UPPSC-AE | 2021

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Mechanical Engineering

Mechanism and Machines

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY
Publications

Mechanism and Machines

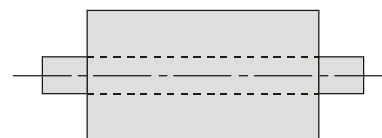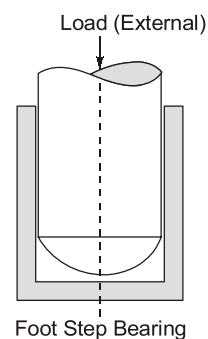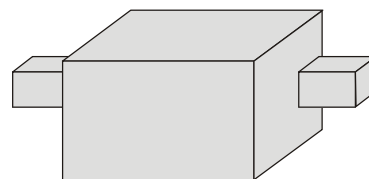# Contents

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Mechanism and Machines

1.1 Introduction

- If a number of bodies are assembled in such a way that the motion of one link causes constrained and predictable motions to the other links is known as a **mechanism**.

- A **machine** is a mechanism or a combination of mechanism which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work.

- Mechanism is neither a source of energy nor a producer of work but helps in proper utilization of the energy.

  Example : A slider crank chain is a mechanism that converts reciprocating motion into rotary motion of the crank or vice-versa. But when it is used in an automobile by adding valve mechanism, cam mechanism etc., it becomes a machine.

1.2 Types of Constrained Motion

(i)   Completely Constrained Motion : When motion between two elements of a pair is in a definite (*single*) direction irrespective of the direction of force applied known as completely constrained motion.
e.g., sliding pair.

(ii) Successfully Constrained Motion : When motion between two element of a pair is *possible* in *more than one* direction but is made to have motion *only in one* direction by using some *external means*, it is called successfully constrained motion.

e.g., a piston in a cylinder of an internal combustion engine is made to have only reciprocating motion due to constrain of the piston pin (external), cam and follower, shaft in foot step bearing.

Load (External)

Foot Step Bearing

(iii) Incompletely Constrained Motion : When the motion between the elements of a pair is possible in *more than one direction* and depends upon the *direction of force applied*, it is known as incompletely constrained motion e.g. cylindrical shaft in round bearing, without collars at ends.
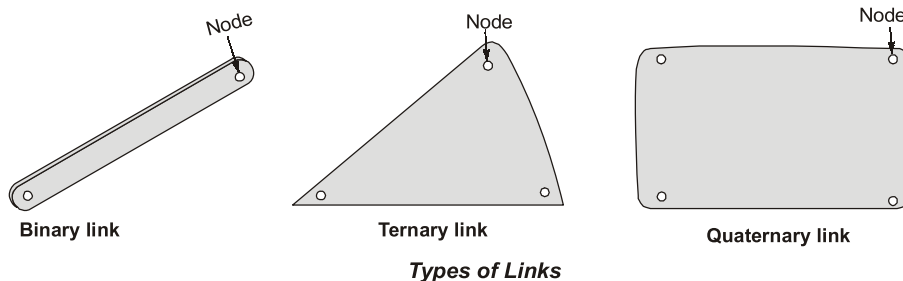
Rigid Body, Resistant Body

- **Rigid body:** It does not suffer any distortion, under the action of force.
- **Resistant body:** Those body which are rigid for the purpose they have to serve for e.g. belt drive, where belt is rigid when subjected to tensile forces.

1.3 Link

A link is define as a member of mechanism connecting other member and having motion relative to them.

Types of Links

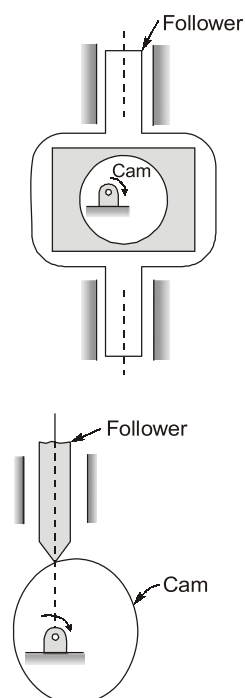## 1.4 Kinematic Pair and Their Classification

A **kinematic pair** or simply a pair is a joint of two links having a constrained relative motion between them.

(i)   According to nature of contact

- **Lower Pair:** A pair of links having surface or area contact between the members is known as lower pair. The contact surfaces of the two links are similar. Example: Nut turning on a screw, shaft rotating in a bearing, universal joint etc.

- **Higher Pair:** When a pair has a point or line contact between the members is known as a higher Pair. The contact surfaces of two links are dissimilar. Example: wheel rolling on a surface, cam and follower pair, tooth gears, ball and roller bearings etc.
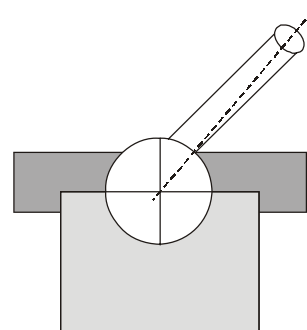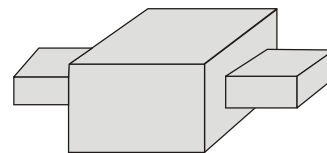
(ii)   According to nature of Mechanical constraint

- **Self Closed Pair/Closed Pair:** When the elements are geometrically identical, one is solid and full and other is hollow or open. The contact between the two can be broken only by destruction of at least one of the members. This means that elements of a pair are held together mechanically and there is a **permanent contact**. All the lower pairs and some of the higher pairs are closed pairs. These are also called self closed pair.

- **Open/forced Closed/Unclosed Pair:** When two links of a pair are in contact either due to forces of gravity or some spring action, is called an **unclosed pair**. Two elements may or may not be geometrically identical. Example : Cam and follower pair, door lock mechanism, automatic clutch operating system.

(iii) According to Nature of Relative Motion

- **Sliding Pair:** If two links have a sliding motion relative to each other, they form a sliding pair. **E.g.:** Such as a rectangular rod in a rectangular hole in a prism is a sliding pair.

- **Turning Pair:** When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair.

  E.g.: In slider-crank mechanism, all pairs except the slider and guide pair are turning pairs.

- **Rolling Pair:** When the links of a pair have a rolling motion relative to each other they form a rolling pair e.g. rolling wheel on a flat surface, ball and roller bearings etc.

- **Screw Pair (Helical Pair):** If two mating links have a turning as well as sliding motion between them they form a screw pair. **E.g.:** The lead screw and the nut of a lathe is a screw pair, pair of nut and bolt.

- **Spherical Pair:** When one link in the form of a sphere turn inside a fixed link, it is a spherical pair.

  E.g.: ball in a socket joint.

NOTE ▶

Sliding pair → Lower Pair;	Screw Pair → Lower Pair
Turning / Revolute / Pin Joint → Lower Pair;	Spherical Pair → Higher Pair
Rolling Pair → Higher Pair	

Kinematic Chain and Mechanism

- A **kinematic chain** is an assembly of links in which the relative motions of the links is possible and the motions are definite and constrained.
- In case the motion of a link results in indefinite motion of other links, it is a **non-kinematic chain**.
- If there is no relative motion of the links, it is a **redundant chain**.

- Kinematic chain $\xrightarrow{\text{One link fixed}}$ Linkage $\xrightarrow{\text{if motion is constrained and definite}}$ Mechanism
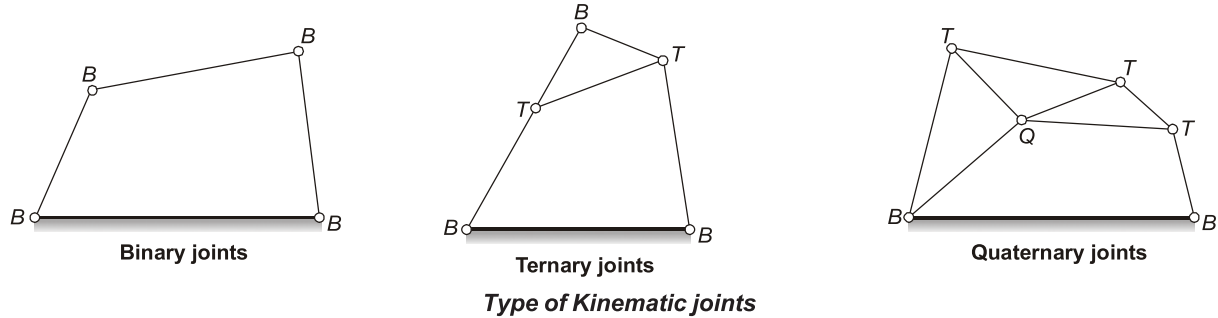
Kinematic Joint

A **kinematic joint** is the connection between two links by a pin. There is ample clearance between the pin and the hole in the ends of the links being connected to provide free motion of the links.

The usual types of joints in a chain are as shown in figure:

- **Binary joint :** Two links are connected at the same joint by a pin.
- **Ternary joint :** Three links are connected at the same joint by a pin.
- **Quaternary joint :** Four links are connected at the same joint by a pin.

  (Binary, Ternary and quaternary joint are shown by B, T and Q respectively in below figure.)

Binary joints   **Ternary joints**   **Quaternary joints**

Type of Kinematic joints

NOTE: If n number of links are connected at a joint, it is equivalent to $(n-1)$ binary joint.

1.5 Degree of Freedom (DOF)

- An unconstrained rigid body can have following independent motions :
  - (i) Translation along x, y and z axis.
  - (ii) Rotation about these axis. (about x, y and z axis)
- So, a rigid body in space can have 6 independent motions or degrees of freedom.
- So, for a pair, DOF is the number of independent relative motions (both translational and rotational) a pair can have.
- The connection of a link with another link imposes certain constraints on their relative motion. Constraints can never be zero (joint is disconnected) or six (rigid joint).

$$\text{DOF} = 6 - \text{Number of restraints}$$

> **NOTE**
>
> (i) **DOF of a system or mechanism** can also be defined as number of independent variables required to define a position or motion of the system.
> (ii) If a link of redundant chain is fixed $\rightarrow$ structure or locked system
> $\therefore$ DOF = 0
> If DOF is negative $\rightarrow$ Superstructure
> (iii) If DOF = 1 $\rightarrow$ Constrained chain
> DOF > 1 $\rightarrow$ Unconstrained chain

1.5.1 Degree of Freedom of Mechanism

- Let, F = Degree of freedom (DOF); L = Total number of links in mechanism;
  P_1 = Number of pair having one DOF; P_2 = Number of pair having two DOF;
  P_3 = Number of pair having three DOF; P_4 = Number of pair having four DOF
  P_5 = Number of pair having five DOF
- If In a mechanism one link is fixed then number of movable links = $N - 1$.
- Number of degrees of freedom of $(N-1)$ movable links = $6(N-1)$.
- Each pair having one degree of freedom imposes 5 restraints on the mechanism reducing its degree of freedom by $5P_1$.
- Each pair having two degrees of freedom will imposes 4 restraints reducing the degrees of freedom of the mechanism by $4P_2$.
- Similarly other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of the mechanism
- Degree of freedom = $6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5$.

1.5.2 Degree of Freedom of Plane (2D) Mechanism (Grubler Criterion)

- Most of the mechanism are two dimensional such as four link or a slider crank mechanism in which displacement is possible along two axes (one restraint) and rotation about only one axis (two restraints). Thus there are three general restraints. Hence.
$$F = 3(N-1) - 2P_1 - P_2.$$
Here, L = Number of link in a mechanism; P_1 = Number of pair having one degree of freedom;
P_2 = Number of pair having two degree of freedom

- **Kutzback's equation**
$$F = 3(L-1) - 2j - h$$
Here, L = Number of link; j = Number of **binary joint**; h = Number of **higher pair**

- **Grubler's equation :** It is for those mechanism which have **single** degree of freedom and **zero** higher pair.
$$3L - 2j - 4 = 0$$
Here, l = Number of links; j = Number of binary joints

- The following relationship holds for a kinematic chain having lower pairs :
$$L = 2P - 4$$
and
$$j = \frac{3}{2}L - 2$$
where L = Number of binary links; j = Number of binary joints; P = Number of lower pairs;
If   LHS > RHS → Locked or redundant chain;   LHS = RHS → Constrained chain;
        LHS < RHS → Unconstrained chain

- We know that
$$3L - 2j - 4 = 0$$
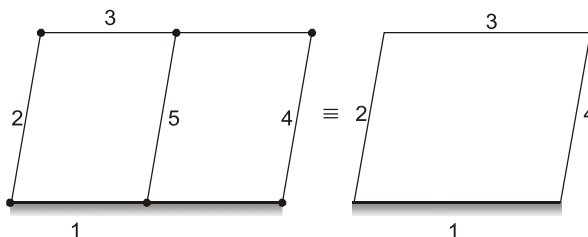$$\Rightarrow \qquad L = \frac{2j+4}{3}$$
∴ to satisfy this condition minimum value of $j = 4$
So, $L_{minimum} = 4$

NOTE: Therefore, minimum number of links to have a mechanism (1 DOF) with only lower pairs is four. But minimum number of links to have a mechanism (1 DOF) with both lower and higher pairs is three.

1.5.3 Redundant Links and Redundant Degree of Freedom :

- Links of a system which do not introduce any extra constraint are called **redundant links.**
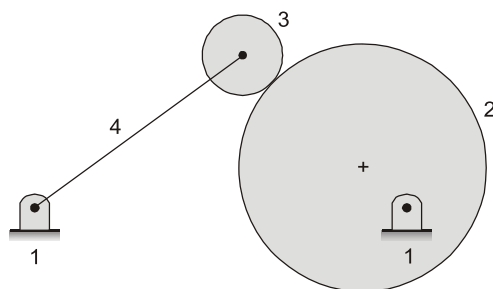- These links do not have any role in mechanism.

Here link 5 do not have any role in mechanism, it is a redundant link.
So, $l = 4, j = 4, h = 0, F_r = 0$
$$\text{Dof} = 3(l-1) - 2j - h - F_r = 3(4-1) - 2 \times 4 - 0 - 0 = 1$$

- Sometimes one or more links of a mechanism can be moved without causing any movement to the rest of the links. Such a link is said to have redundant degree of freedom F_r. e.g.,

(3) can rotate about its axis without causing any movement to other links.

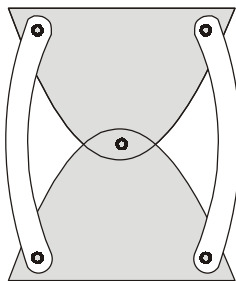$$\therefore \qquad F = 3(N-1) - 2P_1 - P_2 - F_r$$
$$N = 4, P_1 = 3, P_2 = 1$$
$$\therefore \qquad F = 3 \times 3 - 6 - 2 - 1 = 1$$

Example - 1.1 The degrees of freedom of the above chain in planar motion is

(a) −1   (b) 0
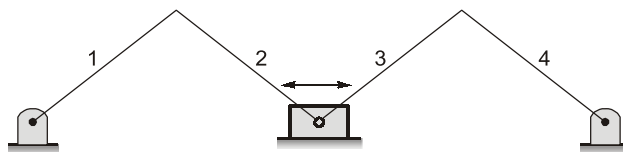(c) 2   (d) 3

Solution: (c)

None of the link is fixed $F = 3N - 2P_1 = 3 \times 4 - 2 \times 5 = 2$

Example - 1.2 The number of degree of freedom for the following mechanism is
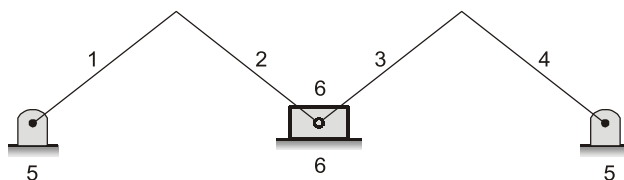

(a) 0   (b) 1
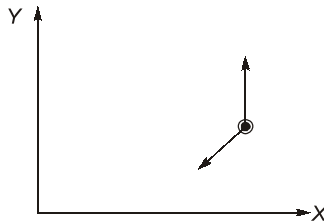(c) 2   (d) 4

Solution: (b)

At slider 3 links are connected $\equiv$ 2 binary joints
$$P_1 = 7$$
$$F = 3(6 - 1) - 2 \times 7 = 15 - 14 = 1$$

Example - 1.3 What is the degree of freedom of the system shown in the figure below :

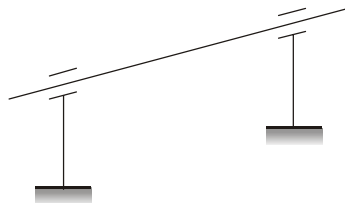(a) 3   (b) 2
(c) 4   (d) 6

Solution: (c)
$$F = 3N - 2P_1 = 3 \times 2 - 2 \times 1 = 4 \qquad\qquad (\because \text{No link is fixed})$$

Example - 1.4 How many degrees of freedom does the mechanism shown below has?

(a) 0   (b) 2
(c) 1   (d) 3

Solution: (a)
Mechanism is given, so one link is fixed.
$$F = 3(N - 1) - 2P_1 - F_r$$
($\because$ rod can be moved without causing any movement to other links)
$$= 3(4 - 1) - 2 \times 8 - 1$$
$$= 0$$

Example - 1.5 The given figure shows a/an

(a) locked chain
(b) constrained kinematic chain
(c) unconstrained kinematic chain
(d) mechanism

Solution: (c)
$$F = 3N - 2P_1 \text{ (no fixed link)}$$
$$= 3 \times 5 - 2 \times 5 = 15 - 10 = 5 > 1$$

$\Rightarrow$ Unconstrained chain

Student's Assignment

Q.1 The degree of freedom of a superstructure is
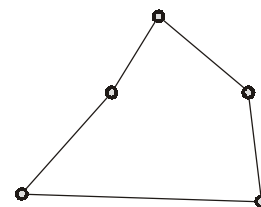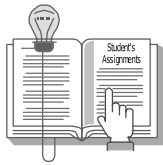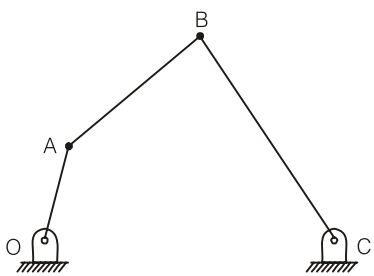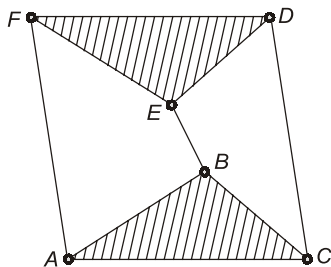(a) 0
(b) positive
(c) negative
(d) none

Q.2 Universal joint is an example of
(i) Lower pair
(ii) Higher pair
(iii) Closed pair
(iv) Unclosed pair
(a) (i) & (iii) correct
(b) (i) & (iv) correct
(c) (ii) & (iii) correct
(d) (ii) & (iv) correct

Q.3 For a 4 bar linkage in toggle position, the value of mechanical advantage is
(a) 0
(b) 0.5
(c) 1
(d) ∞

Q.4 Ball and socket joint is an example of
(a) Rolling pair
(b) Turning pair
(c) Screw pair
(d) Spherical pair

Q.5 Which of the following is an inversion of single-slider-crank chain
(a) Elliptical trammel
(b) Hand pump
(c) Scotch yoke
(d) Oldhman's coupling

Q.6 It is required to connect two parallel shafts, the distance between whose axes is small and variable. They can be coupled by
(a) Universal Joint
(b) Knuckle Joint
(c) Sleeve Coupling
(d) Oldham Coupling

Q.7 The four bar mechanism shown in the figure Given: OA = 3 cm, AB = 5 cm, BC = 6 cm, OC = 7 cm is a

(a) Double crank mechanism
(b) Double rocker mechanism
(c) Crank rocker mechanism
(d) Single slider mechanism

Q.8 The Whitworth quick return mechanism is formed in a slider-crank chain when the

(a) coupler link is fixed
(b) longest link is a fixed link
(c) slider is a fixed link
(d) smallest link is a fixed link

Q.9 Oldham's coupling is used to connect two shafts which are
(a) intersecting
(b) parallel
(c) perpendicular
(d) co-axial

Q.10 The motion of a shaft in a circular hole is an example of
(a) completely constrained motion
(b) incompletely constrained motion
(c) successfully constrained motion
(d) none of the above

Q.11 With the increase in the number of the links the number of access turning pair goes on
(a) decreasing
(b) increasing
(c) remain same
(d) none

Q.12 How many degrees of freedom, a threaded screw has?
(a) 0
(b) 2
(c) 1
(d) Insufficient data

Q.13 For a mechanism shown below, the mechanical advantage for the given configuration is

(a) 0
(b) 0.5
(c) 1
(d) ∞

Q.14 Kinematic pairs are those which have two elements that:
(a) have line contact
(b) have surface contact
(c) permit relative motion
(d) are held together

Q.15 What is the possible minimum number of links required in the planar mechanism of single degree of freedom which have both higher and lower kinetic pairs?
(a) 1
(b) 3
(c) 4
(d) 5

Q.16 A four bar chain has
(a) all turning pairs
(b) one turning pair and the others are sliding pairs
(c) one sliding pair and others are turning pairs
(d) all sliding pairs

Q.17 In a single slider four bar linkage when the slider is fixed, it form a mechanism of
(a) hand pump
(b) reciprocating engine
(c) quick return mechanism
(d) oscillating cylinder

Q.18 The degrees of freedom of the above mechanism are

(a) 0   (b) 1
(c) 2   (d) 3

Q.19 The motion of a piston in the cylinder of a steam engine is an example of
(a) completely constrained motion
(b) incompletely constrained motion
(c) successfully constrained motion
(d) none of the above

Q.20 The motion transmitted between the teeth of gears in mesh is
(a) sliding
(b) rolling
(c) may be rolling or sliding depending upon the shape of teeth
(d) partly sliding and partly rolling

Q.21 The cam and follower without a spring forms a
(a) lower pair   (b) higher pair
(c) self closed pair   (d) force closed pair

Q. 22 The lead screw of a lathe with nut forms a
(a) sliding pair   (b) rolling pair
(c) screw pair   (d) turning pair

Q.23 When the elements of the pair are kept in contact by the action of external forces, the pair is said to be a
(a) lower pair   (b) higher pair
(c) self closed pair   (d) force closed pair

Q.24 A planer mechanism has 8 links and 10 rotary joints. The number of degree of freedom of mechanism, using Gruble'r criterion is _____.
(a) 0   (b) 1
(c) 2   (d) 3

Q.25 A combination of kinematic pairs, joined in such a way that the relative motion between the links is completely constrained, is called a
(a) structure   (b) mechanism
(c) kinematic chain   (d) inversion

Q.26 The relation between the number of pairs (p) forming a kinematic chain and the number of links (l) is
(a) $l = 2p - 2$ (b) $l = 2p - 3$
(c) $l = 2p - 4$ (d) $l = 2p - 5$

Q.27 The relation between the number of links (l) and the number of binary joints (j) for a kinematic chain having constrained motion is given by $j = \frac{3}{2}l - 2$ If the left hand side of this equation is greater than right hand side, then the chain is
(a) locked chain
(b) completely constrained chain
(c) successfully constrained chain
(d) incompletely constrained chain

Q.28 In a kinematic chain, a quaternary joint is equivalent to _____ binary joints.
(a) one   (b) two
(c) three   (d) four

Q.29 If n links are connected at the same joint, the joint is equivalent to
(a) ($n - 1$) binary joints
(b) ($n - 2$) binary joints
(c) ($2n - 1$) binary joints
(d) none of the above

Q.30 In a 4-bar linkage, if the lengths of shortest, longest and the other two links are denoted by s, l, p and q, then it would result in Grashof's linkage provided that
(a) $l + p < s + q$ (b) $l + s < p + q$
(c) $l + p \geq s + q$ (d) none of the

Q.31 A kinematic chain is known as a mechanism when _____ of the links is fixed.
(a) none
(b) one
(c) two
(d) all

Q.32 The Grubler's criterion for determining the degrees of freedom (n) of a mechanism having plane motion is
(a) $n = (l - 1) - j$
(b) $n = 2(l - 1) - 2j$
(c) $n = 3(l - 1) - 2j$
(d) $n = 4(l - 1) - 3j$

Q.33 The mechanism forms a structure, when the number of degrees of freedom (n) is equal to
(a) 0
(b) 1
(c) 2
(d) less than zero

Q.34 In a four bar chain or quadric cycle chain
(a) each of the four pairs is a turning pair
(b) one is a turning pair and three are sliding pairs
(c) three are turning pairs and one is sliding pair
(d) each of the four pairs is a sliding pair.

Q.35 Which of the following is an inversion of double slider crank chain?
(a) Coupling rod of a locomotive
(b) Pendulum pump
(c) Elliptical trammels
(d) Oscillating cylinder engine

Q.36 Which of the following is are the example of completely constrained motion?
(a) motion of a piston in the cylinder of a steam engine
(b) motion of a square bar in a square hole
(c) motion of a shaft with collars at each end in a circular hole
(d) all of the these

ANSWER KEY / STUDENT'S ASSIGNMENT

1. (c)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (b)
6. (d)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (b)
11. (b)	**12.** (c)	**13.** (d)	**14.** (c)	**15.** (b)
16. (a)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (d)
21. (c)	**22.** (c)	**23.** (d)	**24.** (b)	**25.** (c)
26. (c)	**27.** (a)	**28.** (c)	**29.** (a)	**30.** (b)
31. (b)	**32.** (c)	**33.** (a)	**34.** (a)	**35.** (c)
36. (d)				

HINTS & SOLUTIONS / STUDENT'S ASSIGNMENT

1. (c)
For superstructure, DOF < 0 (i.e., negative)

2. (a)
Universal joint is an example of lower and closed pair.

3. (d)
In toggle position, the mechanical advantage = ∞, which can be used in stone crushers.

4. (d)
Ball and socket pair → Spherical pair

5. (b)
Hand-pump is an example of single slider crank mechanism when slider is fixed.

6. (d)
Oldham's coupling is used to connect the shafts having small lateral misalignment.

7. (c)
Since smallest link is adjacent to fixed link and $s + l < p + q$, so the mechanism will be crank-rocker mechanism.

8. (d)
In Whitworth quick return mechanism, crank is fixed.

9. (b)
Oldham's coupling is used to connect two parallel shaft.

10. (b)
When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. A circular bar or shaft in a circular hole, is an example of an incompletely constrained motion as it may either rotate or slide in a hole.

11. (b)
With increase in number of the links the number of access turning pain goes on increasing.

12. (c)

In threaded screw, both rolling and translational motions are there. But these are not independent of each other.
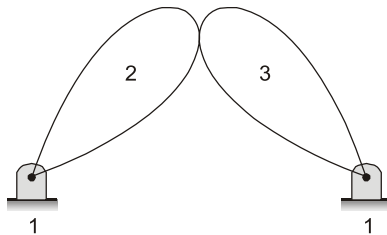
$\therefore \qquad F = 1$

13. (d)

At this instant, output angular velocity is 0.

$\therefore \qquad MA = \infty$

14. (c)

Kinematics pair permit relative motion between two elements.

15. (b)

$N = 3$, $P_1 = 2$, $P_2 = 1$

$F = 3(3 - 1) - 2 \times 2 - 1$

$= 6 - 4 - 1 = 1$

16. (a)

In four bar chain, all links are connected with turning pairs.

17. (a)

In hand pump, slider is fixed link.

18. (a)

$l = 5$, $j = 6$, $h = 0$, $f_r = 0$

Degree of freedom,

$n = 3(l - 1) - 2j - h - F_r$

$= 3(5 - 1) - 2 \times 6 - 0 - 0$

$= 0$

19. (a)

The piston and cylinder in a steam engine form a pair and the motion of the piston is limited to a definite direction (i.e. it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank. Thus forming completely constrained motion.

20. (d)

When the two elements of a pair have a line or point contact when relative motion takes place and the motion between the two elements is partly turning and partly sliding, and in mesh the gears have a point contact. Only at pitch point it is having pure rolling at rest position is forms partial sliding.

21. (c)

When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self closed pair.

22. (c)

When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut, and bolt with a nut is an examples of a screw pair.

23. (d)

When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity. Automatic clutch operating system (by using spring an hydraulic pressure).

24. (b)

$DOF = 3(n - 1) - 2j - g - F_r$

$n = 8$; $j = 10$; $h = 0$; $F_r = 0$

$DOF = 3(8 - 1) - (2 \times 10) - 0 - 0$

$= 1$

25. (c)

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion, it is called a kinematic chain.

26. (c)

The relation between the number of pairs (p) forming a kinematic chain and the number of links (l) may be expressed in the form of an equation:

$l = 2p - 4$