# SSC-JE 2025

**Staff Selection Commission**

Junior Engineer Examination

# Civil Engineering

## Theory of Structures (SOM)

Well Illustrated **Theory** *with*
**Solved Examples** and **Practice Questions**

MADE EASY PUBLICATIONS

# Theory of Structures (SOM)

## Contents

CHAPTER 1

# Properties of Materials

## 1.1 Introduction

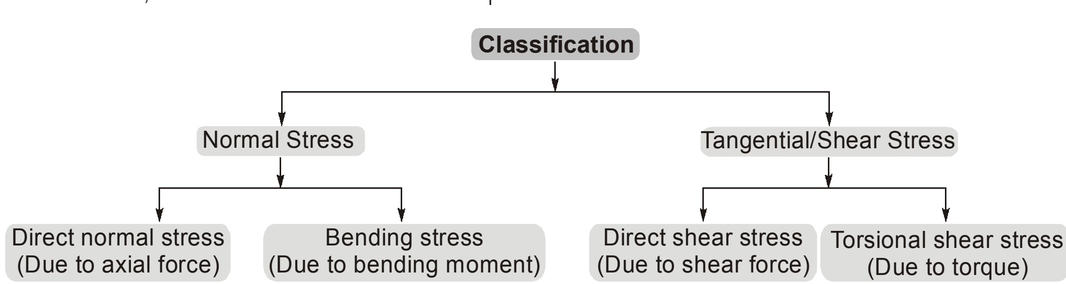
- Strength of material is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading (forces/moment) and internal forces developed due to these loadings.
- So the objective of our analysis is to determine the stresses, strains and deflection produced by the loads in different structures.
- The behaviour of a member subjected to forces depends not only on the fundamental law of Newtonian mechanics that govern the equilibrium of the forces but also on the mechanical characteristics of materials of which the member is fabricated, which could be a rigid material or a deformable material.

## 1.2 Rigid and Deformable Material

- A rigid material is one which does not undergo any change in its geometry, size of shape on the other hand, a deformable material is the one in which change in size, shape or both will occur when it is subjected to a force/moment.
- All materials are actually deformable and the idea of rigid material is only a conceptual idealization.

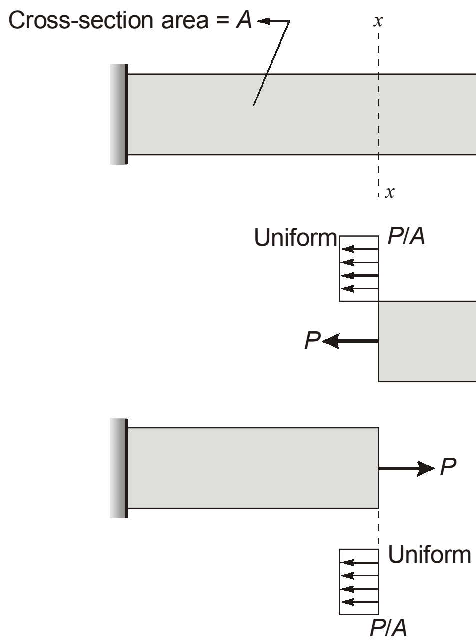
## 1.3 Stress

- Stress is the internal resistance offered by the body against deformation.
- Stresses will develop when strain is resisted. It should be noted that if strain if free to occur in any direction, then stresses will not develop in that direction.



## Direct Normal Stress

- It is the load directed along the axis of the member (i.e., normal to the section).




$$\sigma = \frac{P}{A}$$

- Normal stress could be tensile or compressive.

Sign Convention:

Tensile stress: (+ve)

Compressive stress : (–ve)

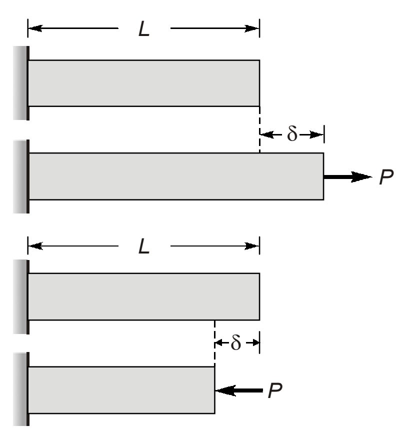
## Strain

- An axially loaded bar undergoes a change in length, becoming longer when in tension and shorter when in compression. Thus the elongation or shortening in axially loaded member per unit length is known as strain.
- It is represented by $\in$
- Mathematically, it can be calculated as

$$\in = \frac{\delta}{L}$$

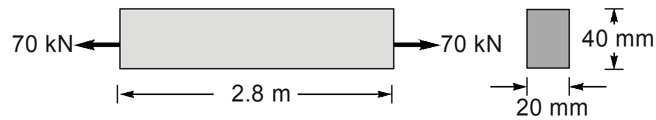
$\delta$ = Change in length

L = Original length




- **Unit:** Strain is dimensionless quantity. It is always expressed in that form of number. If member is in tension, then strain is called tensile strain and if member is in compression, then strain is called compressive strain.

**Example 1.1** A prismatic bar with rectangular cross-section (20 mm × 40 mm) length (L = 2.8 m) is subjected to an axial tensile force of 70 kN. The measured elongation of bar is 1.2 mm. Calculate the tensile stress and strain in the bar.




**Solution:**

Tensile stress, $$\sigma = \frac{P}{A} = \frac{70\times10^3}{20\times40} = 87.5\ \text{N/mm}^2$$

Tensile strain, $$\epsilon = \frac{\delta}{L} = \frac{1.2\times10^{-3}}{2.8} = 4.28\times10^{-4}$$

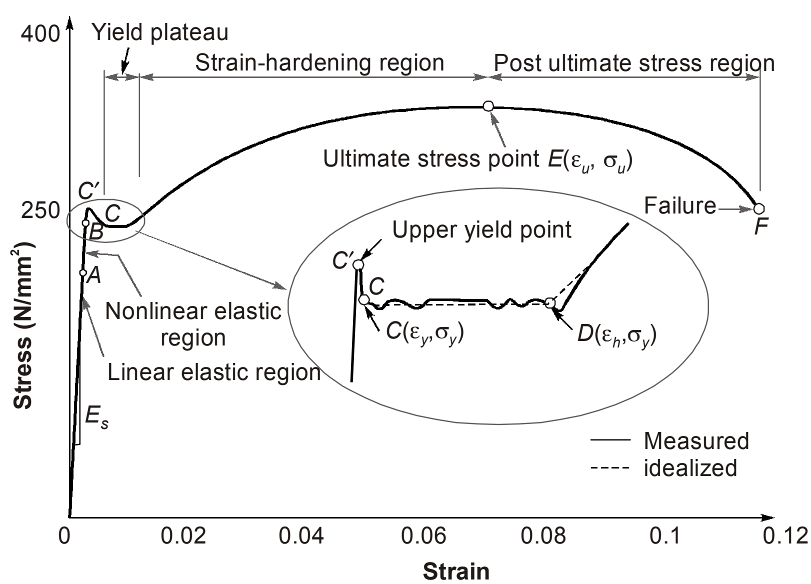
## 1.4 Tension Test Result on Mild Steel

- The mechanical properties of materials used in engineering are determined by experiments performed on small specimen. These experiments are conducted in laboratories equipped with testing machines which is capable of loading in tension or compression.
- The American Society for Testing and Materials (ASTM) has published guidelines for conducting test.
- Tension test is conducted on Universal Testing Machine (UTM).

### Specification of Specimen

- Specimen is solid cylindrical rod.
- Gauge length 2.0″ (inches)
- Die of middle section 0.5″ (inches)
- $\frac{L}{D}$ ratio = 4.0

### Stress-strain Curve for Tension



- **A:** It is called as proportional limit, the curve OA is linear and 'Hooke' law is valid upto this limit only.
- **B:** It is called as elastic limit, after unloading within B, entire strain can be released and the specimen will regain its original dimension.
- **(C′ & C):**
  C′ → upper yield point
  C → lower yield point
  or
  (Actual yield point)
- The fall of stress from C′ to C is due to the slipping of carbon atom in the molecular structure of steel from point C, the stress-strain curve becomes almost horizontal therefore it can be said as from point C without change in stress, strain is continuous. This is known as plastic deformation (start of plastic deformation) therefore, for ductile material the design stresses is the yield stress and for brittle material design stress is ultimate stress.
- **Region (CD):** It is called yield plateau for mild steel, from point C. It is assumed to be the deformation starts plastically therefore the stress corresponding to C is called actual yield stress.
- The strain corresponding to point D is about 1.4% (1.2-1.8%) and corresponding to C is about 0.12% for mild steel.
- Hence, plastic strain is 10 to 15 times of elastic strain.
- **Region (DE):** It is called as strain hardening region. In this region deformation is difficult to achieve due to randomness at the molecular level. The material in this range undergoes change in its elastic and crystalline structure, resulting in increased resistance to further deformation. This portion is not used for structural design.
- **(E):** It is called as ultimate point and the stress corresponding to it is called as ultimate stress.
- **Region (EF):** It is called as necking region. This occurs due to permanent deformation in the specimen hence due to that there will be a region in which cross-section decreases rapidly therefore deformation increases rapidly.
- **(F):** Fracture point.

**NOTE**

- The fracture strain depends on the percentage of carbon, the fracture strain is lower however the strength is higher.
- All the grades of steel will have the same Young's modulus therefore. The initial stress-strain curve will be same for all types of steel.
- A ductile material have very large strain after its elastic limit, however for brittle materials it is very less (<5%).
- The elastic rubber has very large fracture strain but as it posses elastic it fails immediately therefore its behaviour is brittle.

## 1.5 Relation between Actual and Engineering Stresses and Strains

- On the basis of cross-section area considered during calculation of stresses, direct stresses can be of following two types.
  (i) Engineering stresses or nominal stress.

$$\sigma_0 = \frac{P}{A_0}$$

$A_0$ = Original cross-sectional area of specimen

(ii) True stresses or actual stresses

$$\sigma = \frac{P}{A}$$

A = instantenaous cross-section area

$$\Rightarrow \qquad \sigma = \frac{P}{A} = \frac{P}{A_0} \times \frac{A_0}{A}$$

$$\sigma = \sigma_0 \times \frac{A_0}{A} \qquad \text{...(i)}$$

Also, on the basis of length of member used in calculation of strain, strain can be of following two types.

(i) Engineering strain/Nominal strain

$$\in_0 = \frac{\delta}{L_0}$$

$L_0$ = Original length of speciment

(ii) True or actual strain

$$\in = \frac{\delta}{L}$$

L = Instantaneous length

$$L = L_0 \pm d \qquad \text{(+ for tension) (– for compression)}$$

For same volume

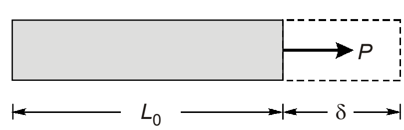
$$A_0 L_0 = AL$$

$$\frac{A_0}{A} = \frac{L}{L_0} = \frac{L_0 \pm \delta}{L_0} = (1 \pm \in_0)$$

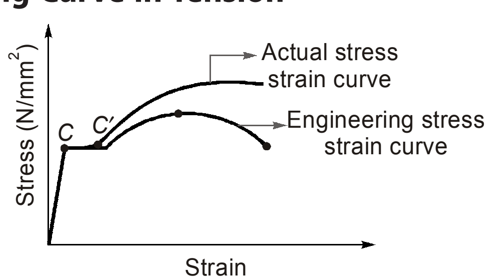
From eq. (i) $\sigma = \sigma_0 (1 \pm \in_0)$ ...(ii)

For tension $\sigma = \sigma_0 (1 + \in_0)$ $(\sigma > \sigma_0)$

For compression $\sigma = \sigma_0 (1 - \in_0)$ $(\sigma < \sigma_0)$

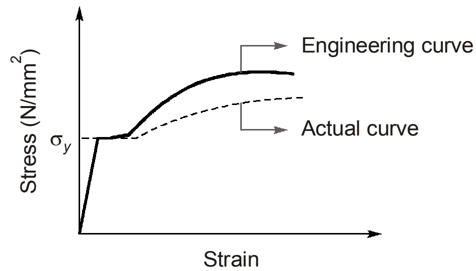



## Actual Curve v/s Engineering Curve in Tension




**Tension curve for mild steel**

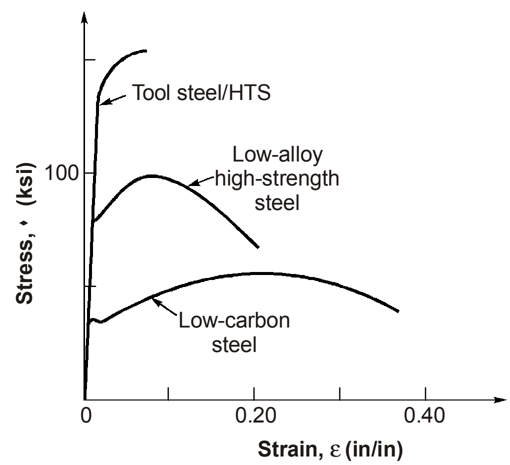
**Actual Curve v/s Engineering Curve in Compression**




**Compression curve for Mild steel**

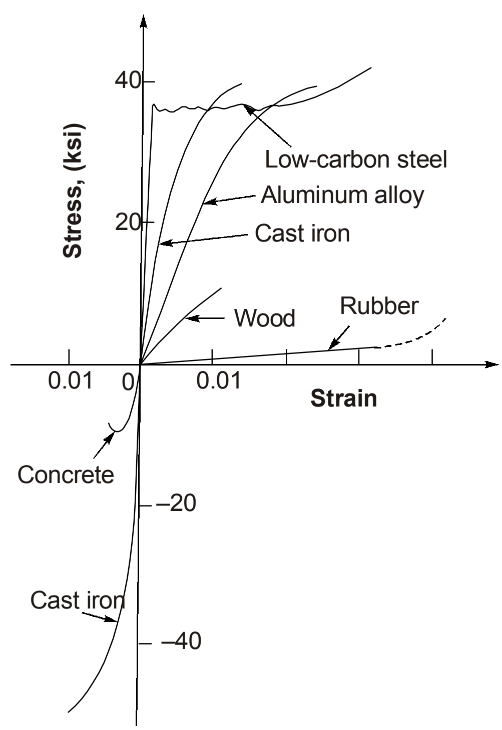
## 1.6 Stress-strain Curve for Different Grades of Steel

- With the increase in grade of steel yield stress increases but strain before fracture decreases. Hence ductility of the material decreases. Modulus of elasticity however for different grades of steel is the same.



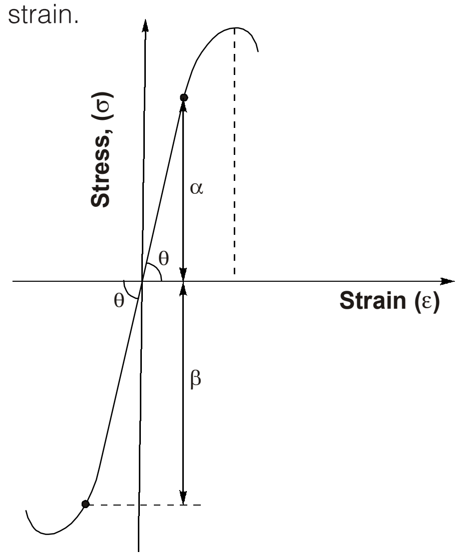

**Tensile stress-strain diagram for different grades of steel**

## 1.7 Stress-strain Curve for Various Materials in Tension




**Stress-strain diagram for different material**

## 1.8 Stress-strain Curve for Brittle Material (Cast Iron)



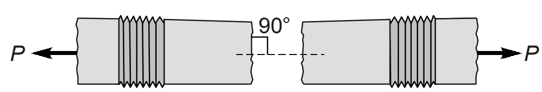

- In case of brittle material fracture takes place at very small strain.
- The fracture strain is elastic.
- The ultimate stress is also equal to fracture stress.
- There is no necking or plastic zone and fracture occur due to normal stress hence fracture plane is 90° plane from the longitudinal axis of the bar.
- Linear elastic range in tension is smaller as compared to that in compression. Also the ultimate strength in compression is larger than the tension.
- Modulus of elasticity in tension and compression is same.
- In brittle materials microscopic cracks and cavity exist in structure which causes weakness in tension however in compression these cracks are bridged hence it has greater strength in compression.

## 1.9 Modes of Failure

- Since brittle material is strong in compression but weak in tension whereas its shearing strength is in between its compressive strength and tensile strength.
- A ductile material are approximately equal strong in tension and compression but weak in shear.

**Case-1: Brittle material under tension**

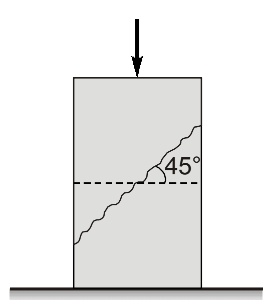
- Brittle material are very weak in tension. Brittle materials fails due to separation of particles along the surface which is at 90° to the direction of load. Failure surface is rough.




**Brittle failure of metal**

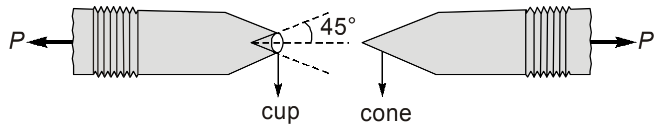
**Case-2: Brittle material under compression**

- Since, brittle material strong in compression therefore, failure is due to shear the plane of failure is at 45° from the axis of shaft.



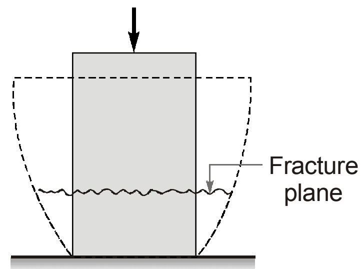

**Case-3: Ductile material in tension**

- Since ductile material strong in tension, therefore failure is due to shear, the plane of failure is at 45° from the axis of shaft, such failure is known as cup and cone failure.



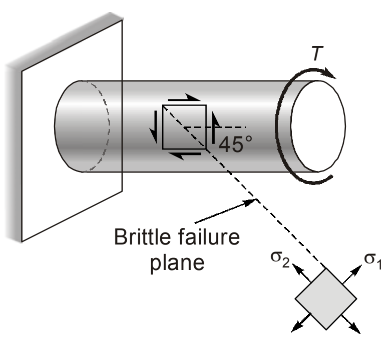

**Case-4: Ductile material in compression**

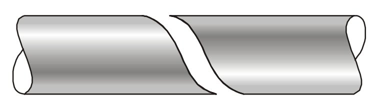
- Short compression members fails in compression yielding. Failure plane is at 90°, to the compressive load. In compression yielding, bulging of material occurs.




**Case-5: Brittle material in torsion**

- Since, brittle materials have minimum tensile strength. Hence, brittle materials fails in tension.
- Hence brittle material subjected to torsion fails at 45° plane (Helicoidal failure).

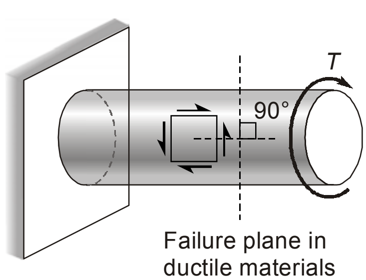


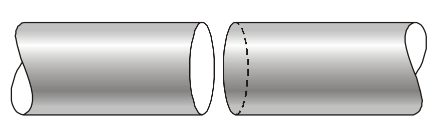



Torsional failure in brittle material

**Case-6: Ductile material in torsion**

- Since, ductile materials are weak in shear. Hence ductile materials failure occurs due to principle shear stress.
- In torsion test maximum shear stress is in the direction perpendicular to longitudinal axis. Hence, ductile failure plane is torsion will be perpendicular to longitudinal axis.



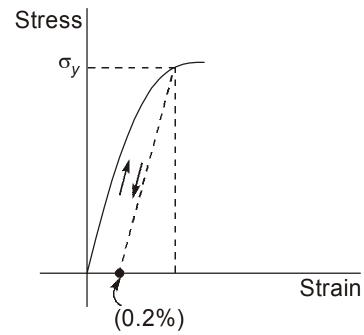
Torsional failure in ductile material

## 1.10 Proof Stress

- The concept of proof stress is used for those ductile materials or metals which do not represent its obvious yield point in testing.

  Ex. Al, Cu, Au, Ag etc. for these ductile material to determine the design stress this concept is used.

- According to that an offset of 0.2% is taken on strain axis, from this point a parallel line is drawn to the slope of initial stress-strain curve, the intersection of this line with the stress-strain curve is said to be design point and the stress corresponding to it is called proof stress (yield stress) or design stress.




## 1.11 Brittle Fracture

- A material which is ductile at normal temperature may become brittle at very low temperature. Similarly, a material which is brittle at normal temperature may become ductile at very high temperature.

  At sub-zero temperature, a structural steel may fail by brittle fracture. In this case modulus of elasticity (E) remains the same but ductility reduces greatly.
- Brittle fracture also occurs at the section where there is irregularity, like presence of notch.

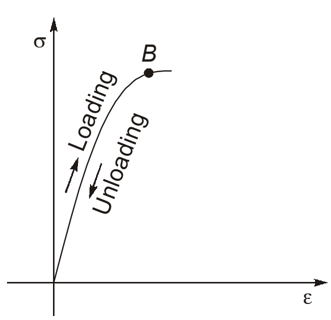
**NOTE**

- Brittleness or ductility is not the absolute property of a material. It depends on temperature, stress condition, rate of loading.
- Behaviour of the material, we are discussing here are for the gradual loading condition only. Under dynamic loading condition behaviour of material may change.

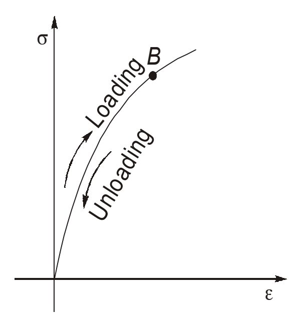
## 1.12 Properties of Material

### Elasticity

- Elasticity is the property by virtue of which material regain its original shape and dimension instantaneously when unloaded before elastic limit.
- Elastic material can be linearly elastic or non-linearly elastic.
- If material is unloaded before elastic limit (B) is reached, the unloading curve will follow the original curve.

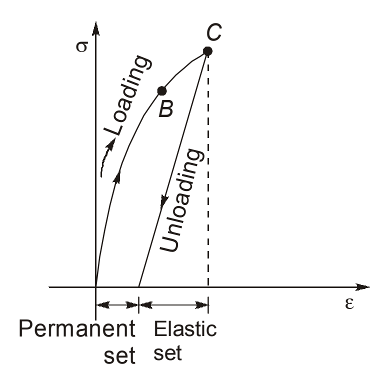



**(a) Linearly elastic material**




**(b) Non-linearly elastic material**

- If material is stressed beyond elastic limit and then unloaded, it will have a residual strain and the unloading curve will be different from original loading curve. The unloading curve will be parallel to the initial portion of the loading curve.
- Residual elongation of bar is called permanent set.




### Plasticity

- The property of material due to which is undergo inelastic strain beyond elastic limit is known as plasticity.

**Ductility**

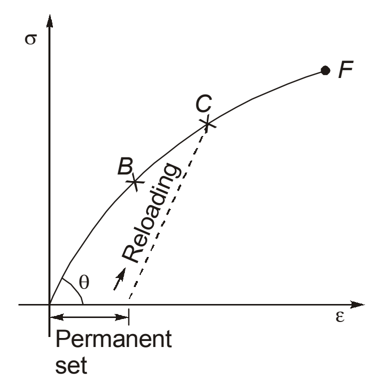
- It is a measure of the amount by which a material can be drawn out in tension before it fractures. Ductility measurement can be done using tension testing.

**Malleability**

- It is that property of metal due to which a piece of metal can be converted into a thin sheet by pressing it.
- A malleable material posses a high degree of plasticity.
- This property is of great use in operations like forging, hot rolling, drop (hammering) etc.

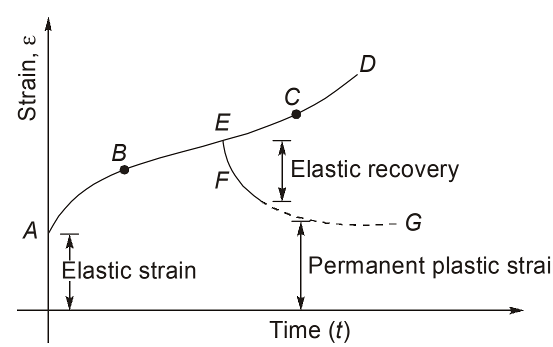
**Reloading**

- During reloading the proportional limit gets extended.
- However, in the reloading stress strain curve ductility of the material decreases.



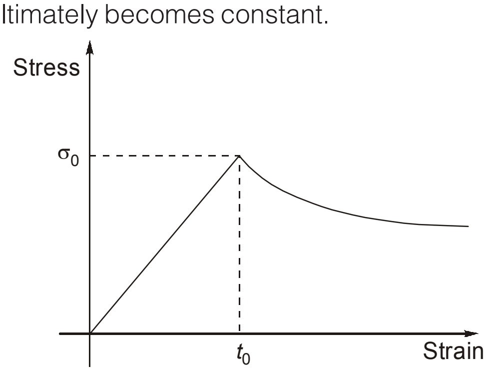

**Creep**

- Creep is the permanent deformation after passage of the time when a constant load is applied under elastic limit.
- Total creep deformation continues to increase with time asymptotically.
- Factors affecting creep.
  - Magnitude of load.
  - Type of loading (static or dynamic).
  - Time or age of loading.
  - Temperature
- At elevated temperature the rate of creep deformation is very high when the temperature becomes half of the melting point (homologous temperature) creep becomes in tolerable.
- Creep strain is due to dead loads or permanent loading only.
- Creep strain is permanent in nature and not 100% recoverable.



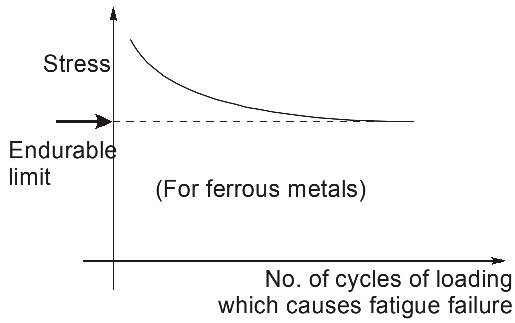

**Relaxation**

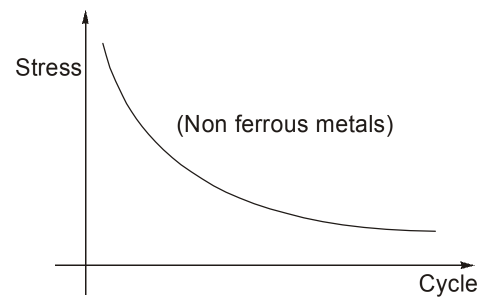
- The decrease in stress in steel as a result of creep within steel under prolonged strain is called relaxation.
- If bar is stretched to $\sigma_0$ stress in time to and thereafter left to bear that stresses then the stress will go on reducing and ultimately becomes constant.



**Fatigue**

- Deterioration of a material under repeated cycles of load resulting in progressive cracking ultimately leading to fracture is called fatigue.
- Magnitude of load causing fatigue failure is less then the load that can be sustained statically.
- Fatigue failure depends on magnitude of loading and number of cycles of loading.



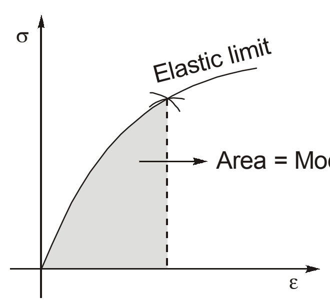




- Fatigue failure occurs because microscopic cracks propagate faster due to reversal of loading.
- For ferrous material endurance limit exist.

  Endurance limit is the stress level below which even infinite number of stress cycles can't produce fatigue failure.
- Normally for steel endurance limit is 50% of ultimate stress.
- Corrosion, surface roughness etc. causes reduction in fatigue strength.

**Resilience**

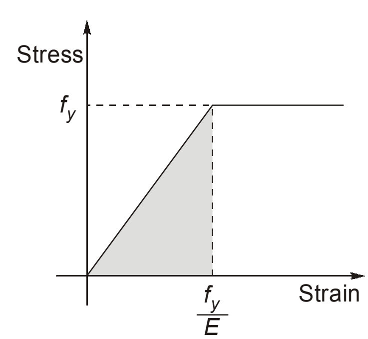
- The property by virtue of which material can absorb energy when deformed elastically is called resilience.
- Area under stress-strain curve upto elastic limit is called modulus of resilience.
- Greater is the modulus of resilience better is the material for making springs.




- For a linearly elastic material strain energy stored per limit volume

$$= \frac{1}{2} \times f_y \times \left(\frac{f_y}{E}\right) = \frac{f_y^2}{2E}$$

$\therefore$ Modulus of resilience $= \dfrac{f_y^2}{2E}$




- Area under load deformation curve within elastic limit is called resilience.

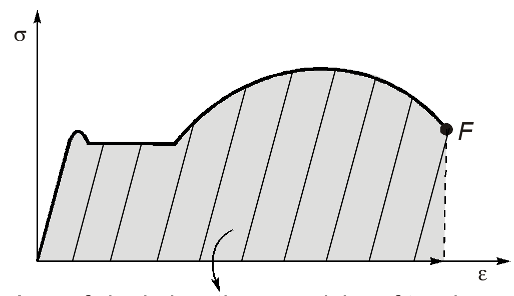
**Toughness**

- Ability to absorb mechanical energy upto fracture.
- It is a desirable properties against impact loading.

- Area under stress-strain curve upto fracture is called modulus of toughness.

$$\text{Modulus of toughness} = \frac{\text{Straight energy stored upto fracture}}{\text{Volume of material}}$$

- Mild steel is more tough as compared to cast iron.




Area of shaded portion = modulus of toughness

**Hardness**

- Ability to resist scratch and abrasion is called hardness. Normally higher the yield stress greater is the hardness.
- Cast iron is harder as compared to mild steel.

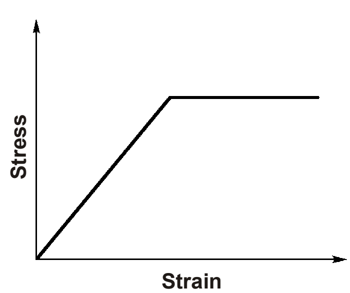
**Tenacity**

- Property of a material to resist fracture under tensile loading is called tenacity.

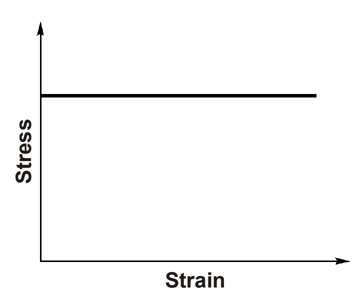
**Viscoelastic Material**

- Viscoelastic material have both viscous and elastic property and exhibits time dependent strain.
- It has following property:
  (i) Hysteresis is seen in the stress-strain curve
  (ii) Stress relaxation occurs
  (iii) Creep occurs

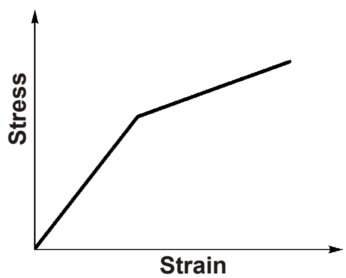
## 1.13 Approximate Stress-Strain Curve



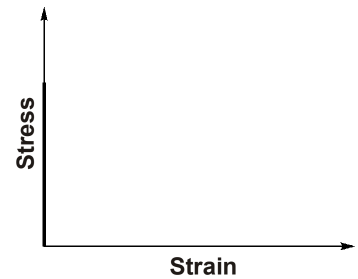

Elasto plastic



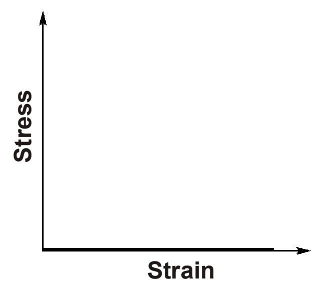

Rigid-plastic



Elasto Plastic with Strain Hardening



Ideal Fluid Behaviour



Ideal Rigid

## STUDENT'S ASSIGNMENTS

**Q.1** True stress $\sigma$ is related with conventional stress $\sigma_0$ as

(a) $\dfrac{\sigma}{\sigma_0} = (1+\varepsilon)^2$   (b) $\dfrac{\sigma}{\sigma_0} = \dfrac{1}{(1+\varepsilon)^2}$

(c) $\dfrac{\sigma}{\sigma_0} = \dfrac{1}{1+\varepsilon}$   (d) $\dfrac{\sigma}{\sigma_0} = (1+\varepsilon)$

**Q.2** Stress-strain curve is always a straight line for
(a) elastic material
(b) material obeying Hooke's law
(c) elastoplastic materials
(d) None of these

**Q.3** Consider the following statements:
The principle of superposition is applied to
1. Linear elastic bodies
2. Bodies subjected to small deformations
Which of these statements is/are correct?
(a) 1 alone   (b) 1 and 2
(c) 2 alone   (d) Neither 1 nor 2

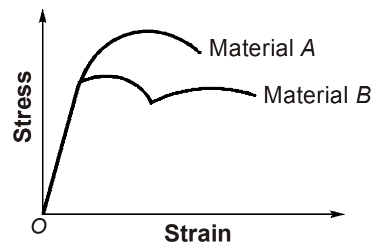
**Q.4** What would be the shape of the failure surface of a standard cast iron specimen subjected to torque?
(a) Cup and cone shape at the centre
(b) Plane surface perpendicular to the axis of the specimen
(c) Pyramid type wedge shaped surface perpendicular to the axis of the specimen
(d) Helicoidal surface at 45° to the axis of the specimen

**Q.5** Steel has its yield strength of 400 N/mm$^2$ and modulus of elasticity of $2 \times 10^5$ MPa. Assuming the material to obey' Hooke's law upto yielding, what is its proof resilience?
(a) 0.8 N/mm$^2$   (b) 0.4 N/mm$^2$
(c) 0.6 N/mm$^2$   (d) 0.7 N/mm$^2$

**Q.6** For linear elastic systems, the type of displacement function for the strain energy is
(a) linear   (b) quadratic
(c) cubic   (d) quartic
[GATE : 2004]

**Q.7** The material that exihibits the same elastic properties in all directions at a point is said to be
(a) homogeneous   (b) orthotropic
(c) viscoelastic   (d) isotropic
[GATE : 2015]

**Q.8** The stress-strain diagram for two material A and B is shown below:




The following statements are made based on t his diagram.

1. Material A is more brittle material B
2. The ultimate strength of material B is more then that of A

With reference to the above statements, which of the following applies?

(a) Both the statements are false
(b) Both the statements are true
(c) I is true but II is false
(d) I is false but II is true

[GATE : 2000]

**Q.9** The property of a material by which it can be between or rolled into plates is called

(a) ductility (b) plasticity
(c) elasticity (d) None of these

[SSC JE : 2018]

**Q.10** The greatest load which a spring can carry without getting permanently distorted is called

(a) Stiffness (b) Proof resilience
(c) Proof stress (d) Proof load

[SSC JE : 2018]

**Q.11** Which of the following statements is false?

(a) mild steel has a yield points
(b) mild steel shows strain hardening
(c) mild steel is a ductile material
(d) None of these

[SSC JE : 2018]

**Q.12** The ratio of longitudinal stress to strain within elastic limit is known as

(a) modulus of elasticity
(b) shear modulus of elasticity
(c) bulk modulus of elasticity
(d) tangent modulus of elasticity

[SSC JE : 2017]

**Q.13** The ratio of shear stress and shear strain of an elastic material is called

(a) Modulus of elasticity
(b) Both modulus of rigidity and shear modulus
(c) Young's modulus
(d) Modulus of rigidity

[SSC JE : 2017]

**Q.14** In compression test, the fracture in cast iron specimen would

(a) occur along the axis of load
(b) occur along an oblique plane
(c) occur at right angles to the axis of specimen
(d) not occur

[SSC JE : 2017]

**Q.15** The phenomenon of slow extension of materials i.e., increasing with time having no constant load is called

(a) Creeping (b) Breaking
(c) Yielding (d) None of these

[SSC JE : 2017]

**Q.16** The stress at which extension of a material takes place more quickly as compared to the increase in load, its called as

(a) Elastic point (b) Plastic point
(c) Breaking point (d) Yielding point

[SSC JE : 2017]

**Q.17** In a tensile test, when the material is stressed beyond elastic limit, the tensile strain ______ as

(a) decreases slowly
(b) increases slowly
(c) decreases more quickly
(d) increases more quickly

[SSE JE : 2017]

**Q.18** With a percentage increase of carbon in steel

(a) strength (b) hardness
(c) brittleness (d) ductility

[SSC JE : 2015, 2017]

**Q.19** The strain energy stored in a body due to external loading, within the elastic limit is known as ______

(a) malleability (b) ductility
(c) toughness (d) resilience
**[SSC JE : 2017]**

**Q.20** Out of the following, which is least elastic?
(a) Silver (b) Rubber
(c) Iron (d) Copper
**[SSC JE : 2012]**

**Q.21** Which of the following material is expected to have the least value of Young's modulus of elasticity?
(a) Wood (b) Copper
(c) Glass (d) Aluminium
**[SSC JE : 2107]**

**Q.22** Creep of a material is
(a) not being ductile
(b) to become brittle
(c) disappearance of deformation on removal of load
(d) continued deformation with time under sustained loading
**[SSC JE : 2015]**

**Q.23** Which of the following material has least carbon content?
(a) Wrought iron (b) Cast iron
(c) Mild steel (d) Pig iron
**[SSC JE : 2014]**

**Q.24** The property of a material by which it can be drawn into smaller section by application of tension is called
(a) Plasticity (b) Ductility
(c) Elasticity (d) Malleability
**[SSC JE : 2007]**

**Q.25** The percentage of elongation of test piece under tension indicates its
(a) brittleness (b) malleability
(c) stiffness (d) ductility
**[SSC JE : 2009]**

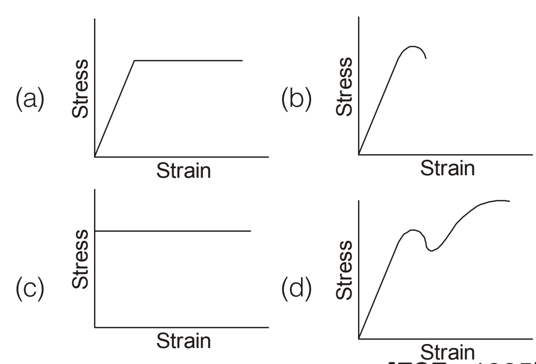
**Q.26** Loss of stress with time at constant strain in steel is called
(a) relaxation (b) creep
(c) shrinkage (d) ductility
**[APPSC : 2016]**

**Q.27** The creep strains are
(a) caused due to dead loads only
(b) caused due to live loads only
(c) caused due to both dead and live load
(d) independent of loads
**[BPSC : 2019]**

**Q.28** The stress developed due to external force is an elastic material
(a) Depends on elastic constant
(b) Does not depends on elastic constant
(c) Depends partially on elastic constants
(d) Depends upon limit of proportionality
**[MPSC : 2017]**

**Q.29** A member which is subjected to reversible tensile or compressive stresses may fail at stresses lower then the ultimate stresses of the material. This property of metal is called
(a) Plasticity of metal
(b) Workability of metal
(c) Fatigue of the metal
(d) Creep of the metal
**[MPPSC : 2014]**

**Q.30** The stress-strain curve for an ideally plastic material is




**[ESE : 1995]**

**Q.31** The stress level, below which a material has a high probability of not failing under reversal of stress is known as
(a) elastic limit (b) endurance limit
(c) proportional limit (d) tolerance limit
**[ESE : 1995]**

**Q.32** Match **List-I** and **List-II** and select the correct answer using the codes below:
**List-I**
A. Ductility

B. Brittleness
C. Tenacity
D. Toughness

List-II
1. Failure without warning
2. Drawn permanently over great changes of shape without rupture
3. Absorption of energy at high stress without rupture
4. High tensile strength

Codes:

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 1 | 2 | 4 | 3 |
| (b) | 1 | 2 | 3 | 4 |
| (c) | 2 | 3 | 4 | 1 |
| (d) | 2 | 1 | 4 | 3 |

[ESE : 1997]

Q.33 Resilience is
(a) maximum strain energy
(b) recoverable strain energy
(c) total potential energy
(d) shear strain energy

[Beyond Hooke's law]

[ESE : 2002]

Q.34 The material in which large deformations is possible before the absolute failure or rupture is termed as
(a) Brittle (b) Elastic
(c) Ductile (d) Plastic

[ESE : 2003]

Q.35 In mild steel specimen's subjected to tensile test cycle, the elastic limit is raised and the elastic limit in compression is lowered. This is called
(a) Annealing effect
(b) Bauschinger effect
(c) Strain rate effect
(d) Fatigue effect

[ESE : 2017]

## ANSWER KEY // STUDENT'S ASSIGNMENTS

| | | | | |
|---|---|---|---|---|
| **1.** (d) | **2.** (b) | **3.** (b) | **4.** (d) | **5.** (b) |
| **6.** (b) | **7.** (d) | **8.** (c) | **9.** (d) | **10.** (d) |
| **11.** (d) | **12.** (a) | **13.** (b) | **14.** (b) | **15.** (a) |
| **16.** (d) | **17.** (d) | **18.** (d) | **19.** (d) | **20.** (b) |
| **21.** (a) | **22.** (d) | **23.** (a) | **24.** (b) | **25.** (d) |
| **26.** (a) | **27.** (a) | **28.** (b) | **29.** (c) | **30.** (c) |
| **31.** (b) | **32.** (d) | **33.** (b) | **34.** (c) | **35.** (b) |

## HINTS & SOLUTIONS // STUDENT'S ASSIGNMENTS

**5. (b)**

Proof resilience

$$= \frac{\sigma_y^2}{2E} = \frac{(400)^2}{2\times 2\times 10^5} = 0.4 \text{ N/mm}^2$$

**6. (b)**

$$\text{Strain energy} = \frac{1}{2}\times\sigma\times\varepsilon = \frac{a^2}{2E}$$

Since strain is directly proportional to displacement so, strain energy is directly proportional to quadratic equation of displacement.

**8. (c)**

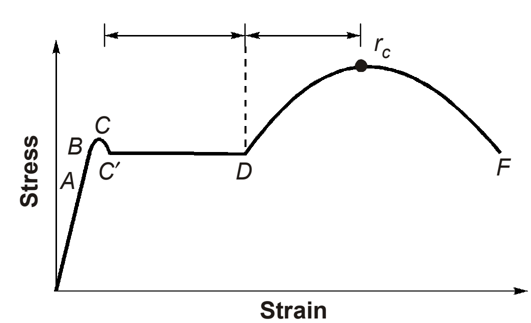
Since strain in material B is more, hence it is more ductile then material A i.e., material A is more brittle then material B. Hence statement 1 is true.

The material A can reach upto higher stress level hence ultimate strength of material A is more then that of material B. Hence statement II is false.

**9. (d)**

The property of a material by which it can be between or rolled into plates is called malleability.

**11. (d)**




A — Proportional limit
B — Elastic limit
C′ — Lower yield point

C — Upper yield point
CD — Plastic zone
DE — Strain hardening zone
E — Ultimate point
F — Fracture point

**16. (d)**

Beyond yield point, as the load increases, elongation of the specimen proceeds at a faster rate.

**17. (d)**

As the material is stressed beyond the elastic limit, the slope between stress and strain decreases rapidly, hence the strain increases more quickly then stress beyond the elastic limit.

**29. (c)**

Due to repeated loading of opposite nature, stress concentration occurs in certain region of the material and it may fall before the load reaches its yield strength. The failure at reduced stress due to repetitive nature is due to fatigue in the material.

**35. (b)**

(Bauschinger Effect)

It refers to property of materials where the materials stress/strain characteristics change as a result of the microscopic stress distribution of the material. Hence, an increase in tensile yield strength occurs at the expense of compressive yield strength.

■■■■

CHAPTER 2

# Simple Stress-strain and Elastic Constants

## 2.1 Hooke's Law

(i) The material is elastic

(ii) The material is homogeneous (it means properties are same at all points).

(iii) Material is isotropic (it means properties are same in all directions).

**Statement:**

Stress is proportional to strain (within proportional limit)

$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

$\sigma$ = stress

$\varepsilon$ = strain

E = a constant of proportionality known as Young's modulus of elasticity

- The slope of stress-strain curve is called modulus of elasticity (E). The modulus of elasticity (E) is the constant of proportionality which is defined as the intensity of stress that causes unit strain. Thus, modulus of elasticity has the same unit as that of stress.

  Also, (shear stress $\propto$ shear strain)

$$\tau \propto \gamma$$

$$\tau = G\gamma$$

G = Constant of proportionality known as modulus of rigidity

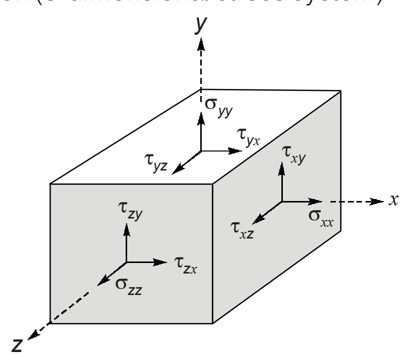
$$G = \frac{\tau}{\gamma}$$

**NOTE**

- The orthotropic material are those which have different properties in mutually perpendicular directions. Ex. boat.
- For isotropic material there are two (E and $\mu$) independent elastic constants.
- For orthographic material there are 9 independent elastic constant.
- For non-isotropic (anisotropic) material there are 21 no. of independent elastic constant.
- In properties are different in all directions then material is called anisotropic.

## 2.2 Stress-strain System

- Under general loading condition (3-dimensional stress system)




$\sigma_{DD}$ = Normal Stress
$Z_{DD}$ = Shear stress

(Planes on which stresses act) (The direction in which stresses act)

Hence, at any point inside the body under general loading condition no. of stresses is 9.

| | | |
|---|---|---|
| $\sigma_{xx}$ | $\tau_{xy}$ | $\tau_{yx}$ |
| $\sigma_{yy}$ | $\tau_{yz}$ | $\tau_{zy}$ |
| $\sigma_{zz}$ | $\tau_{zx}$ | $\tau_{xz}$ |

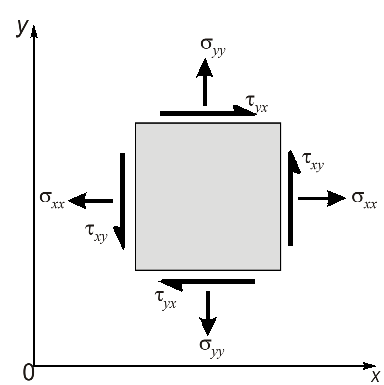
- Shear stress on opposite faces are equal in magnitude and opposite in direction (followed from force equilibrium).
- Shear stresses on adjacent perpendicular planes are equal in magnitude and directed towards each other 'or' away from each other (followed from moment equilibrium)

Hence
$$\tau_{xy} = \tau_{yx}$$
$$\tau_{yz} = \tau_{zy}$$
$$\tau_{xz} = \tau_{xz}$$

### Stress Matrix

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}_{3\times3}$$

(2-dimensional stress system)




Stress matrix, $$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}_{2\times2}$$

For moment equilibrium, $[\tau_{yx} = \tau_{xy}]$

### Properties of Stress Matrix

(i) Stress matrix is always a square matrix.
(ii) The diagonal elements are the normal stress component
(iii) Since the transpose of a matrix remains same
Hence it is a symmetrical matrix.