mm)	Number of Particles
0 – 2	2000
2 – 4	600
4 – 8	140
8 – 12	40
12 – 16	15
16 – 20	5
20 – 24	2

Q.11 The cumulative mass fraction of particles smaller than size d_j for a collection of N_i particles of diameter d_i and mass m_i $(i = 1, 2, 3, ..., \infty)$ is given by

(a) $\dfrac{\sum_{i=1}^{j} N_i d_i^3}{\sum_{i=1}^{\infty} N_i d_i^3}$ (b) $\dfrac{\sum_{i=1}^{j} N_i m_i d_i^3}{\sum_{i=1}^{\infty} N_i m_i d_i^3}$

(c) $\dfrac{\sum_{i=1}^{j} N_i m_i d_j^2}{\sum_{i=1}^{\infty} N_i m_i d_i^2}$ (d) $\dfrac{\sum_{i=1}^{j} N_i m_i d_j}{\sum_{i=1}^{\infty} N_i m_i d_i}$

Q.12 Weight mean diameter is given by

(a) $\frac{\Sigma n_i d_i^4}{\Sigma n_i d_i^3}$ (b) $\left(\frac{\Sigma n_i d_i^3}{\Sigma n_i}\right)^{1/3}$

(c) $\frac{\Sigma n_i d_i^3}{\Sigma n_i d_i^2}$ (d) $\frac{\Sigma n_i d_i^2}{\Sigma n_i d_i}$

ANSWERS

1. (b) **2.** (b) **3.** (a) **4.** (c)

5. (b) **6.** (0.81) **7.** (1235.43, 8.094×10^{-3})

8. (79.11) **9.** (d) **10.** (0.726×10^6)

11. (b) **12.** (c)

Explanation

1. (b)

The volume of cuboid is $5 \times 4 \times 1 = 20$ m^3

and surface area of this cuboid

$$= 2[(5 \times 4) + (4 \times 1) + (5 \times 1)]$$
$$= 2(20 + 4 + 5)$$
$$= 2 \times 29 = 58 \text{ m}^2$$

Let, D_P = Diameter of the equivalent sphere

Then, $\frac{\pi}{6}D_P^3 = 20$

$$D_P = 3.36 \text{ m}$$

Area of sphere $= \pi D_P^2 = 35.63$ m^2

Thus, Spericity $= \frac{35.63}{58} = 0.614$

2. (b)

For a non-spherical particle, the sphericity (ϕ)

$$= \frac{6}{D_P}\left(\frac{V_P}{S_P}\right)_{\text{Particle}}$$

Volume of sphere $= \frac{\pi}{6}D_P^3$

Volume of cylindrical particle $= \frac{\pi}{4}D^2 \times l$

where, $l = D$

$$\frac{\pi}{4}D_P^3 = \frac{\pi}{4}D^3$$

$$D_P = \left(\frac{6}{4}\right)^{1/3} D$$

$$\left(\frac{V_P}{S_P}\right)_{\text{Particle}} = \frac{\frac{\pi}{4}D^3}{\frac{\pi}{2}D^2 + \pi D^2} = \frac{D}{6}$$

$$\phi = \frac{6}{D_P} \times \frac{D}{6} = \left(\frac{4}{6}\right)^{1/3}$$

$$\phi = 0.873$$

Shape factor, $\phi' = \frac{1}{\phi} = \frac{1}{0.873} = 1.145$

3. (a)

For a non-spherical particle, the sphericity (ϕ)

$$= \frac{6}{D_P}\left(\frac{V_P}{S_P}\right)_{\text{Particle}}$$

Vol. of sphere $= \frac{\pi}{6}D_P^3$

Volume of cylindrical particle $= \frac{\pi}{4}D^2 \times l$

where, $l = D$

$$\frac{\pi}{6}D_P^3 = \frac{\pi}{4}D^3$$

$$D_P = \left(\frac{6}{4}\right)^{1/3} D$$

$$\left(\frac{V_P}{S_P}\right)_{\text{Particle}} = \frac{\frac{\pi}{4}D^3}{\frac{\pi}{2}D^2 + \pi D^2} = \frac{D}{6}$$

$$\phi = \frac{6}{D_P} \times \frac{D}{6} = \left(\frac{4}{6}\right)^{1/3}$$

$$\phi = 0.873$$

4. (c)

Sphericity $\phi = \frac{6}{D_p}\left(\frac{V_p}{S_p}\right)_{\text{particle}}$

Volume of sphere = Volume of cylindrical particle

$$\frac{\pi}{6}D_p^3 = \frac{\pi}{4}D^2 \times l$$

where, $l = 2D$

$$\frac{\pi}{6}D_p^3 = \frac{\pi}{4} \times 2D^3$$