UPPSC-AE 2021

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Electrical Engineering

Control Systems

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

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Control Systems

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CHAPTER 1

Introduction

The control system is that means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in accordance with desired manner. For example, the driving system of an automobile. Speed of the automobile is a function of the position of its accelerator. The desired speed can be maintained (or a desired change in speed can be achieved) by controlling pressure on the accelerator pedal. This automobile driving system (accelerator, carburetor and engine-vehicle) constitutes a control system.

An Example of a Control Action

- Control of a room temperature is achieved by switching ON and OFF of a supply to a heating appliance. Thus power supply to an appliance is switched ON, when the room temperature is felt low and switched OFF, when the desired temperature is reached.
- The above system can be modified, if the duration of application of power is predetermined to achieve the room temperature within desired limits.
- However, a further refinement can be made by measuring the difference between the actual room temperature and the desired room temperature and this difference being the error is used to control the element which in turn controls the output i.e. room temperature.
- The above description indicates that in the former case the output (room temperature) has no control on the input and the control action is purely based on a sort of predetermined calibration only, whereas in the latter case the control action is affected by a feedback received from the output to the input.

1.1 Open Loop and Closed Loop Systems

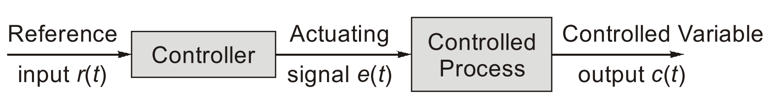
1.1.1 Open Loop Systems

- This is the simplest and most economical type of control system and does not have any feedback arrangement.

  Examples for open loop systems are:

  (a) Traffic Light Controller (b) Electric Washing Machine

  (c) Automatic Coffee Server (d) Bread Toaster
- As shown in the block diagram below, an input or command signal r(t) is applied to a controller which may be an amplifier or signal conditioner and whose output e(t) acts as an actuating signal. The actuating signal then actuates the controlled process and drives the controlled variable c(t) to a desired value.



- The output is normally predetermined by calibration and the input control may be accompanied by some form of calibration chart. The actual output obtained depends on the validity of calibration and the environment like temperature, humidity etc.

Advantages

(a) Simple and economic
(b) No stability problem

Disadvantages

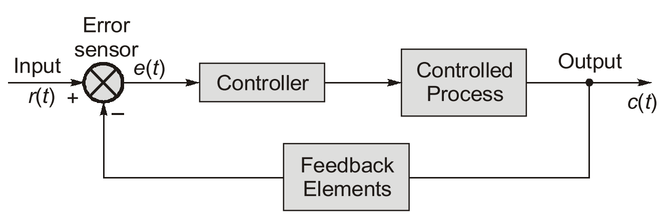
(a) Inaccurate
(b) Unreliable
(c) The effect of parameter variation and external noise is more.

NOTE

- No performance analysis is required for open loop control systems.
- Feedback is not used for improving stability.
- An open loop stable system may become unstable when negative feedback is applied.
- Except oscillators, in positive feedback, we have always unstable systems.

1.1.2 Closed Loop System or Feedback System

- The closed loop system has the same basic feature as of open loop system and an additional feedback feature. The actual output is measured and a signal corresponding to this measurement is feedback to the input section, where it is compared with the input to obtain the desired output. In this case, the actuating signal e(t) is the error or difference between the reference input r(t) and a signal which is a function of actual output. Human beings are probably the most complex and sophisticated feedback control system in existence.



- As the closed loop system automatically attempts to correct any difference between the desired and actual output, the actual output is independent of calibration and of fluctuations in environmental conditions of operation. The block diagram in figure below describes a closed loop system.

Examples of closed loop systems are:
(a) Electric iron
(b) DC motor speed control
(c) A missile launching system (direction of missile changes with the location of target)
(d) Radar tracking system
(e) Human respiratory system
(f) Autopilot system
(g) Economic inflation

Advantages

(a) Accurate and reliable.
(b) Reduced effect of parameter variation.
Example: RLC network.
(c) Bandwidth of the system can be increased.
(d) Reduced effect of non-linearities.

Disadvantages

(a) The system is complex and costly.
(b) System may become unstable.
(c) Gain of the system reduces with negative feedback.

A comparison between the two types of control systems are:

Open Loop System

1. So long as the calibration is good, an open -loop system performance will be accurate.
2. Organisation is simple and easy to construct.
3. Generally stable in operation.
4. If non linearity is present, system operation non-linearity.

Closed Loop System

1. Due to feedback, the performance of closed-loop system is accurate.
2. Complicated and difficult.
3. Stability depends on system components.
4. Comparatively the performance is better than open-loop system, if non-linearity is present.

Example - 1.1 Match List-I (Physical action or activity) with List-II (Category of system) and select the correct code:

List-I
A. Human respiration system
B. Pointing of an object with a finger
C. A man driving a car
D. A thermostatically controlled room heater

List II
1. Man-made control system
2. Natural including biological control system
3. Control system whose components are both man-made and natural

Codes:

	A	B	C	D
(a)	2	2	3	1
(b)	3	1	2	1
(c)	3	2	2	3
(d)	2	1	3	3

***Solution:* (*a*)**

1.2 Use of Laplace Transformation in Control Systems

Laplace Transform

In order to transform a given function of time f(t) into its corresponding Laplace transform first multiply $f(t)$ by e^{-st}, s being a complex number ($s = \sigma + j\omega$). Integrate this product with respect to time with limits from zero to ∞. This integration results in Laplace transform of $f(t)$, which is denoted by $F(s)$ or $Lf[(t)]$.

The mathematical expression for Laplace transform is,

$$L[f(t)] = F(s),\ t \geq 0$$

or,
$$F(s) = \int_0^{\infty} f(t)\cdot e^{-st}dt \quad \text{... (i)}$$

The original time function $f(t)$ is obtained back from the Laplace transform by a process called inverse Laplace transformation and denoted as L^{-1} thus,

$$L^{-1}[Lf(t)] = L^{-1}[F(s)] = f(t)$$

The time function $f(t)$ and its Laplace transform $F(s)$ form a transform pair.

Table of Laplace Transform Pairs

S.No.	$f(t)$	$F(s) = L[f(t)]$
1.	$\delta(t)$ unit impulse at $t = 0$	1
2.	$u(t)$ unit step at $t = 0$	$\frac{1}{s}$
3.	$u(t-T)$ unit step at $t = T$	$\frac{1}{s}e^{-sT}$
4.	t	$\frac{1}{s^2}$
5.	$\frac{t^2}{2}$	$\frac{1}{s^3}$
6.	t^n	$\frac{n!}{s^{n+1}}$
7.	e^{at}	$\frac{1}{s-a}$
8.	e^{-at}	$\frac{1}{s+a}$
9.	te^{at}	$\frac{1}{(s-a)^2}$
10.	te^{-at}	$\frac{1}{(s+a)^2}$
11.	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
12.	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
13.	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$

1.2.1 Basic Laplace Transform Theorems

Basic theorems of Laplace transform are given below:

(a) Laplace Transform of Linear Combination:

$$L[af_1(t) + bf_2(t)] = aF_1(s) + bF_2(s)$$

where $f_1(t)$, $f_2(t)$ are functions of time and a, b are constants.

(b) If the Laplace Transform of $f(t)$ is $F(s)$, then:

(i) $$L\left[\frac{df(t)}{dt}\right] = [sF(s) - f(0^+)]$$

(ii) $$L\left[\frac{d^2f(t)}{dt^2}\right] = [s^2F(s) - sf(0^+) - f'(0^+)]$$

(iii) $$L\left[\frac{d^3f(t)}{dt^3}\right] = [s^3F(s) - s^2f(0^+) - sf'(0^+) - f''(0^+)]$$

where $f(0^+)$, $f'(0^+)$, $f''(0^+)$ ... are the values of $f(t)$, $\frac{df(t)}{dt}$, $\frac{d^2f(t)}{dt^2}$ ... at $t = (0^+)$.

(c) If the Laplace Transform of $f(t)$ is $F(s)$, then:

(i) $$L\left[\int f(t)\right] = \left[\frac{F(s)}{s} + \frac{f^{-1}(0^+)}{s}\right]$$

(ii) $$L\left[\iint f(t)\right] = \left[\frac{F(s)}{s^2} + \frac{f^{-1}(0^+)}{s^2} + \frac{f^{-2}(0^+)}{s}\right]$$

(iii) $$L\left[\iiint f(t)\right] = \left[\frac{F(s)}{s^3} + \frac{f^{-1}(0^+)}{s^3} + \frac{f^{-2}(0^+)}{s^2} + \frac{f^{-3}(0^+)}{s}\right]$$

where $f^{-1}(0^+)$, $f^{-2}(0^+)$, $f^{-3}(0^+)$ ... are the values of $\int f(t)$, $\iint f(t)$, $\iiint f(t)$ ... at $t = (0^+)$.

(d) If the Laplace Transform of $f(t)$ is $F(s)$, then:

$$L[e^{-at} f(t)] = F(s + a)$$

(e) If the Laplace Transform of $f(t)$ is $F(s)$, then:

$$L[t\, f(t)] = -\frac{d}{ds}F(s)$$

(f) Initial Value Theorem:

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sL[f(t)]$$

or $$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

(g) Final Value Theorem:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sL[f(t)]$$

or $$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

The final value theorem gives the final value ($t \to \infty$) of a time function using its Laplace transform and as such very useful in the analysis of control systems. However, if the denominator of $sF(s)$ has any root having real part as zero or positive, then the final value theorem is not valid.

Example - 1.2 Laplace transform of $\sin(\omega t + \alpha)$ is

(a) $\dfrac{s\cos\alpha + \omega \sin\alpha}{s^2 + \omega^2}$ (b) $\dfrac{\omega}{s^2 + \omega^2}\cos\alpha$

(c) $\dfrac{s}{s^2 + \omega^2}\sin\alpha$ (d) $\dfrac{s\sin\alpha + \omega\cos\alpha}{s^2 + \omega^2}$

Solution: (d)

$$\sin(\omega t + \alpha) = \sin\omega t\cos\alpha + \cos\omega t\sin\alpha$$

$$\mathcal{L}\{\sin(\omega t + \alpha)\} = \frac{\omega\cos\alpha}{s^2 + \omega^2} + \frac{s\sin\alpha}{s^2 + \omega^2}$$

$$= \frac{s\sin\alpha + \omega\cos\alpha}{s^2 + \omega^2}$$

Student's Assignment

Q.1 Given the Laplace transform of $f(t) = F(s)$, the Laplace transform of $[f(t)\, e^{-at}]$ is equal to

(a) $F(s + a)$ (b) $\dfrac{F(s)}{(s + a)}$

(c) $e^{as} F(s)$ (d) $e^{-as} F(s)$

Q.2 A forcing function $(t^2 - 2t)\, u(t - 1)$ is applied to a linear system. The $\mathcal{L}$-transform of the forcing function is

(a) $\dfrac{2 - s}{s^3} e^{-2s}$ (b) $\left(\dfrac{1 - s^2}{s}\right) e^{-s}$

(c) $\dfrac{1}{s} e^{-s} - \dfrac{1}{s^2} e^{-2s}$ (d) $\left(\dfrac{2 - s^2}{s^3}\right) e^{-s}$

Q.3 The Laplace transform of $f(t) = t^n e^{-\alpha t} u(t)$ is

(a) $\dfrac{(n+1)!}{(s+\alpha)^{n+1}}$ (b) $\dfrac{n!}{(s+\alpha)^n}$

(c) $\dfrac{(n-1)!}{(s+\alpha)^{n+1}}$ (d) $\dfrac{n!}{(s+\alpha)^{n+1}}$

ANSWER KEY — STUDENT'S ASSIGNMENT

1. (a) **2.** (d) **3.** (d)

HINTS & SOLUTIONS — STUDENT'S ASSIGNMENT

2. (d)

$$f(t) = (t^2 - 2t)\, u(t - 1)$$
$$= [(t - 1)^2 - 1]\, u(t - 1)$$
$$= (t - 1)^2 u(t - 1) - u(t - 1)$$

$$\therefore \quad F(s) = \frac{2e^{-s}}{s^3} - \frac{e^{-s}}{s} = \left(\frac{2 - s^2}{s^3}\right) e^{-s}$$

3. (d)

$$L(t^n u(t)) = \frac{n!}{s^{n+1}}$$

$$L(e^{-\alpha t} t^n u(t)) = \frac{n!}{(s + \alpha)^{n+1}}$$

■■■■

CHAPTER 2

Modelling of Control Systems

2.1 Mathematical Modelling

A control system has to be modelled mathematically as per the physical laws governing its elements. A control system is an interconnection of physical elements arranged in a planned manner.

2.2 Mechanical Systems

All mechanical systems are divided into two parts:

2.2.1 Mechanical Translational System

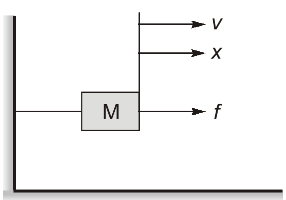
Input = Force (F),
Output = Linear displacement (x) or Linear velocity (v)
The three ideal elements are:

(a) Mass Element:

$$F = M\frac{d^2x}{dt^2}$$

or

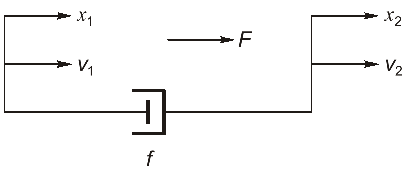
$$F = M\frac{dv}{dt}$$



(b) Damper Element:

$$F = f\frac{d}{dt}(x_1 - x_2) = f\frac{dx}{dt}$$

where, $x_1 - x_2 = x$
or $F = f(v_1 - v_2) = fv$
where, $v = v_1 - v_2$



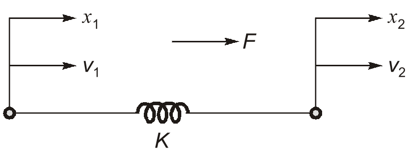
(c) Spring Element:

$$F = K(x_1 - x_2) = Kx$$

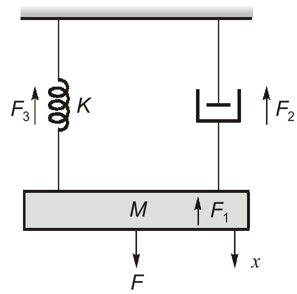
where, $x_1 - x_2 = x$

or $F = K\int(v_1 - v_2) = K\int v$

where, $v = v_1 - v_2$



Consider,



$$F = F_1 + F_2 + F_3$$

$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx \quad \text{... (i)}$$

2.2.2 Mechanical Rotational System

Input = Torque (τ),

Output = Angular displacement (θ) or Angular velocity (ω)

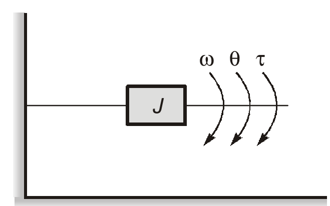
The three ideal elements are

(a) Inertial Element:

$$\tau = J\, d^2\theta/dt^2$$

or

$$\tau = J\, d\omega/dt$$



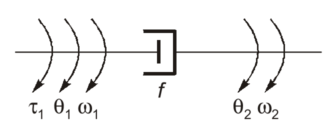
(b) Torsional Damper Element:

$$\tau = f\frac{d}{dt}(\theta_1 - \theta_2) = f\frac{d\theta}{dt},$$

where, $\theta = \theta_1 - \theta_2$

or $\tau = f(\omega_1 - \omega_2) = f\omega$

where, $\omega = \omega_1 - \omega_2$

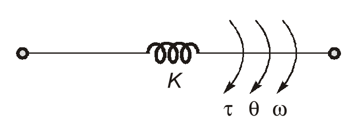


(c) Torsional Spring Element:

$$\tau = K\theta$$

or

$$\tau = K\int \omega dt$$



Consider,

$$\tau = \tau_1 + \tau_2 + \tau_3$$

$$\tau = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} + K\theta \quad \text{... (ii)}$$